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WELTY ET AL

SOLUTIONS TO 5<sup>th</sup> EDITION PROBLEMS

CHAPTERS 1-25

# CHAPTER 1

1.1  $n = 4 \times 10^{20}$  MOLECULES/ $\text{in}^3$

$$\bar{V} = \sqrt{k_B T} = 1.32 \times 10^4 \text{ m/s}$$

$$A = \pi/4 (10^{-3} \text{ in})^2$$

$$NA = \frac{1}{4} n \bar{V} A = 1.04 \times 10^{18} \text{ m/s}$$

## 1.2 FLOW PROPERTIES:

VELOCITY

PRESSURE GRADIENT

STRESS

## FLUID PROPERTIES:

PRESSURE

TEMPERATURE

DENSITY

SPEED OF SOUND

SPECIFIC HEAT

1.3 MASS OF SOLID =  $\rho_s V_s$

" " FLUID =  $\rho_f V_f$

$$x = \frac{\rho_s V_s}{\rho_s V_s + \rho_f V_f}$$

$$\Rightarrow \frac{V_f}{V_s} = \frac{1-x}{x} \frac{\rho_s}{\rho_f}$$

$$\rho_{\text{mix}} = \frac{\rho_s V_s + \rho_f V_f}{V_s + V_f} = \frac{\rho_s + \rho_f (V_f/V_s)}{1 + V_f/V_s}$$

$$= \frac{\rho_s \rho_f}{x \rho_f + (1-x) \rho_s}$$

1.4 GIVEN  $\frac{P+B}{P+B} = \left(\frac{\rho}{\rho_1}\right)^7$

FOR  $P_1 = 1 \text{ ATM}$   $\frac{\rho}{\rho_1} = 1.01$

$$P = 3001 (1.01)^7 - 3000$$

$$= 217 \text{ ATM}$$

## 1.5 AT CONSTANT TEMPERATURE

$$P/\rho T = \text{CONST.} \Rightarrow P/\rho = \text{CONST.}$$

FOR 10% INCREASE IN  $\rho$

$P$  MUST ALSO INCREASE BY 10%

## 1.6 SINCE DENSITY VARIES AS

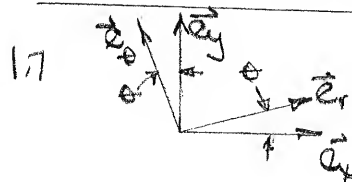
$$\rho = k P$$

$$\rho_{250,000 \text{ FT}} = \rho_{\text{SL}} \frac{P_{250,000 \text{ FT}}}{P_{\text{SL}}}$$

$$\rho = n M \quad (M = \text{MOLECULAR WT})$$

$$\therefore n_{250,000} = n_{\text{SL}} \left[ \frac{1.5 \times 10^{-7}}{2.378 \times 10^{-3}} \right]$$

$$= 4 \times 10^{20} [\text{in}^3] = 2.5 \times 10^{16}$$



$$\vec{e}_r = |\vec{e}_r|_x \vec{e}_x + |\vec{e}_r|_y \vec{e}_y$$

$$= \cos \theta \vec{e}_x + \sin \theta \vec{e}_y$$

$$\vec{e}_\theta = |\vec{e}_\theta|_x \vec{e}_x + |\vec{e}_\theta|_y \vec{e}_y$$

$$= -\sin \theta \vec{e}_x + \cos \theta \vec{e}_y$$

Q.E.D.

$$1.8 \quad \frac{d\vec{e}_r}{d\theta} = -\sin\theta \vec{e}_x + \cos\theta \vec{e}_y \\ = \vec{e}_\theta$$

$$\frac{d\vec{e}_\theta}{d\theta} = -\cos\theta \vec{e}_x - \sin\theta \vec{e}_y \\ = -\vec{e}_r$$

Q.E.D.

1.9 TRANSFORMATION FROM  $(x, y)$  TO  $(r, \theta)$

$$\frac{\partial}{\partial x} = \frac{\partial r}{\partial x} \frac{\partial}{\partial r} + \frac{\partial \theta}{\partial x} \frac{\partial}{\partial \theta}$$

$$\frac{\partial}{\partial y} = \frac{\partial r}{\partial y} \frac{\partial}{\partial r} + \frac{\partial \theta}{\partial y} \frac{\partial}{\partial \theta}$$

$$r^2 = x^2 + y^2 \quad \theta = \tan^{-1} y/x$$

$$\text{so, } \frac{\partial r}{\partial x} = \frac{x}{(x^2 + y^2)^{1/2}} = \frac{r \cos\theta}{r} = \cos\theta$$

$$\frac{\partial \theta}{\partial x} = -\frac{y}{x^2 + y^2} = -\frac{r \sin\theta}{r^2} = -\frac{\sin\theta}{r}$$

$$\frac{\partial r}{\partial y} = \sin\theta \quad \frac{\partial \theta}{\partial y} = \frac{\cos\theta}{r}$$

$$\Rightarrow \frac{\partial}{\partial x} = \cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r} \frac{\partial}{\partial \theta}$$

$$\frac{\partial}{\partial y} = \sin\theta \frac{\partial}{\partial r} + \frac{\cos\theta}{r} \frac{\partial}{\partial \theta}$$

$$1.10 \quad \nabla = \frac{\partial}{\partial x} \vec{e}_x + \frac{\partial}{\partial y} \vec{e}_y + \frac{\partial}{\partial z} \vec{e}_z \\ = \left( \cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r} \frac{\partial}{\partial \theta} \right) \vec{e}_x \\ + \left( \sin\theta \frac{\partial}{\partial r} + \frac{\cos\theta}{r} \frac{\partial}{\partial \theta} \right) \vec{e}_y \\ + \frac{\partial}{\partial z} \vec{e}_z$$

1.10 CONTINUED --

$$= (\vec{e}_x \cos\theta + \vec{e}_y \sin\theta) \frac{\partial}{\partial r} \\ + \frac{1}{r} (-\vec{e}_x \sin\theta + \vec{e}_y \cos\theta) \frac{\partial}{\partial \theta} \\ + \vec{e}_z \frac{\partial}{\partial z}$$

THUS:

$$\nabla = \vec{e}_r \frac{\partial}{\partial r} + \frac{1}{r} \vec{e}_\theta \frac{\partial}{\partial \theta} + \vec{e}_z \frac{\partial}{\partial z}$$

$$1.11 \quad \nabla P = \frac{\partial P}{\partial x} \vec{e}_x + \frac{\partial P}{\partial y} \vec{e}_y$$

$$\nabla P(a, b) = 8\pi a^2 b^2 \left\{ \left[ \frac{1}{a} \cos 1 \sin 1 + 2 \right] \vec{e}_x \right. \\ \left. + \frac{1}{b} (\sin 1 \cos 1) \vec{e}_y \right\} \\ = 8\pi a^2 b^2 \left\{ \left[ \frac{1}{a} \frac{\sin 2}{2} + 2 \right] \vec{e}_x \right. \\ \left. + \frac{1}{b} \left( \frac{\sin 2}{2} \right) \vec{e}_y \right\}$$

$$1.12 \quad \nabla T(x, y) = T_0 \vec{e}^{-1/4} \left[ \frac{1}{a} \left( \cosh \frac{x}{a} \cosh \frac{y}{b} \right) \vec{e}_x \right. \\ \left. + \frac{1}{b} \left( \sinh \frac{x}{a} \sinh \frac{y}{b} \right) \vec{e}_y \right]$$

$$\nabla T(a, b) = T_0 \vec{e}^{-1/4} \left[ \frac{1}{a} (\cosh 1) \vec{e}_x \right. \\ \left. + \frac{1}{b} (\sinh 1) \vec{e}_y \right] \\ = T_0 \vec{e}^{-1/4} \left[ \frac{\cosh 1 (e + e^{-1})}{2a} \vec{e}_x \right. \\ \left. + \frac{\sinh 1 (e - e^{-1})}{2b} \vec{e}_y \right] \\ = T_0 \frac{\vec{e}^{-5/4}}{2} \left[ \frac{\cosh 1}{a} (1 + \vec{e}^2) \vec{e}_x \right. \\ \left. + \frac{\sinh 1}{b} (1 - \vec{e}^2) \vec{e}_y \right]$$

1.13 In Prob 1.12  $T(x,y)$  is dimensionally homogeneous (D.H.)

$P(x,y)$  in Prob 1.11 will be D.H. if

$$P_p \sim \frac{P}{U_p^2} \quad \text{LBFS}^2 / \text{ft}^4$$

OR USING THE CONVERSION FACTOR  $g_c$

1.14  $\phi = 3x^2y + 4y^2$

a)  $\nabla\phi = (6xy)\vec{e}_x + (3x^2 + 8y)\vec{e}_y$

$\nabla\phi(3,5) = 90\vec{e}_x + 67\vec{e}_y$

b)  $\nabla\phi \cdot \vec{e}_s = [6xy\vec{e}_x + (3x^2 + 8y)\vec{e}_y] \cdot [\cos\theta\vec{e}_x + \sin\theta\vec{e}_y]$

AT POINT (3,5)

$$\begin{aligned} \nabla\phi \cdot \vec{e}_s &= (90\vec{e}_x + 67\vec{e}_y) \cdot [\cos(-60)\vec{e}_x + \sin(-60)\vec{e}_y] \\ &= 45 - 58.02 = -13.02 \end{aligned}$$

1.15 FOR AN IDEAL GAS

$$P = \frac{\rho RT}{M}$$

FROM PROB. 1.3  $\rho = \frac{\rho_m(1-x)}{1 - \frac{\rho_m}{\rho_s}x}$

$$\therefore P = \frac{\rho_m(1-x)}{1 - \frac{\rho_m}{\rho_s}x} \cdot \frac{RT}{M}$$

1.16  $\phi = Ar \sin\theta \left(1 - \frac{a^2}{r^2}\right)$

a)  $\nabla\phi = \frac{\partial\phi}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\vec{e}_\theta$   
 $= A \sin\theta \left(1 - \frac{a^2}{r^2}\right)\vec{e}_r$

b)  $|\nabla\phi| = A \left[ \sin^2\theta \left(1 + \frac{a^2}{r^2}\right)^2 + \cos^2\theta \left(1 - \frac{a^2}{r^2}\right)^2 \right]^{1/2}$

$|\nabla\phi|_{\max}$  IS GIVEN BY  $\frac{\partial}{\partial r}|\nabla\phi| = 0$

OR  $\frac{\partial}{\partial r}|\nabla\phi| dr + \frac{\partial}{\partial\theta}|\nabla\phi| d\theta = 0$

REQUIREMENT  $\frac{\partial}{\partial r}|\nabla\phi| = \frac{\partial}{\partial\theta}|\nabla\phi| = 0$

FOR  $\frac{\partial}{\partial r}|\nabla\phi| = 0$ :

$$-\sin^2\theta \left(1 + \frac{a^2}{r^2}\right) + \cos^2\theta \left(1 - \frac{a^2}{r^2}\right) = 0 \quad (1)$$

FOR  $\frac{\partial}{\partial\theta}|\nabla\phi| = 0$

$$\sin\theta \cos\theta \left[ \left(1 + \frac{a^2}{r^2}\right)^2 - \left(1 - \frac{a^2}{r^2}\right)^2 \right] = 0 \quad (2)$$

FROM EQ 2:  $\sin\theta \cos\theta \cdot 4a^2/r^2 = 0$

IF  $a \neq 0, r \neq 0$  THEN  $\sin\theta \cos\theta = 0$

FOR WHICH  $\theta = 0, \pi/2$  (3)

SUBST INTO EQ 1  $\theta = 0, 1 - \frac{a^2}{r^2} = 0$

GIVING  $a = r$

FOR  $\theta = \pi/2$   $1 + \frac{a^2}{r^2} = 0$  ~ IMPOSSIBLE

THUS CONDITIONS FOR  $|\nabla\phi|_{\max}$  ARE

$$\theta = 0 \quad r = a$$

$$1.17 \quad P = P_0 + \frac{1}{2} \rho U_w^2 \left[ \frac{2xy^2}{L^3} + 3\left(\frac{x}{L}\right)^2 + \frac{U_w^2}{L} \right]$$

$$\frac{\partial P}{\partial x} \vec{e}_x = \frac{1}{2} \rho U_w^2 \left[ \frac{2y^2}{L^3} + \frac{6x}{L^2} \right] \vec{e}_x$$

$$\frac{\partial P}{\partial y} \vec{e}_y = \frac{1}{2} \rho U_w^2 \left[ \frac{2x}{L^3} \right] \vec{e}_y$$

$$\frac{\partial P}{\partial z} \vec{e}_z = \frac{1}{2} \rho U_w^2 \left[ \frac{2xy}{L^3} \right] \vec{e}_z$$

$$\nabla P = \frac{1}{2} \rho U_w^2 \left[ \left( \frac{2xz}{L^3} + \frac{6x}{L^2} \right) \vec{e}_x + \frac{2xz}{L^3} \vec{e}_y + \frac{2xy}{L^3} \vec{e}_z \right]$$

1.18 VERTICAL CYLINDER  $d = 10 \text{ m}$   
 $h = 6 \text{ m}$

$$V = \frac{\pi}{4} (10 \text{ m})^2 (6 \text{ m}) = 471.2 \text{ m}^3$$

@  $20^\circ \text{C}$   $\rho_w = 998.2 \text{ kg/m}^3$

$$m = \rho_w V = (998.2)(471.2) = 470350 \text{ kg}$$

@  $80^\circ \text{C}$   $\rho_w = 971.8 \text{ kg/m}^3$

$$m = (971.8)(471.2) = 457910 \text{ kg}$$

$$\Delta m = 12440 \text{ kg}$$

1.19 Liquid -  $V = 1200 \text{ cm}^3$  @  $1.25 \text{ MPa}$

$$V = 1188 \text{ cm}^3$$
 @  $2.5 \text{ MPa}$

$$\beta = -V \left( \frac{\partial P}{\partial V} \right)_T \approx -V \frac{\Delta P}{\Delta V}$$

$$V = 1194 \text{ cm}^3 = 1.194 \times 10^{-3} \text{ m}^3$$

$$\Delta V = -12 \text{ cm}^3 = -1.2 \times 10^{-7} \text{ m}^3$$

$$\beta = -1.194 \times 10^{-3} \left[ \frac{1.25 \text{ MPa}}{-1.2 \times 10^{-7}} \right]$$

$$= +12440 \text{ MPa} = +12.44 \text{ MPa}$$

$$1.20 \quad \beta = -V \left( \frac{\partial P}{\partial V} \right)_T \approx -V \frac{\Delta P}{\Delta V}$$

$$V = 0.25 \text{ m}^3$$

$$\Delta V = -0.005 \text{ m}^3$$

$$\Delta P = 10 \text{ MPa}$$

$$\beta = -0.25 \left[ \frac{10}{-0.005} \right] = 500 \text{ MPa}$$

1.21 for  $\text{H}_2\text{O}$  -  $\beta = 2.205 \text{ GPa}$

$$\frac{\Delta V}{V} = -0.0075$$

$$\beta \approx -V \frac{\Delta P}{\Delta V} \quad \text{or} \quad \Delta P = \beta \frac{\Delta V}{V}$$

$$\Delta P = (2.205 \text{ GPa})(0.0075)$$

$$= 0.0165 \text{ GPa} = 16.5 \text{ MPa}$$

1.22  $\text{H}_2\text{O}$ :  $P_1 = 100 \text{ kPa}$   $P_2 = 120 \text{ MPa}$   
 $\beta = 2.205 \text{ GPa}$

$$\beta \approx -V \frac{\Delta P}{\Delta V} \quad \text{or} \quad \frac{\Delta V}{V} = \frac{\Delta P}{\beta}$$

$$\frac{\Delta V}{V} = \frac{\Delta P}{\beta} = \frac{(120000 - 100) \text{ kPa}}{2.205 \times 10^6 \text{ kPa}}$$

$$= 0.999 \times 10^{-3}$$

$$= 0.0999 \text{ percent}$$

1.23  $H_2O @ 68^\circ C$  (341 K)

$$\sigma = 0.123 \left[ 1 - 0.00139(341) \right]$$

$$= 0.0647 \text{ N/m}$$

IN A CLEAN TUBE -  $\theta = 0^\circ$

$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

$$= \frac{2(0.0647)}{979(9.81)(0.2875 \times 10^{-2}/2)}$$

$$= 9.37 \times 10^{-3} \text{ m} = 9.37 \text{ mm}$$

1.24 PARALLEL GLASS PLATES

- GAP = 1.625 mm

$$\sigma = 0.0735 \text{ N/m}$$

FOR A UNIT DEPTH:

$$\text{SURFACE TENSION FORCE} = 2(1)\sigma \cos\theta$$

$$\text{WEIGHT OF } H_2O = \rho g h (1)(1.625 \times 10^{-3})$$

FOR CLEAN GLASS  $\cos\theta = 1$

EQUATING FORCES:

$$2(1)(\sigma) = \rho g h (1)(1.625 \times 10^{-3})$$

$$h = \frac{2(0.0735)}{(1000)(9.81)(1.625 \times 10^{-3})}$$

$$= 0.00922 \text{ m} = 9.22 \text{ mm}$$

1.25 GLASS TUBE -  $d_i = 0.25 \text{ mm}$

$$d_o = 0.35 \text{ mm}$$

$$\theta = 130^\circ$$

SURFACE TENSION FORCE -

$$\text{INSIDE} - 2\pi r_i \sigma \cos\theta$$

$$\text{OUTSIDE} - 2\pi r_o \sigma \cos\theta$$

TOTAL UPWARD FORCE -

$$F = 2\pi \sigma \cos\theta (r_i + r_o)$$

$$= 2\pi (0.144) (\cos 130^\circ) \left( \frac{0.25 + 0.35}{2} \times 10^{-3} \right)$$

$$= 5.33 \times 10^{-4} \text{ N}$$

1.26  $H_2O$ -AIR-GLASS INTERFACE @  $40^\circ C$

TUBE RADIUS = 1 mm

$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

$$\sigma = 0.123 \left[ 1 - 0.00139(313) \right] = 0.0695 \text{ N/m}$$

$$h = \frac{2(0.0695)}{(993)(9.81)(1 \times 10^{-3})}$$

$$= 0.0143 \text{ m} \quad (1.43 \text{ cm})$$

1.27 SOAP BUBBLE -  $T = 20^\circ C$   $d = 4 \text{ mm}$

$$\sigma = 0.025 \text{ N/m} \quad (\text{TABLE 1.2})$$

FORCE BALANCE FOR BUBBLE:

$$\pi r^2 \Delta p = 2\pi r \sigma$$

$$\Delta p = \frac{2\sigma}{r} = \frac{2(0.025)}{2 \times 10^{-3}}$$

$$= 25 \text{ N/m}^2 \sim 25 \text{ Pa}$$

1.28 GLASS TUBE IN Hg (S.G. = 13.6)

For Hg / glass -  $\sigma = 0.44 \text{ N/m}$

$$\theta = 130^\circ$$

$$h = \frac{2\sigma}{\rho g r} \quad r = 3 \text{ mm}$$

$$= \frac{2(0.44)}{13.6(1000)(1.5 \times 10^{-3})}$$

$$= -0.0277 \text{ m}$$

$$= 2.77 \text{ cm DEPRESSION}$$

1.29 @  $60^\circ\text{C}$   $\sigma_{\text{H}_2\text{O}} = 0.0662 \text{ N/m}$

$$\sigma_{\text{Hg}} = 0.44$$

TUBE DIAMETER = 0.55 mm

$$h = \frac{2\sigma \cos \theta}{\rho g r}$$

For  $\text{H}_2\text{O}$ :

$$h = \frac{2(0.0662)(\cos 0^\circ)}{983(9.81)(0.55 \times 10^{-3}/2)}$$

$$= 0.0499 \text{ m (4.99 cm Rise)}$$

For Hg:

$$h = \frac{2(0.44)(\cos 130^\circ)}{13.6(983)(9.81)(\frac{0.55 \times 10^{-3}}{2})}$$

$$= -0.0157 \text{ m}$$

$$(1.57 \text{ cm DEPRESSION})$$

1.30  $\text{H}_2\text{O} / \text{GLASS}$  INTERFACE

$$T = 30^\circ\text{C}$$

$$\sigma = 0.123[1 - 0.00139(303)]$$

$$= 0.0712 \text{ N/m}$$

$$\rho = 996 \text{ kg/m}^3$$

$$h \leq 1 \text{ mm}$$

$$h = \frac{2\sigma \cos \theta}{\rho g r}$$

$$r = 2\sigma / \rho g h$$

$$= \frac{2(0.0712)}{996(9.81)(1 \times 10^{-3})}$$

$$= 0.0146 \text{ m (1.451 cm)}$$

$$d = 2r = 2.915 \text{ cm}$$

## CHAPTER 2

2.1 ASSUME IDEAL GAS BEHAVIOR

$$\frac{dp}{dy} = -\rho g = -\frac{\rho g}{RT}$$

$$\text{for } T = a + by$$

$$\Rightarrow T = 530 - 24 y/h$$

$$\frac{dp}{p} = -\frac{g}{R} \frac{dy}{530 - 24(y/h)}$$

$$\int_{p_0}^p \frac{dp}{p} = \frac{g}{24R} \int_0^h \frac{-24}{530 - 24(y/h)} d(y/h)$$

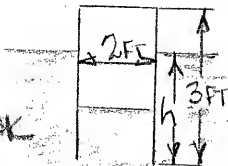
$$\ln \frac{p}{p_0} = \frac{g}{24R} \ln \frac{506}{530}$$

$$\text{WITH } p = 10.6 \text{ PSIA}, p_0 = 30.1 \text{ inHg}$$

$$h = 9192 \text{ FT}$$

2.2

$$\sum F_y = 0 \text{ ON TANK}$$



$$\frac{\rho \pi d^2}{4} - \frac{\rho_{\text{atm}} \pi d^2}{4} - 250 = 0 \quad (1)$$

AT H<sub>2</sub>O LEVEL IN TANK:

$$p = p_{\text{atm}} + \rho_w g (h - y) \quad (2)$$

$$\text{FROM (1) \& (2) } h - y = 1.275 \text{ FT} \quad (3)$$

FOR ISOTHERMAL COMPRESSION OF AIR

$$p_{\text{atm}} V_{\text{TANK}} = p (V_{\text{AIR}})$$

$$p = \frac{3}{3-y} p_{\text{atm}} \quad (4)$$

COMBINING (1) \& (4)

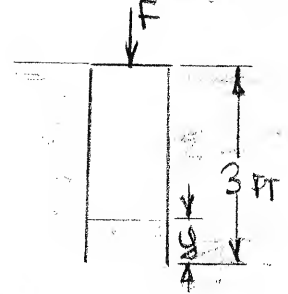
$$y = 0.12 \text{ FT}$$

$$h = 1.395 \text{ FT.}$$

2.2 CONT.

FOR TOP OF TANK FLUSA WITH H<sub>2</sub>O LEVEL

$$\sum F_y = 0$$



$$p = p_{\text{atm}} + \frac{250 + F}{\pi d^2/4}$$

AT H<sub>2</sub>O LEVEL IN TANK:

$$p = p_{\text{atm}} + \rho_w g (3 - y)$$

COMBINING EQUATIONS:

$$F = 196 (3 - y) - 250$$

FOR ISOTHERMAL COMPRESSION OF AIR:  
(AS IN PART (a))

$$3 - y = 2.8 \text{ FT}$$

$$\Rightarrow F = 196 (2.8) - 250 = 293.6 \text{ Lbf}$$

2.3 WHEN NET FORCE ON TANK = 0

$$W_T = \text{BUOYANT FORCE} = 250 \text{ Lbf}$$

$$V_{\text{W DISPLACED}} = 250 / \rho_w g = 4.01 \text{ FT}^3$$

ASSUMING ISOTHERMAL COMPRESSION

$$p_{\text{atm}} (3 \text{ FT}) = p (4.01 \text{ FT}^3) \\ = (p_{\text{atm}} + \rho_w g y) (4.01)$$

$$y = 45.88 \text{ FT}$$

$$\text{TOP IS AT LEVEL: } y = \frac{4.01}{\pi d^2/4}$$

OR AT 44.6 FT BELOW SURFACE

2.4.

$$\frac{dP}{dy} = \rho g = \rho_0 e^{\Delta P/\beta}$$

$$\int_0^{\Delta P} \frac{-\Delta P/\beta}{e^{\Delta P/\beta}} = \int_0^y -\frac{\rho_0 g \Delta y}{\beta}$$

$$e^{-\Delta P/\beta} = 1 - \frac{\rho_0 g y}{\beta}$$

$$\Delta P = -\beta \ln\left(1 - \frac{\rho_0 g y}{\beta}\right)$$

$$= 300,000 \ln(1 - 0.0462)$$

$$= 14190 \text{ psi}$$

DENSITY RATIO:

$$\frac{\rho}{\rho_0} = e^{-\Delta P/\beta} = 1.0484$$

$$\text{so } \rho = 1.0484 \rho_0$$

2.5. BUOYANT FORCE:

$$F_B = \rho V = \frac{PV}{RT}$$

FOR CONSTANT VOLUME:

F VARIES INVERSELY WITH T

2.6 SEA H<sub>2</sub>O:  $\rho_g = 1.025$ AT DEPTH  $y = 185 \text{ m}$ 

$$\rho_g = 1.025 \rho_w g y$$

$$= 1.025 (1000)(9.81)(185)$$

$$= 1.86 \times 10^6 \text{ Pa}$$

$$= 1.86 \text{ MPa}$$

2.7

r MEASURED FROM EARTH'S SURFACE

R = RADIUS OF EARTH

$$\frac{dP}{dr} = \rho g = \rho_0 \frac{r}{R}$$

$$P - P_{\text{atm}} = \frac{\rho_0 r^2}{2R}$$

AT CENTER OF EARTH --  $r = R$ 

$$P_{\text{ctr}} - P_{\text{atm}} = \frac{\rho_0 R}{2}$$

SINCE  $P_{\text{ctr}} \gg P_{\text{atm}}$ 

$$P_{\text{ctr}} \approx \frac{\rho_0 R}{2} = \frac{(5.67)(1000)(9.81)(6380 \times 10^3)}{2}$$

$$= 176 \times 10^9 \text{ Pa}$$

$$= 176 \text{ MPa}$$

2.8

$$\frac{dP}{dy} = -\rho g$$

$$\int_{P_{\text{atm}}}^P dP = -\rho g \int_0^h dy$$

$$P - P_{\text{atm}} = \rho g (+h)$$

$$= (1050)(9.81)(11034)$$

$$= 113.7 \text{ MPa}$$

$$\approx 1122 \text{ ATMOSPHERES}$$

2.9 AS IN PREVIOUS PROBLEM

$$P - P_{\text{atm}} = \rho g h$$

$$\text{FOR } P - P_{\text{atm}} = 101.33 \text{ kPa}$$

$$h = 101.33 / \rho g$$

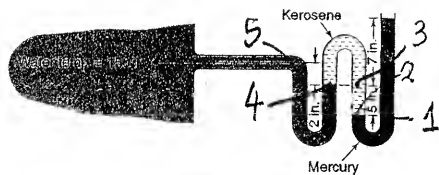
2.9 (CONT.)

for  $H_2O$ :  $h = \frac{101.33}{(1000)(9.81)} = 10.33 \text{ m}$

SEA  $H_2O$ :  $h = \frac{101.33}{(1.025)(1000)(9.81)} = 10.08 \text{ m}$

$Hg$   $h = \frac{101.33}{13.6(1000)(9.81)} = 0.80 \text{ m}$

2.10



$$P_1 = P_{atm} + \rho_{Hg} g (12") \quad P_1 = P_2$$

$$P_2 = P_3 + \rho_K g (5") \quad P_3 = P_4$$

$$P_4 = P_A + \rho_W g (2") \quad P_4 = P_5$$

$$P_{atm} + \rho_{Hg} g (12) = P_A + \rho_W g (2) + \rho_K g (5)$$

$$P_A = P_{atm} + \rho_W g [(13.6)(12) - 2 - 0.75(5)]$$

$$= P_{atm} + 5.81 \text{ psi} = 5.81 \text{ psig}$$

2.11 FORCE BALANCE ON LIQUID COLUMN:

$A = \text{AREA OF TUBE}$

$$-3A + 14.7A - \rho g h A = 0$$

$$h = \frac{11.7(144)}{62.4(12.2)}$$

$$= 26.6 \text{ IN.}$$



2.12

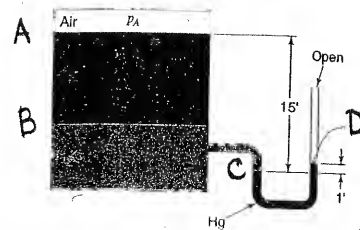
$$P_A = P_B - \rho_o g (10 \text{ ft})$$

$$P_C = P_B + \rho_W g (5 \text{ ft})$$

$$P_D = P_C - \rho_{Hg} g (1 \text{ ft})$$

$$P_A - P_D = \rho_{Hg} g (1) - \rho_W g (5) - \rho_o g (10)$$

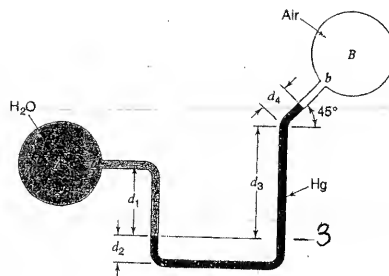
$$P_A - P_{atm} = \rho_W g (13.6 \times 1 - 5 - 0.8 \times 10 \times 1) = 37.4 \text{ Lbf/ft}^2$$



2.13

$$P_3 = P_A - d_1 g \rho_W$$

$$= P_B + (\rho_{Hg} g) \times (d_3 + d_4 \sin 45)$$



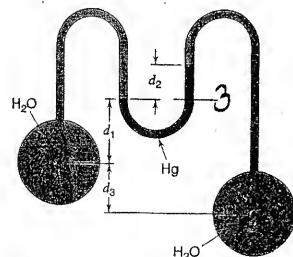
$$P_A - P_B = \frac{(62.4)(32.2)}{32.2} [(2.4 + 4 \sin 45) (13.6 - 2)]$$

$$= 245 \text{ Lbf/ft}^2 = 1.70 \text{ psi}$$

2.14

$$P_3 = P_A - \rho_W g d_1$$

$$P_3 = P_B - \rho_W g (d_1 + d_2 + d_3) + \rho_{Hg} g d_2$$



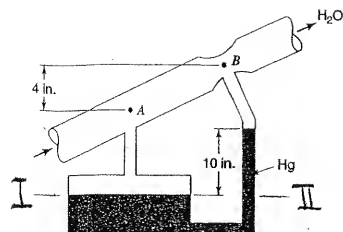
EQUATING:

$$P_A - P_B = \rho_{Hg} g d_2 - \rho_W g (d_2 + d_3)$$

$$= \rho_W g [(13.6)(1/12) - 7/12]$$

$$= 32.8 \text{ Lbf/ft}^2 = 0.227 \text{ psi}$$

2.15



$$P_I = P_A + \rho_w g (10")$$

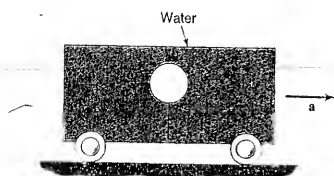
$$P_{II} = P_B + \rho_w g (4") + \rho_{Hg} g (10")$$

$$P_I = P_{II}$$

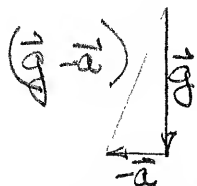
$$P_A - P_B = \rho_w g [-6 + 13.6(10)]$$

$$= 56.3 \text{ psi}$$

2.16



PRESSURE GRADIENT IS IN DIRECTION OF  $\vec{g} - \vec{a}$  & ISOBARS ARE PERPENDICULAR TO  $(\vec{g} - \vec{a})$



SPRING WILL ASSUME THE  $(\vec{g} - \vec{a})$  DIRECTION & BALLOON WILL MOVE FORWARD.

2.17

AT REST:

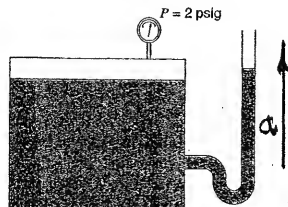
$$P = \rho g y_0$$

ACCELERATING:

$$P = \rho |(\vec{g} - \vec{a})| = \rho (g + a) y_a$$

EQUATING:  $y_a = \frac{g}{g+a}$  WHICH  $< y_0$

LEVEL GOES DOWN



$$2.18 \quad F = P_{G.C.} A - P_{ATM} A = \rho g h (\pi r^2)$$

$$h = 2 \text{ m} \quad r = 0.3 \text{ m}$$

$$F = 5546 \text{ N}$$

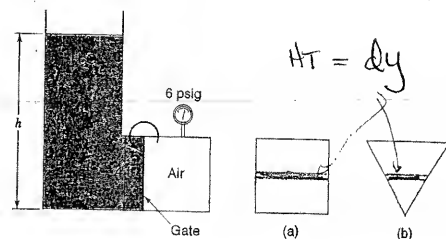
$$y_{c.p.} = \bar{y} + I_{bb}/A\bar{y}$$

FOR A CIRCLE:  $I_{bb} = \pi r^4/4$

$$y_{c.p.} = 2 \text{ m} + \frac{\pi (0.3 \text{ m})^4}{4\pi (0.3 \text{ m})^2 (2 \text{ m})}$$

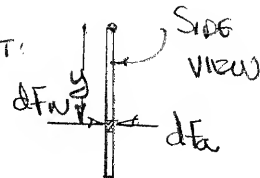
$$= 2.011 \text{ m}$$

2.19



HEIGHT OF H<sub>2</sub>O COLUMN ABOVE DIFFERENTIAL ELEMENT:

$$= h - 4 + y$$



FOR (a) - RECTANGULAR GATE -  $dA = 4 dy$

$$dF_w = [\rho_w g (h - 4 + y) + P_{ATM}] dA$$

$$dF_a = [P_{ATM} + (6 \text{ psig})(1.44)] dA$$

$$\sum M_o = \int_A y (dF_w - dF_a) = 0$$

$$\int_0^4 y [\rho g (h - 4 + y) - 864] (4 dy) = 0$$

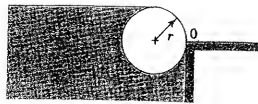
$$h = 15.18 \text{ FT}$$

FOR (b):  $dA = (4 - y) dy$

$$\int_0^4 y [\rho g (h - 4 + y) - 864] (4 - y) dy = 0$$

$$h = 15.85 \text{ FT}$$

2.20



PER UNIT DEPTH:

$$\sum F_y = 0$$

$$F_{y \text{ up}} = \rho_w g \pi r^2 / 2 \quad \{\text{BOUOYANCY}\}$$

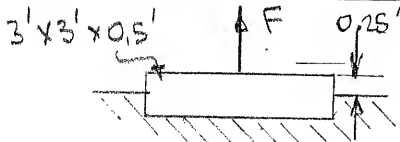
$$F_{y \text{ down}} = \rho_g \pi r^2 + \rho_w g \left( r^2 - \frac{\pi r^2}{4} \right)$$

EQUATING:

$$\frac{\rho_w g \pi r^2}{2} = \rho_g \pi r^2 + \rho_w g r^2 \left( 1 - \frac{\pi}{4} \right)$$

$$\begin{aligned} \rho &= \rho_w \left( \frac{\pi}{2} - 1 + \frac{\pi}{4} \right) / \pi \\ &= \rho_w \left( \frac{3}{4} - \frac{1}{\pi} \right) = 0.432 \rho_w \\ &= 432 \text{ kg/m}^3 \end{aligned}$$

2.21



a) TO LIFT BLOCK FROM BOTTOM

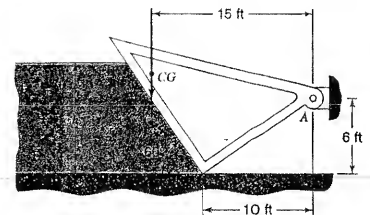
$$\begin{aligned} F &= \{ \text{WT OF CONCRETE} \} + \{ \text{WT OF H}_2\text{O} \} \\ &= \rho_c g V + [\rho_w g (22.75') + P_{\text{atm}}] A \\ &= (150) g (3 \times 3 \times 0.5) \\ &\quad + [62.4 g (22.75) + 14.7 (144)] \\ &\quad \times (3 \times 3) \\ &= 675 + 31828 = 32503 \text{ Lbf} \end{aligned}$$

2.21 (CONT)

b) TO MAINTAIN BLOCK IN FREE POSITION:

$$\begin{aligned} F &= \{ \text{WT OF CONCRETE} \} - \{ \text{BOUOYANT FORCE OF H}_2\text{O} \} \\ &= 675 - \rho_w g V \\ &= 675 - [62.4 g (3 \times 3 \times 0.5)] \\ &= 675 - 281 = 394 \text{ Lbf} \end{aligned}$$

2.22.

DISTANCE Z  
MEASURED ALONG  
GATE SURFACE  
FROM BOTTOM

$$\sum M_A = 500(15) - \int_0^{h/\sin 60} z \rho g (h - z \sin 60) dz = 0$$

$$\rho g \int_0^{h/\sin 60} (zh - z^2 \sin 60) dz = 7500$$

$$\rho g \left[ hz^2/2 - \frac{z^3}{3} \sin 60 \right]_0^{h/\sin 60} = 7500$$

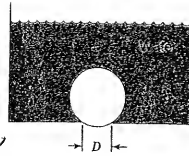
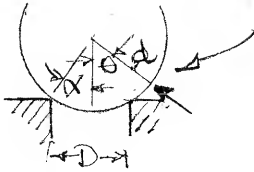
$$(62.4) g \left[ \frac{h^3}{(\sin 60)^2} \left( \frac{1}{2} - \frac{1}{3} \right) \right] = 7500$$

$$h^3 = \frac{7500 (6) (\sin 60)^2}{62.4 g} = 541$$

$$h = 8.15 \text{ FT}$$

2.23

↓ y



USING SPHERICAL COORDINATES FOR A PATCH  
AT  $y = \text{CONSTANT}$ :

$$dA = 2\pi r^2 \sin\theta d\theta$$

$$P = \rho g [h - r \cos\theta + r \cos\theta]$$

$$dF_y = dF \cos\theta$$

$$F_y = \int \rho g (h - r \cos\theta + r \cos\theta) \underbrace{(\pi r^2 \sin\theta \cos\theta d\theta)}_{dA}$$

$$= \underbrace{2\pi \rho g r^2}_C \int_0^\pi (h - r \cos\theta + r \cos\theta) \sin\theta \cos\theta d\theta$$

$$= C \left[ \int_0^\pi (h - r \cos\theta) \sin\theta \cos\theta d\theta + r \int_0^\pi \sin\theta \cos^2\theta d\theta \right]$$

$$= C \left[ (h - r \cos\theta) \sin^2\theta \Big|_0^\pi + r \left( -\frac{1}{3} \cos^3\theta \right) \Big|_0^\pi \right]$$

$$= C \left[ (h - r \cos\theta) (1 - \sin^2\theta) - \frac{r}{3} (0 - \cos^3\theta) \right]$$

Now - for  $F_y = 0$

$$\sin\theta = \frac{D}{d} \quad \cos\theta = \left[ 1 - \left( \frac{D}{d} \right)^2 \right]^{1/2}$$

$$\frac{1}{2} r = \frac{d}{2}$$

2.23 CONT.

$$0 = \left( h - \frac{d}{2} \cos\theta \right) \left( \cos^2\theta \right) + \frac{d}{6} \cos^3\theta$$

$$= h - \frac{d}{2} \cos\theta + \frac{d}{6} \cos\theta$$

GIVING  $h = \frac{d}{3} \cos\theta$

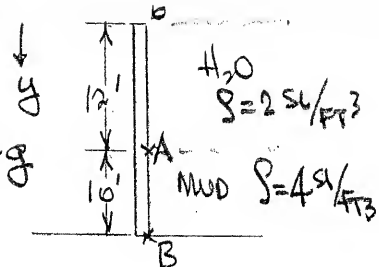
$$\frac{h}{d} = \frac{\cos\theta}{3} = \frac{1}{3} \left[ 1 - \left( \frac{D}{d} \right)^2 \right]^{1/2}$$

For  $d = 0.6 \text{ m}$

$$h = \frac{0.6}{3} \left[ 1 - \left( \frac{50}{3} \right)^2 \right]^{1/2}$$

$$= \frac{1}{5} \left[ 1 - \left( \frac{50}{3} \right)^2 \right]^{1/2}$$

2.24



$$P_A - P_{\text{atm}} = \rho_w g (12) = 24g$$

$$P_B - P_{\text{atm}} = 24g + 40g = 64g$$

BETWEEN 0 & A:  $P - P_{\text{atm}} = \rho_w g y$

" A & B:  $P = \rho_w g (12) + \rho_m g (y - 12)$

PER UNIT DEPTH:

$$F = \int (P - P_{\text{atm}}) dA$$

$$= \int_0^{12} \rho_w g y dy + \int_{12}^{22} [\rho_w g 12 + \rho_m g (y - 12)] dy$$

$$= \rho_w g (192) + \rho_m g (50)$$

$$= 18790 \text{ Lbf}$$

2.24 CONT.

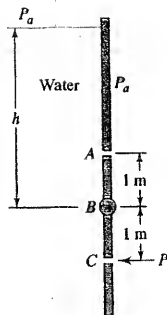
FORCE LOCATION:

$$\begin{aligned}
 Fx y &= \int_0^{22} y(P - P_{atm}) dA \\
 &= \int_0^{12} \rho_w g y^2 dy + \int_{12}^{22} \rho_w g (12y) dy \\
 &\quad + \int_{12}^{22} \rho_m g (y^2 - 12y) dy \\
 &= \rho_w g (576 + 2040) \\
 &\quad + \rho_m g (2973 - 2040) \\
 &= 288400 \text{ Ft Lbf}
 \end{aligned}$$

$$\bar{y} = \frac{288400}{18790} = 15.35 \text{ ft}$$

2.25

$$\begin{aligned}
 \text{FORCE ON GATE} &= \rho g \bar{y} A \\
 &= (1000)(9.81)(12) \frac{\pi}{4} (2)^2 \\
 &= 369,8 \text{ kN}
 \end{aligned}$$

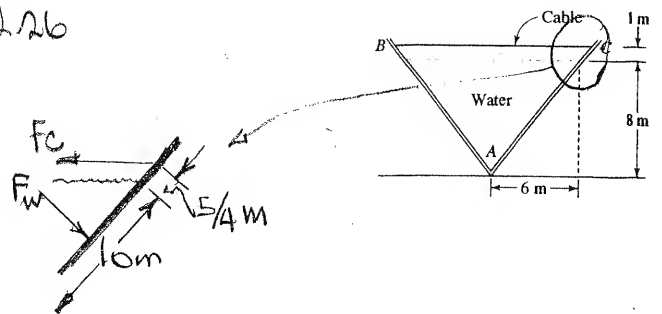


$$y_{cp} = \frac{I_{xx}}{\bar{y} A} = \frac{(\frac{\pi}{4})(1)^4}{(12)(\frac{\pi}{4})(2)^2} = 0.0208 \text{ m (BELOW AXIS B)}$$

$$\sum M_B = 0$$

$$\begin{aligned}
 P(1) &= (369,8 \times 10^3)(0.0208) \\
 &= 7.70 \text{ kN}
 \end{aligned}$$

2.26



$$\begin{aligned}
 F_w &= \rho g \bar{y} A \\
 &= (1000)(9.81)(4)(10)(1) = 392 \text{ kN}
 \end{aligned}$$

$y_{cp}$  IS  $\frac{2}{3}$  DISTANCE FROM WATER LINE TO A

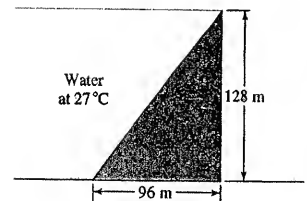
~ 6.66 m DOWN FROM H<sub>2</sub>O LINE  
3.33 m UP FROM A

$$\sum M_A = F_C(9) = 392(3.33)$$

$$F_C = 145,2 \text{ kN}$$

2.27 WIDTH = 100 m

H<sub>2</sub>O @ 27°C  $\rho = 997 \text{ kg/m}^3$



$$\begin{aligned}
 F &= \rho g \bar{y} A \\
 &= (997)(9.81)(64) \\
 &\quad \times (100)(100)
 \end{aligned}$$

$$= 10.016 \times 10^9 \text{ N} = 10.02 \times 10^3 \text{ MN}$$

FOR A FREE H<sub>2</sub>O SURFACE

$$y_{cp} = \frac{2}{3}(128 \text{ m}) = 85,3 \text{ m} \left\{ \begin{array}{l} \text{BELOW} \\ \text{H}_2\text{O} \\ \text{SURFACE} \end{array} \right\}$$

$$= 106,7 \text{ m} \left\{ \begin{array}{l} \text{MEASURED ALONG} \\ \text{DAM SURFACE} \end{array} \right\}$$

## 2.28 SPHERICAL FLOAT

UPWARD FORCES  $\sim F + F_{\text{BOYANT}}$

DOWNWARD "  $\sim W$

$$W = \rho g V = \rho g \left( \frac{4}{3} \pi R^3 \right)$$

$$F_b = \rho_w g V z = \rho_w g \left( \frac{4}{3} \pi R^3 \right) z$$

$z$  = FRACTION SUBMERGED

$$F = \rho g \left( \frac{4}{3} \pi R^3 \right) - \rho_w g z \left( \frac{4}{3} \pi R^3 \right)$$

$$z = \frac{\rho g \left( \frac{4}{3} \pi R^3 \right) - F}{\rho_w g \left( \frac{4}{3} \pi R^3 \right)}$$

## 2.29 (CONT.)

$$M = \rho g L^4 \sin \theta \left[ -\frac{1}{60} + 0.045 \right]$$

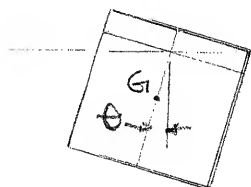
$$= \rho g L^4 \sin \theta (0.02833)$$

$$= \underline{0.00556 \rho g L^4}$$

## 2.29

CUBE -

LENGTH OF SIDE = L



$$\theta = \tan^{-1} \frac{0.1}{0.5}$$

G IS CENTER OF MASS OF SOLID

$$\sum M_G = 2 \left[ \frac{1}{2} \left( \frac{L}{2} \right) (0.1L) (L) \rho g \left( \frac{2}{3} \frac{L}{2} \sin \theta \right) - (0.9L) (L) (L) \rho g (0.05L \sin \theta) \right]$$

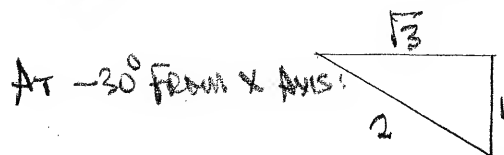
+ M

{ PART OF ORIGINAL SUBMERGED VOLUME IS NOW OUT OF H<sub>2</sub>O -  
PART THAT WAS ORIGINALLY OUT IS NOW SUBMERGED }

# CHAPTER 4

4.1  $\vec{v} = 10\vec{e}_x + 7x\vec{e}_y$

At (2,2)  $\vec{v} = 10\vec{e}_x + 14\vec{e}_y$



At  $-30^\circ$  from x axis:

UNIT VECTOR:  $\vec{e} = \frac{\sqrt{3}}{2}\vec{e}_x - \frac{1}{2}\vec{e}_y$

Along this direction the component is  $\vec{e} \cdot \vec{v}$ :

$$\vec{e} \cdot \vec{v} = \left(\frac{\sqrt{3}}{2}\vec{e}_x - \frac{1}{2}\vec{e}_y\right) \cdot (10\vec{e}_x + 14\vec{e}_y)$$

$$= 5\sqrt{3} - 7 = 1.66 \text{ m/s}$$

4.2  $\vec{v} = 10\vec{e}_x + 7x\vec{e}_y$

$$\frac{dy}{dx} = \frac{v_y}{v_x} = \frac{7x}{10}$$

$$10 \int dy = 7 \int x dx$$

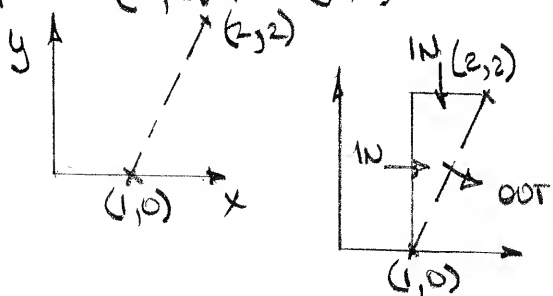
$$10y = 7\frac{x^2}{2} + C$$

At (2,1)  $C = 10 - 14 = -4$

Eqn is:  $7\frac{x^2}{2} - 10y + C = 0$

OR  $x^2 - \frac{10}{7}y - \frac{8}{7} = 0$  (a)

ACROSS THE SURFACE CONNECTING POINTS (1,0) AND (2,2):



4.2 (CONT)

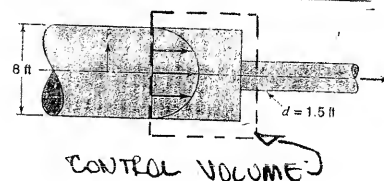
$$\dot{V} = \int_0^2 \vec{v} \cdot \vec{e}_x dy + \int_1^2 \vec{v} \cdot \vec{e}_y dx$$

$$= 10y \Big|_0^2 + 7\frac{x^2}{2} \Big|_1^2$$

$$= 20 + \frac{7}{2}(3) = 20 + 10.5$$

$$= 30.5 \text{ m}^3/\text{s} \quad (b)$$

4.3



$$\iint_{CS} \rho(\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{CV} \rho dV = 0$$

$$\iint_{CS} \rho(\vec{v} \cdot \vec{n}) dA = \rho v_{2,avg} A_2$$

$$- \int_0^R \rho g \left(1 - \frac{r^2}{16}\right) 2\pi r dr = 0$$

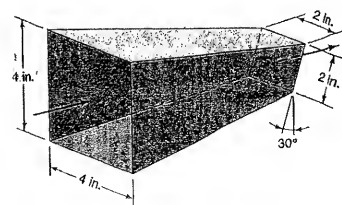
$$v_{2,avg} = 18\pi \int_0^4 \left(r - \frac{r^3}{16}\right) dr$$

$$= 18\pi \left[ \frac{r^2}{2} - \frac{r^4}{64} \right]_0^4 = 72\pi$$

$$v_{2,avg} = \frac{72\pi}{\pi(3/4)^2} = 128 \text{ ft/s}$$

4.4

$$\iint_{CS} \rho(\vec{v} \cdot \vec{n}) dA = 0$$



$$= \int_{A_o} \rho v \cos 30 dA - \int_{A_i} \rho v dA = 0$$

4.4 - CONTINUED

$$\rho_{IN} = \rho_{OUT} \quad A_{IN} = 4 A_{OUT}$$

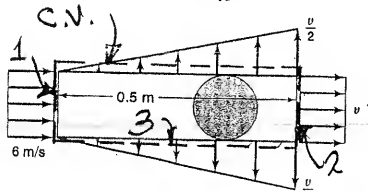
$$V_{OUT} = \frac{A_1 V_1}{A_2 \cos 30^\circ}$$

$$= \frac{4(10)}{\cos 30^\circ} = \underline{\underline{46.2 \text{ FT/s}}}$$

$$\dot{V} = A V = 10 \left( \frac{1}{3} \times \frac{1}{3} \right)$$

$$= \underline{\underline{1.11 \text{ FT}^3/\text{s}}}$$

4.5 Steady Flow:  $\iint_{C.S.} \rho (\vec{V} \cdot \vec{n}) dA = 0$



$$-\rho V_1 A_1 + \rho V_2 A_2 + \int_{A_3} \rho (\vec{V} \cdot \vec{n}) dA = 0$$

$\rho = \text{CONST.}$

$$V_1 A_1 = V_2 A_2 + \int_0^L \frac{V_2}{2} \frac{\pi D}{L} dx$$

$$= V_2 A_2 + \frac{V_2}{2} \frac{\pi D}{L} \frac{x^2}{2} \bigg|_0^L$$

$$= V_2 \frac{\pi D^2}{4} + \frac{V_2}{2} \frac{\pi D L}{2}$$

$$= \frac{V_2 \pi D}{4} (D + L)$$

$$\frac{6\pi}{4} (0.2)^2 = \frac{V_2 \pi}{4} [0.2(0.2 + 0.5)]$$

$$V_2 = \frac{6(0.04)}{(0.2)(0.7)}$$

$$= \underline{\underline{1.71 \text{ m/s}}}$$

4.6 FOR STEADY, INCOMPRESSIBLE FLOW:

$$\dot{V} = A V_{AVG} = \sum_i V_i \Delta A_i$$

FROM GIVEN DATA SET:

DIST FROM CENTER IN	$V_i$ FT/S	$\Delta A_i$ IN <sup>2</sup>	$V_i \Delta A_i$ FT <sup>3</sup> /S
0	7.5	7.844	0.4084
3.16	7.10	37.64	1.856
4.45	6.75	31.96	1.498
5.48	6.42	32.10	1.431
6.33	6.15	31.48	1.344
7.07	5.81	29.85	1.204
7.75	5.47	33.22	1.262
8.37	5.10	33.22	1.176
8.94	4.50	31.44	0.982
9.49	3.82	31.57	0.838
10.1	2.40	15.82	0.264

$$\sum \rightarrow 316.14 \quad 18.263$$

$$\sum \Delta A_i = 316.14 \text{ IN}^2 \quad \left\{ \begin{array}{l} \text{EXACT AREA} \\ = 314.16 \text{ IN}^2 \end{array} \right.$$

$$= 2.195 \text{ FT}^2$$

$$\dot{V} = \sum V_i \Delta A_i = \underline{\underline{18.26 \text{ FT}^3/\text{s}}}$$

$$V_{AVG} = \frac{\dot{V}}{A} = \frac{18.26}{2.195} = \underline{\underline{8.32 \text{ FT/s}}}$$

4.7 INFLOW:  $\dot{V} = 2 \text{ gal/m} = 19.2 \text{ LB/m}$

OUTFLOW:  $\dot{V} = 19.2 \text{ LB/m}$

~ STEADY FLOW ~

$$\text{FOR TOTAL FLOW: } \iint_{C.S.} \rho (\vec{V} \cdot \vec{n}) dA = \dot{m}_{OUT} - \dot{m}_{IN} = 0$$

TOTAL MASS IN TANK = M

MASS OF SALT IN TANK = S

FOR SALT -  $\dot{m}_{OUT} = 19.2 \text{ (S/M)} \quad \text{LB/M}$

$\dot{m}_{IN} = 2(9.92) \quad "$

4.7 - CONTINUED

For Salt: LEAKS OF MASS

$$19.2 \frac{S}{M} - 3.84 + \frac{dS}{dt} = 0$$

$$\frac{dS}{dt} = 3.84 - \frac{19.2}{M} S$$

$$= A - BS$$

$$A = 3.84$$

$$B = \frac{19.2}{M} = \frac{(19.2)(7.48)}{(100)(62.4)}$$

$$= 0.0230$$

$$\int_0^S \frac{dS}{A - BS} = \int_0^t dt$$

$$-\frac{1}{B} \ln \frac{A - BS}{A} = t$$

$$\ln \left[ 1 - \frac{BS}{A} \right] = -Bt$$

$$\text{OR } S = \frac{A}{B} \left[ 1 - e^{-Bt} \right]$$

For  $t = 100$  m

$$S = \frac{3.84}{0.0230} (1 - e^{-2.30})$$

$$\approx \underline{150 \text{ lb}_m} \quad (a)$$

For  $t = \infty$   $S = 167 \text{ lb}_m$  (b)

For  $S = 100 \text{ lb}_m$

$$t = \frac{1}{0.023} \ln \left[ 1 - \frac{0.023}{3.84} (100) \right]$$

4.7 - CONTINUED

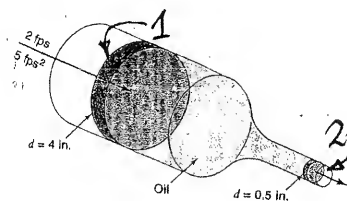
$$t = 39.8 \text{ m} \quad \text{for } S = 100 \text{ lb}_m$$

From (a)  $t = 100$  m for  $S \approx 150 \text{ lb}_m$

For  $S$  from 100 To 150

$$\Delta t = 100 - 39.8 = \underline{60.2 \text{ m}} \quad (c)$$

4.8



For PISTON & CYLINDER SHOWN:

$$\text{At 1} \quad U = U_1 = 2 \text{ ft/s} \quad a = a_1 = 5 \text{ ft/s}^2$$

$$A_1 U_1 = A_2 U_2$$

$$U_2 = \frac{A_1}{A_2} U_1 = \left( \frac{d_1}{d_2} \right)^2 U_1$$

$$= \left( \frac{4}{0.5} \right)^2 (2) = \underline{128 \text{ ft/s}}$$

$$a_2 = a_1 \left( \frac{d_1}{d_2} \right)^2 = 5 \left( \frac{4}{0.5} \right)^2 = \underline{320 \text{ ft/s}^2}$$

4.9 For STEADY FLOW:

$$\oint_{c.s.} \rho (\vec{v} \cdot \vec{n}) dA = 0$$

$$\text{OR } \rho v A = \text{CONSTANT}$$

$$\frac{d(\rho v A)}{\rho v A} = \frac{d\rho}{\rho} + \frac{dv}{v} + \frac{dA}{A} = 0$$

Q.E.D.

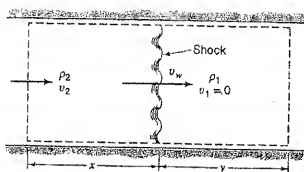
$$4.10 \quad \iint_{CS} \rho(\vec{v} \cdot \vec{n}) dA = \iint_{CS} d\dot{m}$$

$$\rho(\vec{v} \cdot \vec{n}) dA = d\dot{m}$$

$$\frac{\partial}{\partial t} \iiint_{CV} \rho dV = \frac{\partial}{\partial t} M$$

$$\therefore \frac{\partial M}{\partial t} + \iint_{CS} d\dot{m} = 0 \quad \text{Q.E.D.}$$

4.11



FOR THE C.V. SHOWN

$$\iint_{CS} \rho(\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{CV} \rho dV = 0$$

O - STY FLOW

C.V. MOVES TO RIGHT WITH  $V = V_w$   
THUS:

$$-\rho_1 A V_w + \rho_2 A (V_w - V_2) = 0$$

$$V_2 = V_w (1 - \rho_1 / \rho_2)$$

4.12

$$V_{avg} = \frac{1}{A} \int_A v dA$$

$$= \frac{V_{max}}{\pi R^2} \int_0^R 2\pi r \left[1 - \frac{r}{R}\right]^{1/4} dr$$

FOR  $z = r/R \quad dz = dr/R$

$$V_{avg} = 2V_{max} \int_0^1 z(1-z)^{1/4} dz$$

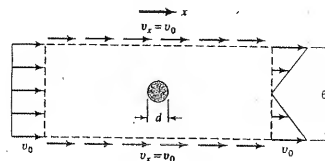
FOR  $\xi = 1-z \quad d\xi = -dz$

$$V_{avg} = -2V_{max} \int_1^0 (1-\xi)^{1/4} \xi d\xi$$

4.12 - CONTINUED

$$V_{avg} = \frac{49}{60} V_{max} = 0.817 V_{max}$$

4.13



$$\iint_{CS} \rho(\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{CV} \rho dV = 0$$

O - STY FLOW

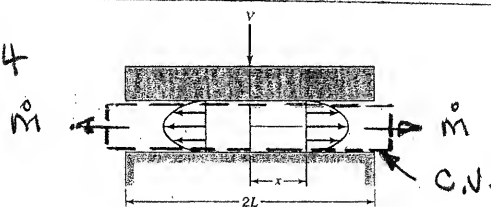
$$\iint_{CS} \rho(\vec{v} \cdot \vec{n}) dA = -\rho v_0 (bd) + \dot{m}_{horiz}$$

$$+ 2 \int_0^{3d} \rho \frac{v_0}{3d} y dy$$

$$\dot{m}_{horiz} = \rho v_0 (bd) - \rho v_0 (3d)$$

$$= \rho v_0 (3d)$$

4.14



$$\frac{\partial m}{\partial t} + \int d\dot{m} = 0$$

$$M = \rho(2L)(b)(1) \quad \frac{\partial m}{\partial t} = 2\rho L \dot{b}$$

WHERE  $\dot{b} = -v$

$$\int d\dot{m} = 2 \dot{m}_{side} = 2 \int_0^b \rho v(1) dy$$

GIVING:  $-2\rho L v + 2\rho \int_0^b v dy = 0$

OR  $LV = \int_0^b v dy$

4.14 - CONTINUED

(a) For  $V = V_{avg}$  (A CONSTANT)

$$LV = V_{avg} b$$

$$\underline{V_{avg} = \frac{L}{b} V}$$

(b)  $V = ay + by^2$

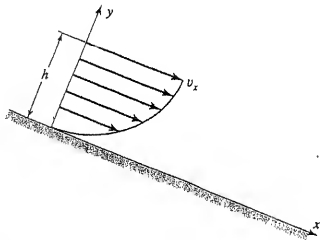
WITH  $V(b) = 0$   $V(\frac{b}{2}) = V_{max}$

$$V = 4V_{max} \left[ \frac{y}{b} - \left( \frac{y}{b} \right)^2 \right]$$

$$LV = 4V_{max} \int \left[ \frac{y}{b} - \left( \frac{y}{b} \right)^2 \right] dy$$

$$\underline{V_{max} = \frac{3}{2} \frac{LV}{b}}$$

4.15

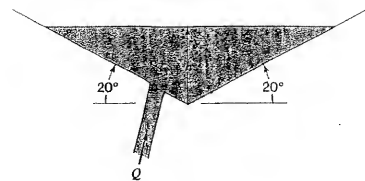


$$\begin{aligned} \dot{V} &= \int_A V_x dA \\ &= W \int_0^h V_0 \left( 2 \frac{y}{h} - \frac{y^2}{h^2} \right) dy \\ &= \frac{2}{3} W V_0 h \end{aligned}$$

$$2000 \frac{cm^3}{m} = \frac{2}{3} (10) V_0 (2)$$

$$\underline{V_0 = 150 \text{ cm/m} = 2.5 \text{ cm/s}}$$

4.16



$$V = WA = W h^2 \cot 20^\circ$$

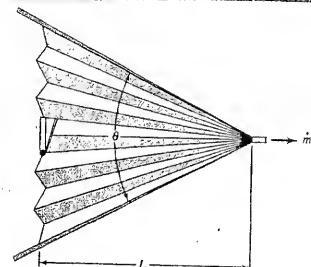
$$\dot{V} = W \cot 20^\circ \frac{dh^2}{dt}$$

$$\int_{h_1}^{h_2} dh^2 = \frac{\dot{V} \tan 20^\circ}{W} \int_0^t dt$$

$$h^2 \Big|_{h_1}^{h_2} = \frac{\dot{V} \tan 20^\circ}{W} t$$

$$\underline{t = (h_2^2 - h_1^2) \left[ \frac{W}{\dot{V}} \cot 20^\circ \right]}$$

4.17



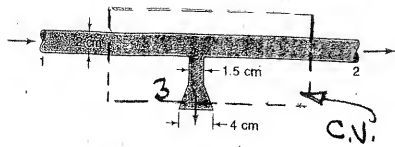
$$V = WA = W L^2 \tan \frac{\theta}{2}$$

$$\begin{aligned} \dot{m} &= \rho \dot{V} = \rho W L^2 \frac{d}{dt} \left( \tan \frac{\theta}{2} \right) \\ &= \rho W L^2 \sec^2 \frac{\theta}{2} \frac{\dot{\theta}}{2} \end{aligned}$$

$$\cos^2 \frac{\theta}{2} = \frac{1 + \cos \theta}{2}$$

$$\underline{\dot{m} = \frac{\rho W L^2 \dot{\theta}}{1 + \cos \theta} = \frac{\rho W L^2 \dot{\theta}}{2 \cos^2 \frac{\theta}{2}}}$$

4.18



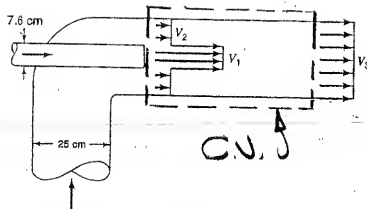
$$\text{STEADY FLOW} - \iint_{\text{C.S.}} \rho (\vec{U} \cdot \vec{n}) dA = 0$$

$$\dot{\rho} \dot{V}_1 = \dot{\rho} A_2 U_2 + \dot{\rho} A_3 U_3$$

$$1.3 \times 10^{-3} = \frac{\pi}{4} (0.02)^2 (2.1) + (100) \frac{\pi}{4} (10^{-3})^2 U_3$$

$$U_3 = 8.15 \text{ m/s}$$

4.19



$$\text{STEADY FLOW} - \iint_{\text{C.S.}} \rho (\vec{U} \cdot \vec{n}) dA = 0$$

$$\dot{\rho} A_3 U_3 - \dot{\rho} A_1 U_1 - \dot{\rho} A_2 U_2 = 0$$

$$U_3 = \frac{A_1 U_1 + A_2 U_2}{A_3}$$

$$= \left[ \frac{\pi}{4} (0.076)^2 (40) + \frac{\pi}{4} (0.25^2 - 0.076^2) (3) \right]$$

$$\frac{\pi}{4} (0.25^2)$$

$$U_3 = 5.15 \text{ m/s}$$

4.20

VOLUME DISPLACED BY PLUNGER

$$\dot{V} = A_p U_p = \frac{\pi}{4} d_p^2 U$$

VOLUME OF H<sub>2</sub>O MOVING PAST P:

$$\dot{V} = (A - A_p) U = \frac{\pi}{4} (D^2 - d_p^2) U$$

IN STEADY STATE OPERATION THESE MUST BE EQUAL:

$$\frac{\pi}{4} d_p^2 U = \frac{\pi}{4} (D^2 - d_p^2) U$$

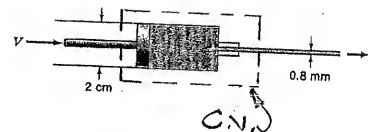
$$U = V \frac{d_p^2}{D^2 - d_p^2} \quad (a)$$

RELATIVE TO PLUNGER -

$$U_R = U + V$$

$$= V \left[ \frac{d_p^2}{D^2 - d_p^2} + 1 \right] \quad (b)$$

4.21

CONS. OF MASS - CONSTANT  $\rho$ 

$$\dot{V}_{\text{OUT}} = 6 \text{ cm}^3/\text{s} - \text{CONSTANT}$$

FOR NO LEAKAGE

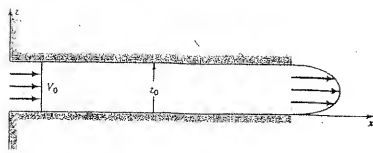
$$\dot{V} = A_p V = \frac{\pi}{4} (2)^2 V = 6$$

$$V = 1.91 \text{ cm/s}$$

FOR LEAKAGE -  $\dot{V} = 6 + 0.6$ 

$$\dot{V} = \frac{6.6}{\frac{\pi}{4} (2^2)} = 2.1 \text{ cm/s}$$

4.22



PARALLEL PLATES -

INCOMPRESSIBLE, STEADY FLOW -

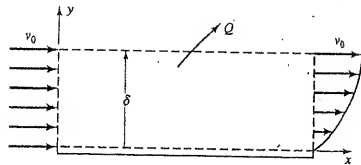
 $\dot{V} = \text{CONSTANT}$ 

$$\begin{aligned} V_0 2z_0 &= \int U dA \\ &= a \int_0^{z_0} 2(z_0 - z) dz \\ &= a \frac{2z_0^3}{6} \Rightarrow a = \frac{6V_0}{z_0^2} \end{aligned}$$

U IS MAX AT  $z = z_0/2$ 

$$\begin{aligned} U &= 6 \frac{V_0}{z_0^2} \left[ z_0 - \frac{z_0}{2} \right] \frac{z_0}{2} \\ &= 6V_0/4 = \underline{12 \text{ cm/s}} \end{aligned}$$

4.23



FOR STEADY INCOMPRESSIBLE FLOW:

$$\dot{V}_{\text{OUT}} - \dot{V}_{\text{IN}} = 0$$

$$Q + b \int_0^{\delta} U_0 \left( \frac{3\eta - \eta^3}{2} \right) dy = U_0 \delta b$$

$$\begin{aligned} \int_0^{\delta} \frac{3\eta - \eta^3}{2} dy &= \frac{\delta}{2} \int_0^1 \frac{3\eta - \eta^3}{2} d\eta \\ &= 5/8 \delta \end{aligned}$$

$$Q = U_0 \delta b \left( 1 - 5/8 \right) = \underline{\underline{\frac{3}{8} U_0 b \delta}}$$

4.24

SEE SKETCH FOR PROB 4.14

PLATES ARE CIRCULAR

$$\frac{\partial M}{\partial t} + \int \dot{m} = 0$$

$$M = 8b\pi L^2$$

$$\frac{\partial M}{\partial t} = 8\pi L^2 \dot{b} = -8\pi L^2 V$$

$$\dot{m}_{\text{EXIT}} = 8 \cdot 2\pi L b U_{\text{EXIT}} = 8\pi L^2 V$$

$$\Rightarrow \underline{\underline{U_{\text{EXIT}} = LV/2b}} \quad (a)$$

AS IN PROB 4.14 - PARABOLIC EXIT PROFILE

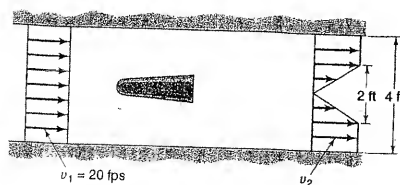
$$U_{\text{EXIT}} = ay + by^2$$

$$= 4U_{\text{MAX}} \left[ \frac{y}{b} - \left( \frac{y}{b} \right)^2 \right]$$

$$\begin{aligned} \dot{m}_{\text{EXIT}} &= 8 \int_0^b U_{\text{EXIT}} 2\pi L dy \\ &= 8 \frac{4}{3} \pi L b U_{\text{MAX}} \end{aligned}$$

$$\therefore \underline{\underline{U_{\text{MAX}} = \frac{3}{4} \frac{L}{b} V}}$$

5.1



CONS. OF MASS:  $\iint_{C.S.} \rho(\vec{v} \cdot \vec{n}) dA = 0$

$$\rho u_1 A_1 = 2 \left[ \int_0^1 \rho u_2 y dy + \int_1^2 \rho u_2 dy \right]$$

$$4u_1 = 2u_2 \left[ \frac{y^2}{2} \Big|_0^1 + u_2 y \Big|_1^2 \right]$$

$$= 3u_2$$

$$u_2 = \frac{4}{3} u_1 = \underline{26.7 \text{ ft/s}}$$

5.2 SYSTEM SHOWN IN PROB 5.1

$$\sum F_x = \iint_{C.S.} u_x \rho(\vec{v} \cdot \vec{n}) dA$$

— ASSUMING UNIT DEPTH —

$$F_x + (P_1 - P_2)A$$

$$= 2\rho \left[ \int_0^1 (u_2 y)^2 dy + \int_1^2 u_2^2 dy - u_1^2 \int_0^2 dy \right]$$

$$= 2\rho \left[ u_2^2 \frac{y^3}{3} \Big|_0^1 + u_2^2 y \Big|_1^2 - u_1^2 y \Big|_0^2 \right]$$

$$= 2\rho u_2^2 \left[ \left( \frac{1}{3} + 1 \right) \right] - 4\rho u_1^2$$

From 5.1 -  $u_2 = \frac{4}{3} u_1$

5.2 - CONTINUED -

$$F_x + (P_1 - P_2)A = \frac{20}{27} \rho u_1^2$$

$$F_x = -800 \text{ N/m} = 52.8 \text{ lbf/ft}$$

$$P_1 - P_2 = \frac{1}{4} \left[ \frac{20}{27} \rho u_1^2 + 52.8 \right]$$

$$= 157 \text{ lbf/ft}^2$$

$$\approx \underline{7500 \text{ Pa} = 7.5 \text{ kPa}}$$

5.3 SAME GENERAL CONFIGURATION EXCEPT EXIT VELOCITY DISTRIBUTION IS

$$u = u_2 \left( 1 - \cos \frac{\pi y}{4} \right)$$

AS IN 5.1 THE EXPRESSION TO BE USED IS:

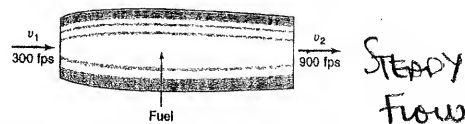
$$u_1 A_1 = 2 \left[ \int_0^2 u_2 \left( 1 - \cos \frac{\pi y}{4} \right) dy \right]$$

$$4u_1 = 2u_2 \left[ 2 - 4/\pi \right]$$

$$u_2 = \frac{2u_1}{2 - 4/\pi} = \frac{u_1}{1 - 2/\pi}$$

$$= 55 \text{ ft/s}$$

5.4



$$F_x = \iint_{C.S.} u_x \rho(\vec{v} \cdot \vec{n}) dA$$

$$= \rho_2 u_2^2 A_2 - \rho_1 u_1^2 A_1$$

$$= \dot{m}_2 u_2 - \dot{m}_1 u_1$$

$$= \dot{m}_1 (1.02 u_2 - u_1)$$

5.4 - CONTINUOUS -

$$\begin{aligned}\dot{m} &= \rho A V_1 \\ &= \left(0.0805 \frac{\text{lbm}}{\text{ft}^3}\right) (10.8 \text{ ft}^2) \left(300 \frac{\text{ft}}{\text{s}}\right) \\ &= 260.8 \text{ lbm/s}\end{aligned}$$

$$\begin{aligned}F_x &= \left(260.8 \frac{\text{lbm}}{\text{s}}\right) \left[1.02 \left(900 \frac{\text{ft}}{\text{s}}\right) - 300 \frac{\text{ft}}{\text{s}}\right] \\ &= \underline{5005 \text{ lbf}}\end{aligned}$$

5.5



STEADY INCOMPRESSIBLE FLOW:

$$F_x = \iint_{CS} V_x \rho (\vec{V} \cdot \vec{n}) dA$$

$$F_y = \iint_{CS} V_y \rho (\vec{V} \cdot \vec{n}) dA$$

IN X-DIRECTION:

$$\begin{aligned}F_x &= (\rho V A) [V_2 \cos(30) - (-V_1)] \\ &= \dot{m} [V(0.866) + 1]\end{aligned}$$

$$\begin{aligned}\dot{m} &= \rho A V = 62.4 \frac{\text{lbm}}{\text{ft}^3} (3 \text{ ft}^3/\text{s}) \\ &= 187.2 \text{ lbm/s}\end{aligned}$$

$$\begin{aligned}F_x &= \frac{(187.2 \text{ lbm/s})(25 \text{ ft/s})(1.866)}{32.2 \text{ lbm ft/s}^2 \text{ lbf}} \\ &= \underline{271.2 \text{ lbf}} \quad \text{FORCE ON BLADE IS IN } (-X)\end{aligned}$$

5.5 - CONTINUOUS

$$\begin{aligned}F_y &= \dot{m} V_2 \sin(-30) \\ &= \frac{(187.2)(25)(-0.5)}{32.2}\end{aligned}$$

$$= -72.7 \text{ lbf}$$

{ FORCE ON BLADE IS IN +Y DIRECTION }

PART (b) - BLADE MOVES TO RIGHT AT 15 FT/S

RELATIVE TO BLADE:  $V_1 = -40 \text{ ft/s}$

$V_2 = 40 \text{ ft/s}$  @  $-30^\circ$

ABSOLUTE VELOCITY OF LEAVING JET

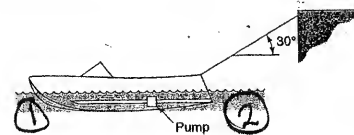
$$= \vec{V}_{\text{RELATIVE TO BLADE}} + \vec{V}_{\text{BLADE}}$$

$$= (34.64 \vec{e}_x - 20 \vec{e}_y) + 40 \vec{e}_x$$

$$= \underline{74.64 \vec{e}_x - 20 \vec{e}_y}$$

$$|V_{\text{EXIT}}| \approx \underline{77.3 \text{ ft/s}}$$

5.6



C.V. AROUND BOAT - STEADY INCOMPRESSIBLE FLOW

$$\begin{aligned}\sum F_x &= \iint_{CS} V_x \rho (\vec{V} \cdot \vec{n}) dA \\ &= \dot{m} (V_2 - V_1)\end{aligned}$$

5.6 - CONTINUED

$$F_x = \dot{m} \dot{V} \left( \frac{1}{A_2} - \frac{1}{A_1} \right)$$

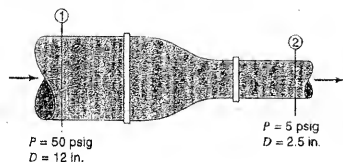
$$= \frac{(62.4)(6)^2 \left( \frac{1}{0.15} - \frac{1}{0.25} \right)}{32.2}$$

$$= 186 \text{ lbf}$$

$$\text{TENSION IN ROPE} = F_x / \cos 30^\circ$$

$$= \underline{215 \text{ lbf}}$$

5.7



Flow is Steady, Incompressible

$$\sum F_x = \iint_{C.S.} v_x \rho (\vec{v} \cdot \vec{n}) dA$$

$$= \dot{m} (v_2 - v_1)$$

$$\dot{m} = \rho \dot{V} = 0.8(62.4)(3 \text{ ft}^3/\text{s})$$

$$= 149.8 \text{ lbm/s}$$

$$\sum F_x = F_x + P_1 A_1 - P_2 A_2$$

{ ATMOSPHERIC PRESSURE }  
CANCELS

EQUATING:

$$F_x + P_1 \frac{\pi D_1^2}{4} - P_2 \frac{\pi D_2^2}{4} = \dot{m} \dot{V} \left( \frac{1}{A_2} - \frac{1}{A_1} \right)$$

$$F_x = \frac{\pi}{4} (P_2 D_2^2 - P_1 D_1^2)$$

$$+ \dot{m} \dot{V} \frac{4}{\pi} \left( \frac{1}{D_2^2} - \frac{1}{D_1^2} \right)$$

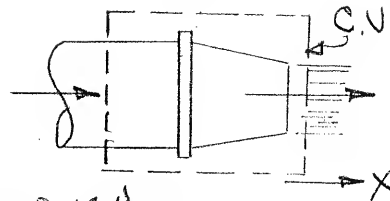
5.7 - CONTINUED

$$P_1 = 50 \text{ psig} \quad P_2 = 5 \text{ psig}$$

$$F_x = -5630 + 392$$

$$= \underline{-5238 \text{ lbf}}$$

5.8



fluid is  $H_2O$

$$P_1 = 60 \text{ psig} \quad P_2 = 14.7 \text{ psia}$$

$$D_1 = 0.25 \text{ ft} \quad D_2 = \frac{1.5}{12} \text{ ft}$$

$$\dot{V} = 400 \text{ gal/min} = 0.892 \text{ ft}^3/\text{s}$$

Steady, Incompressible Flow

$$\iint_{C.S.} v_x \rho (\vec{v} \cdot \vec{n}) dA = 0$$

$$v_1 = \frac{\dot{V}}{A_1} = \frac{0.892}{\frac{\pi}{4} (0.25)^2} = 18.17 \text{ ft/s}$$

$$v_2 = v_1 \frac{D_1^2}{D_2^2} = 18.17 \left( \frac{0.25(12)}{1.5} \right)^2 = 72.7 \text{ ft/s}$$

$$F_x + P_1 A_1 - P_2 A_2 = \dot{m} (v_2 - v_1)$$

$$F_x = \rho \dot{V} (v_2 - v_1) - P_1 A_1 + P_2 A_2$$

$$= [62.4(0.892)(72.7 - 18.17)] / 32.2$$

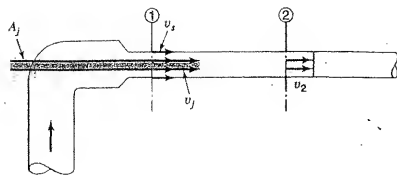
$$- (60 + 14.7)(144) \frac{\pi}{4} (0.25)^2$$

$$+ (14.7)(144) \frac{\pi}{4} \left( \frac{1.5}{12} \right)^2$$

$$= 94.3 - 528.0 + 25.4$$

$$= \underline{-408 \text{ lbf}}$$

5.9



$H_2O$  - Flow is Steady, Incompressible

$$\sum F_x = \iint_{CS} \rho v_x (\vec{v} \cdot \vec{n}) dA$$

$$\sum F_x = P_1 A_1 - P_2 A_2$$

$$\iint_{CS} \rho \vec{v} \cdot \vec{n} dA = \rho A_2 v_2^2 - \rho (A_1 v_1^2 + A_2 v_2^2)$$

EQUATING!

$$P_1 - P_2 = \rho \left[ v_2^2 - \frac{A_1}{A_2} v_1^2 - \frac{A_2}{A_2} v_2^2 \right]$$

By Conservation of Mass:

$$\iint_{CS} \rho (\vec{v} \cdot \vec{n}) dA = 0$$

$$A_2 v_2 - A_1 v_1 - A_2 v_2 = 0$$

$$v_2 = \frac{A_1}{A_2} v_1 + \frac{A_2}{A_2} v_2$$

$$= \frac{0.54}{0.60} (10) + \frac{0.06}{0.60} (90)$$

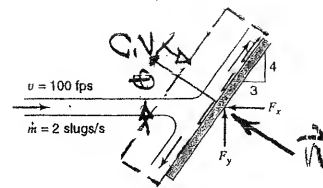
$$= 18 \text{ ft/s} \quad (a)$$

$$P_1 - P_2 = \frac{62.4}{32.2} \left[ (18)^2 - \frac{0.54}{0.6} (10)^2 - \frac{0.06}{0.6} (90)^2 \right]$$

$$= -1116 \text{ lbf/ft}^2 = -7.75 \text{ psi}$$

$$P_2 - P_1 = 7.75 \text{ psi}$$

5.10



Flow is Steady, Incompressible, Frictionless

For Frictionless Flow -

NO DRAG ON PLATE -

$$\sum F_n = \iint_{CS} \rho v_n (\vec{v} \cdot \vec{n}) dA = 0$$

$$F_n = -v_j \rho (v \cos \theta) A_j$$

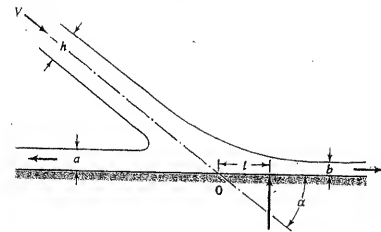
$$= -\dot{m} v \cos \theta$$

$$= -2 (100) (4/5) = -160$$

$$F_x = (-160) (4/5) = -128 \text{ lbf}$$

$$F_y = (-160) (3/5) = -96 \text{ lbf}$$

5.11



Steady, Incompressible Frictionless Flow

In X-DIRECTION:

$$\sum F_x = \iint_{CS} \rho v_x (\vec{v} \cdot \vec{n}) dA$$

$$= \rho v^2 b - \rho v^2 a - \rho v^2 h \cos \alpha = 0$$

Cons. of Mass:  $\rho v h = \rho v (a+b)$

$$h = a+b$$

$$a = \frac{h}{2} (1 - \cos \alpha) \quad b = \frac{h}{2} (1 + \cos \alpha)$$

# 5.11 - CONTINUED

$$\begin{aligned}\sum F_y &= \iint_{CS} \rho v_y (\vec{v} \cdot \vec{n}) dA \\ &= \rho v^2 h \sin \alpha \quad (a)\end{aligned}$$

Part (b):

$$\sum M_z = F_y L = \iint_{CS} (\vec{r} \times \vec{v})_z \rho (\vec{v} \cdot \vec{n}) dA$$

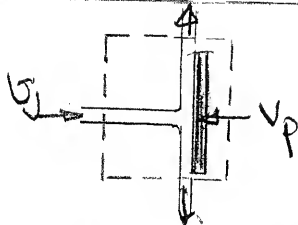
$$F_y L = \frac{a}{2} v (\rho v a) - \frac{b}{2} v (\rho v b)$$

$$\rho v^2 h (\sin \alpha) L = \frac{a^2 \rho v^2}{2} - \frac{b^2 \rho v^2}{2}$$

$$L = \frac{a^2 - b^2}{2 h \sin \alpha}$$

$$= \frac{h^2 \cot \alpha}{2 h \sin \alpha} = \frac{h \cot \alpha}{2}$$

5.12



Flow is Steady, Incompressible,  
Frictionless -

Atmospheric Pressure Cancels

CV. Moves To Left With Velocity,  $v_p$

$$\sum F_x = \iint_{CS} v_x \rho (\vec{v} \cdot \vec{n}) dA$$

$$F_x = \rho A_0 (v_0 + v_p)^2$$

$$= \frac{(62.4)}{32.2} \frac{3}{30} (5 + 30)^2$$

$$= 237.4 \text{ lbf}$$

FOR MOVING PLATE

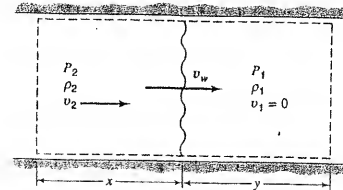
# 5.12 - CONTINUED

For  $v_p = 0$

$$F_x = \frac{62.4}{32.2} \frac{3}{30} (30)^2$$

$$= 174.4 \text{ lbf}$$

5.13



Cons. of Mass: for unit cross section

$$\frac{\partial M}{\partial t} + \iint_{CS} \dot{m} = 0$$

$$M = \rho_2 x + \rho_1 y \quad \int \dot{m} = -\rho_2 v_2$$

$$\rho_2 \dot{x} + \rho_1 \dot{y} - \rho_2 v_2 = 0$$

$$\text{Since } \dot{x} = v_w \quad \dot{y} = -v_w$$

$$\rho_2 (v_w - v_2) - \rho_1 v_w = 0 \quad (1)$$

X-Momentum:

$$\sum F_x = \iint_{CS} v_x \rho (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{C.V.} v_x \rho dV$$

$$p_2 - p_1 = -v_2 \rho_2 v_2 + \frac{\partial}{\partial t} v_2 \rho_2 x$$

$$= -\rho_2 v_2^2 + \rho_2 v_2 v_w$$

$$= \rho_2 v_2 [v_w - v_2]$$

$$\text{From (1): } \rho_2 v_2 (v_w - v_2) = \rho_1 v_2 v_w$$

$$\text{GIVEN } p_2 - p_1 = \rho_1 v_2 v_w$$

Q.E.D.

5.14 FOR SITUATION CONSIDERED IN Prob 5.13

(a) Air  $V_w = 1130 \text{ ft/s}$   
 $S = 0.00238 \text{ slug/ft}^3$

$$P_2 - P_1 = \rho_1 V_2 V_w$$

$$= (0.00238)(10)(1130)$$

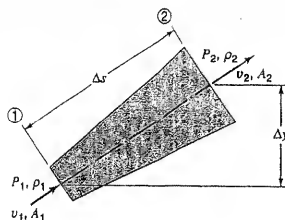
$$= 26.9 \text{ PSF} = 0.187 \text{ PSI}$$

(b)  $H_2O$   $V_w = 4700 \text{ ft/s}$   
 $\rho = 1.938 \text{ slug/ft}^3$

$$\Delta P = (1.938)(10)(4700)$$

$$= 91,080 \text{ PSF} = 633 \text{ PSI}$$

5.15.



CONS. OF MASS:

TECHNIQUE IS TO LET

$$P_2 = P_1 + \frac{\partial P}{\partial s} \Delta s$$

$$V_2 = V_1 + \frac{\partial V}{\partial s} \Delta s$$

ETC

BY CONSERVATION OF MASS -  $\Delta s = \Delta y \neq 0$

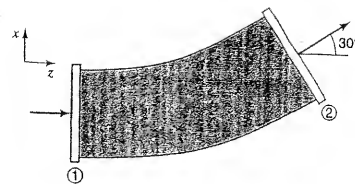
$$\frac{\partial}{\partial s} (\rho A V) = 0$$

BY MOMENTUM THEOREM, USING  
 CONS. OF MASS RESULT:

$$\Delta P + \rho V \Delta V + \rho g \Delta y = 0$$

- MESSY -

5.16



$$D_1 = 0.3 \text{ m}$$

$$D_2 = 0.38 \text{ m}$$

$$V_1 = 12 \text{ m/s}$$

$$P_2 = 145 \text{ kPa}$$

$$P_1 = 128 \text{ kPa}$$

$$V_2 = 7.48 \text{ m/s}$$

$$A_1 = \frac{\pi}{4} (0.3 \text{ m})^2 = 0.0707 \text{ m}^2$$

$$A_2 = \pi = 0.1134 \text{ m}^2$$

$$\dot{V} = A_1 V_1 = (0.0707)(12) = 0.8484 \text{ m}^3/\text{s}$$

IN X DIRECTION:  $\sum F_x = \iint_{C.S.} V_x \rho (\vec{V} \cdot \vec{n}) dA$

$$F_x + P_1 A_1 - P_2 A_2 \cos \theta = \dot{m} (V_2 \cos \theta - V_1)$$

$$F_x = (1000)(0.8484) [7.48 (\cos 30^\circ) - 12]$$

$$- (1000) [128(0.0707) + (145)(0.1134) \cos 30^\circ]$$

$$= -505.5 \text{ N}$$

IN y DIRECTION:  $\sum F_y = \iint_{C.S.} V_y \rho (\vec{V} \cdot \vec{n}) dA$

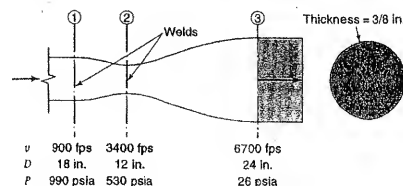
$$F_y - P_2 A_2 \sin \theta = \dot{m} (V_2 \sin \theta)$$

$$F_y = (1000)(0.8484) (7.48 \sin 30^\circ)$$

$$+ (1000)(145)(0.1134) (\sin 30^\circ)$$

$$= 11395 \text{ N} = 11.395 \text{ kN}$$

5.17



STEADY INCOMPRESSIBLE FLOW:

$$\sum F_x = \iint_{C.S.} V_x \rho (\vec{V} \cdot \vec{n}) dA$$

5.17 - CONTINUED -

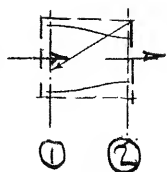
$$A_1 = \pi/4 (1.5)^2 = 1.767 \text{ ft}^2$$

$$A_2 = \pi/4 (1)^2 = 0.785 "$$

$$A_3 = \pi/4 (2)^2 = 3.142 "$$

For C.V. BETWEEN

① & ② :



$$\sum F_x = \iint_{C.S.} \rho u_x (\vec{v} \cdot \vec{n}) dA$$

$$F_x + F_2 + P_1 A_1 - P_2 A_2 = \dot{m} (v_2 - v_1)$$

$$F_x = \frac{770(3400 - 900)}{32.2}$$

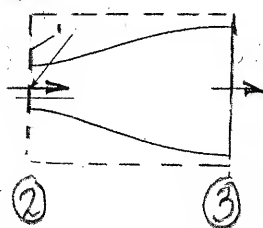
$$+ (530 - 14.7)(144)(0.785)$$

$$- (990 - 14.7)(144)(1.767) - F_2$$

$$= -130,000 \text{ lbf} - F_2$$

For C.V. BETWEEN

② & ③ :



$$F_x + P_2 A_2 - P_3 A_3 = \dot{m} (v_3 - v_2)$$

$$F_x = \frac{770}{32.2} (6700 - 3400)$$

$$+ (26 - 14.7)(144)(3.14)$$

$$- (530 - 14.7)(144)(0.785)$$

$$= 25777 \text{ lbf}$$

5.17 - CONTINUED -

STRESS AT 2:

$$\sigma = \frac{F}{A} = \frac{25777 \text{ lbf}}{\pi (12)(3/8)} = 1823 \text{ PSI}$$

(COMPRESSION)

At 1

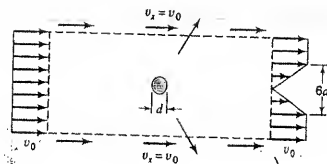
$$F_1 = -130,000 + 25777$$

$$= -104220 \text{ lbf}$$

$$\sigma = \frac{104220}{\pi (18)(3/8)} = 4915 \text{ PSI}$$

TENSION

5.18



FLOW IS STEADY & INCOMPRESSIBLE  
NO NET PRESSURE FORCE

$$\sum F_x = \iint_{C.S.} \rho u_x (\vec{v} \cdot \vec{n}) dA$$

$$F_x = \int_{\text{OUT}} \rho u_x^2 dA - \rho v_0^2 A_{\text{IN}}$$

$$= 2 \int_0^{3d} \rho v_0^2 \left(\frac{y}{3d}\right)^2 dy$$

$$+ \rho v_0^2 (3d) - \rho v_0^2 (6d)$$

↑  
MOMENTUM OUT TOP & BOTTOM

$$F_x = 2 \rho v_0^2 \frac{1}{9d^2} \frac{y^3}{3} \Big|_0^{3d} + \rho v_0^2 (3d - 6d)$$

$$= -\rho v_0^2 d$$

$$\text{FORCE ON CYLINDER} = \underline{\underline{\rho v_0^2 d}}$$

5.19 FLUID IS  $H_2O$

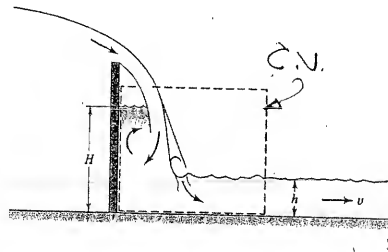
$$V_{\text{sonic}} = 1433 \text{ m/s}$$

THIS IS JUST LIKE PROB 5.13

FOR AN OBSERVER MOVING WITH  $H_2O$  STREAM:  $U = 3 \text{ m/s}$

$$\begin{aligned} \text{THEN } \Delta P &= \rho U_w \Delta U \\ &= (1000)(1433-3)(3) \\ &= \underline{4287 \text{ kPa}} \end{aligned}$$

5.20



STATIC PRESSURE OF  $H_2O$ :

$$\text{ON LEFT} - P = \rho g H$$

$$\text{ON RIGHT} - P = \rho g h$$

$$\sum F_x = \iint_{C.S.} U_x \rho (\vec{v} \cdot \vec{n}) dA$$

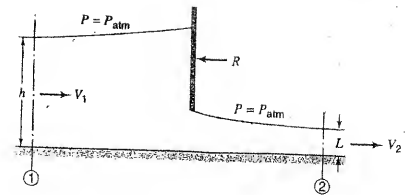
$$\frac{\rho g H^2}{2} - \frac{\rho g h^2}{2} = (\rho U h) U$$

$$H^2 = \frac{2}{\rho g} \left[ \rho h U^2 + \frac{\rho g h^2}{2} \right]$$

$$= \frac{2 h U^2}{g} + h^2$$

$$H = \left[ \frac{2 h U^2}{g} + h^2 \right]^{1/2}$$

5.21



CONSERVATION OF MASS:

$$\iint_{C.S.} \rho (\vec{v} \cdot \vec{n}) dA = 0$$

$$- \rho h V_1 + \rho L V_2 = 0$$

$$V_2 = \frac{h V_1}{L} \quad (a)$$

x - MOMENTUM:

$$\sum F_x = \iint_{C.S.} U_x \rho (\vec{v} \cdot \vec{n}) dA$$

$$P_1 A_1 - P_2 A_2 + F_x = \dot{m} (U_2 - U_1)$$

$$F_x = \dot{m} (U_2 - U_1) + P_2 A_2 - P_1 A_1$$

$$= \rho U_1 h (U_2 - U_1) + \rho g \frac{L^2}{2} - \rho g \frac{h^2}{2}$$

$$= \rho U_1^2 h \left( \frac{h}{L} - 1 \right) + \frac{\rho g}{2} (L^2 - h^2) \quad (b)$$

5.22



CONSERVATION OF MASS:

$$U_1 h_1 = U_2 h_2$$

MOMENTUM THM:

$$\sum F_x = \iint_{C.S.} U_x \rho (\vec{v} \cdot \vec{n}) dA$$

$$P_1 h_1 - P_2 h_2 = \dot{m} (U_2 - U_1)$$

$$P_1 = \rho g \frac{h_1^2}{2} \quad P_2 = \rho g \frac{h_2^2}{2}$$

5.22 - CONTINUED

$$\frac{\rho g h_1^2}{2} - \frac{\rho g h_2^2}{2} = \rho v_1 h_1 (v_2 - v_1)$$

from CONS. OF MASS:  $v_2 = v_1 h_1 / h_2$

$$\frac{g}{2} (h_1^2 - h_2^2) = v_1^2 h_1 \frac{h_1 - h_2}{h_2}$$

factoring & canceling  $h_1 - h_2$

$$\frac{g h_2}{2} (h_1 + h_2) = v_1^2 h_1$$

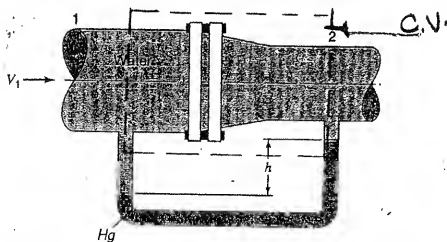
$$h_2^2 + h_1 h_2 - \frac{2 v_1^2 h_1}{g} = 0$$

$$h_2 = \frac{h_1}{2} \left[ \sqrt{1 + \frac{8 v_1^2}{g h_1}} - 1 \right]$$

from CONTINUITY

$$v_2 = \frac{g h_1}{4 v_1} \left[ 1 + \sqrt{1 + \frac{8 v_1^2}{g h_1}} \right]$$

5.23



$$D_1 = 8 \text{ cm}$$

$$D_2 = 5 \text{ cm}$$

$$v_1 = 5 \text{ m/s}$$

$$P_2 = 1 \text{ atm}$$

$$h = 58 \text{ cm}$$

$$A_1 = \frac{\pi}{4} (8 \text{ cm})^2 = 50.3 \text{ cm}^2$$

$$A_2 = \frac{\pi}{4} (5 \text{ cm})^2 = 19.6$$

$$v_2 = 5 \text{ m/s} \left( \frac{50.3}{19.6} \right) = 12.83 \text{ m/s}$$

5.23 - CONTINUED

X-MOMENTUM:  $\sum F_x = \iint_{c.s.} v_x \rho (\vec{v} \cdot \vec{n}) dA$

$$F_x + P_1 A_1 - P_2 A_2 = \rho V (v_2 - v_1)$$

$$\begin{aligned} P_1 - P_2 &= \rho g h [13.6 - 1] \\ &= (1000)(9.81)(0.58)(12.6) \\ &= 71.69 \text{ kPa} \end{aligned}$$

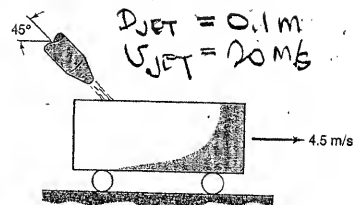
SINCE  $P_2 = 1 \text{ atm}$

$$P_1 = 71.69 \text{ kPa gauge}$$

$$F_x + P_1 A_1 = (1000)(50.3 \times 10^{-4})(5)(12.83 - 5)$$

$$\begin{aligned} F_x &= 197 - 71.69(50.3 \times 10^{-4})(1000) \\ &= -163.7 \text{ N} \end{aligned}$$

5.24



X-MOMENTUM

$$\sum F_x = \iint_{c.s.} v_x \rho (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint v_x \rho dV$$

$$\begin{aligned} F_x &= -\rho_w A_j v_j (v_j \cos \theta) + \rho_w A_j v_j v_c \\ &= \rho_w A_j v_j (v_c - v_j \cos \theta) \\ &= 1000 \left( \frac{\pi}{4} \right) (0.1)^2 (20) [4.5 - 20 \cos 45^\circ] \\ &= -1515 \text{ N} \end{aligned}$$

FORCE ON GEAR BY JET:  $F_x = 1515 \text{ N}$

5.24 - CONTINUOUS -

y Momentum:

$$\sum F_y = \iint_{C.S.} \rho v_y (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{C.V.} \rho v_y dV$$

$$F_y = -\rho v_j \sin \theta S (v_j A_j) + 0$$

$$= -(20 \sin 45)(1000)(-20)$$

$$\times \pi/4 (0.1)^2$$

$$= +2220 \text{ N}$$

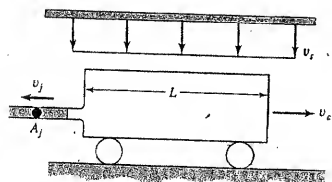
FORCE EXERTED BY A20:

$$F_y = -2220 \text{ N}$$

TOTAL FORCE -

$$\vec{F} = 1515 \vec{e}_x - 2220 \vec{e}_y \text{ N}$$

5.25



COORDINATES FIXED TO CART  
~ MOVING TO RIGHT AT  $v_c$

MOMENTUM FLOW IN X-DIRECTION

$$\sum F_x = \rho A_j v_j (-v_j)$$

$$- \rho A_s v_s (-v_c)$$

$$F_x = \rho A_s v_s v_c - \rho A_j v_j^2$$

IN y-DIRECTION

$$F_y = \rho A_j v_j (0) - \rho A_s v_s (-v_s)$$

$$= \rho A_s v_s^2$$

FORCE OF FLUID ON CART IS NEGATIVE  
OF THESE.

31

5.26

for C.V. SHOWN:

$$\int_{C.S.} d\dot{m} + \frac{\partial}{\partial t} M = 0$$

$$M = \rho A h$$

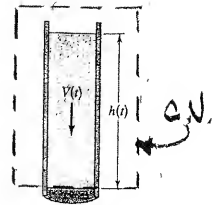
$$d\dot{m} = \rho A \dot{h}$$

$$\sum F_y = \iint_{C.S.} \rho v_y (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{C.V.} \rho v_y dV$$

$$- \rho g A h = + \rho A \dot{h}^2 + \rho A \frac{\partial}{\partial t} (h \dot{h})$$

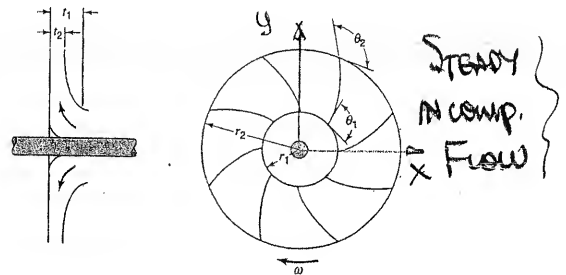
$$-gh = -\dot{h}^2 + \frac{\partial}{\partial t} (h \dot{h})$$

$$\Rightarrow \underline{\underline{\ddot{h} = -g}}$$



5.27

$\omega = 1180 \text{ rpm}$   $r_2 = 0.6 \text{ in.}$   
 $r_1 = 2 \text{ in.}$   $\theta_2 = 135^\circ$   
 $r_2 = 8 \text{ in.}$   $t_1 = 0.8 \text{ in.}$



ROTATION IS ABOUT z-AXIS -

$$\sum M_z = \iint_{C.S.} (\vec{r} \times \vec{v})_z \rho (\vec{v} \cdot \vec{n}) dA$$

$$= \begin{vmatrix} \vec{e}_r & \vec{e}_\theta & \vec{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} \dot{m}_{out}$$

$$= (r_x v_y - r_y v_x) \dot{m}_{out}$$

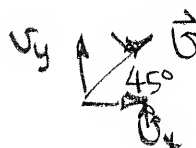
AT POSITION ON x-AXIS -  $r_x = r_2$   
 $r_y = 0$

$$\dot{V} = 800 \left( \frac{1}{7.48} \right) \left( \frac{1}{60} \right) = 1.783 \text{ FT}^3/\text{s}$$

$$v_x = \frac{1.783}{\pi (8/12)(2)(0.6/12)} = 851 \text{ FT/s}$$

5.27 - CONTINUED

AT THIS LOCATION -



$$U_{tan} = U_y = U_x = 8.51 \text{ ft/s}$$

ABS. VELOCITY @  $r_2$

$$\begin{aligned} U_y &= -r\omega + U_{tan} \\ &= -\left(\frac{8}{12}\right)\left(\frac{1180 \times 2\pi}{60}\right) + U_{tan} \\ &= -82.38 + 8.51 = -73.87 \text{ ft/s} \end{aligned}$$

NOW - IN MOMENTUM EXPRESSION:

$$\begin{aligned} M_z &= (r_2 U_y) \dot{V} \\ &= \frac{8}{12} \frac{(-73.87)(64)(1.783)}{32.2} \\ &= 174 \text{ ft-lbf} \end{aligned}$$

$$\begin{aligned} \text{POWER} &= M_z \omega \\ &= 174 \left( \frac{1180 \times 2\pi}{60} \right) \left( \frac{1}{550} \right) \\ &= 39.1 \text{ hp} \end{aligned}$$

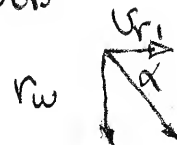
5.28 FOR CONFIGURATION OF  
PROB 5.28

$$\dot{V} = 1.783 \text{ ft}^3/\text{s}$$

AT INLET -  $U_r = \frac{\dot{V}}{2\pi r_1 t_1}$

$$= \frac{1.783}{2\pi \left(\frac{2}{12}\right) \left(\frac{6.8}{12}\right)} = 25.54 \text{ ft/s}$$

5.28 - CONTINUED



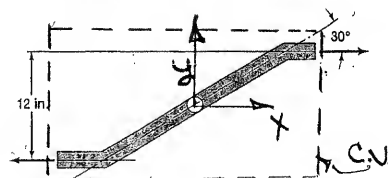
$$\begin{aligned} r\omega &= -U_{r1} = -\left(\frac{2}{12}\right)\left(\frac{1180 \times 2\pi}{60}\right) \\ &= -20.6 \text{ ft/s} \end{aligned}$$

$$\alpha = \tan^{-1} \frac{r\omega}{U_{r1}} = \tan^{-1} \frac{20.6}{25.54} = 38.9^\circ$$

PART (B):  $\sum F_x = \iint_{CS} U_x \rho (\vec{U} \cdot \vec{n}) dA$

$$\begin{aligned} F_x &= -\rho U_z (-U_z) A_1 \\ &= \rho U_z^2 = \rho \dot{V} \frac{\dot{V}}{A_1} \\ &= \frac{(64)(1.783)^2}{\pi \left(\frac{2}{12}\right)^2 (32.2)} \\ &= 70.6 \text{ lbf} \end{aligned}$$

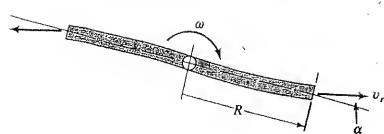
5.29



FOR C.V. SHOWN:  $\sum M_z = \iint_{CS} (\vec{r} \times \vec{U})_z \rho (\vec{U} \cdot \vec{n}) dA$

$$\begin{aligned} M_z &= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ r_x & r_y & r_z \\ U_x & U_y & U_z \end{vmatrix}_z \dot{m} \\ &= 2 \dot{m} (U_x r_y) \\ &= \frac{2(644)(\pi/4) \left(\frac{0.5}{12}\right)^2 (20)^2 (6/12)}{32.2} \\ &= 1.057 \text{ ft-lbf} \end{aligned}$$

5,30



$$\sum M_A = \iiint_{C.S.} (\vec{r} \times \vec{v})_z \rho (\vec{v} \cdot \vec{n}) dA$$

$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} (\dot{m})$$

$$M_A = 2 (-r_y v_x) \dot{m}$$

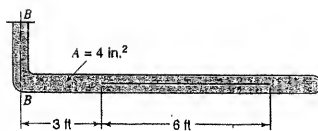
$$r_y = R \sin \alpha$$

$$v_x = v_r \sin \alpha - R \omega$$

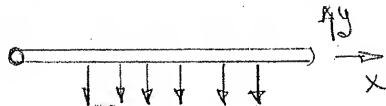
$$M_A = 2 \dot{m} [-R \sin \alpha (v_r \sin \alpha - R \omega)]$$

$$\omega = \frac{M_A}{2 \dot{m} R^2 \sin \alpha} + \frac{v_r \sin \alpha}{R}$$

5,31



Top View



$$\sum M_z = \iiint_{C.S.} (\vec{r} \times \vec{v})_z \rho (\vec{v} \cdot \vec{n}) dA$$

$$= \int_3^9 \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} \rho v_y t dx$$

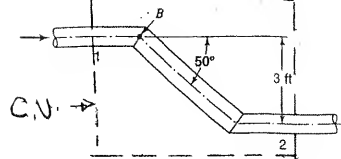
$$= \int_3^9 -8t v^2 x dx = -8t^2 \left( \frac{x^2}{2} \right) \Big|_3^9$$

$$v = \frac{8}{6(0.25/12)} = 64 \text{ ft/s}$$

$$M = - \frac{624(64)^2(0.25)}{32.2} \left( \frac{81-9}{2} \right)$$

$$= -5950 \text{ FT LBF}$$

5,32



$$\sum M_B = \iiint_{C.S.} (\vec{r} \times \vec{v})_z \rho (\vec{v} \cdot \vec{n}) dA$$

$$M_B = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} \dot{m} + r_z p_2 A_2$$

$$= (r_x v_y - r_y v_x) \dot{m} + r_z p_2 A_2$$

$$\dot{V} = \left( 30 \frac{\text{gal}}{\text{min}} \right) \left( \frac{1}{7.48 \times 60} \right) = 0.00668 \text{ ft}^3/\text{s}$$

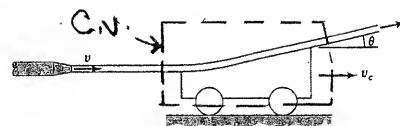
$$v_2 = \frac{\dot{V}}{A_2} = \frac{0.00668}{\frac{\pi}{4} \left( \frac{0.75}{12} \right)^2} = 21.79 \text{ ft/s}$$

$$M_B = + \frac{3(21.79)(62.4)(0.00668)}{32.2}$$

$$+ (3)(24) \left( \frac{\pi}{4} \right) (0.75)^2$$

$$= 8.46 + 31.81 = 40.3 \text{ FT LBF}$$

5,33



LINEAR MOMENTUM! COORDINATE SYSTEM MOVES WITH CART

$$\sum F_x = \iiint_{C.S.} v_x \rho (\vec{v} \cdot \vec{n}) dA$$

$$F_x = 8A [(v - v_c) \cos \theta] (v - v_c) + 8A (v - v_c)^2$$

$$P = v_c F_x = 8A (v - v_c)^2 [\cos \theta - 1] v_c$$

$$\text{For } m = \frac{v_c}{v} \quad P = 8A \left[ \frac{v_c^3}{v^3} m (1 - m)^2 \right]$$

5.33 - CONTINUED

For  $P = P_{\max}$   $\frac{dP}{dm} = 0$

$$\therefore \frac{dP}{dm} = 8A [\dots] U^3 \frac{d}{dm} (m - 2m^2 + m^3)$$

$$= \{ \} (1 - 4m + 3m^2) = 0$$

$$m = 1, 1/3$$

$$m = 1 \rightarrow \text{MINIMUM}$$

$$m = 1/3 \rightarrow \text{MAXIMUM}$$

$$\therefore m = \frac{U_c}{U} = 1/3$$

Part (b) Rotation About Z-axis-

$$M_z = \iint_{O.S.} (\vec{r} \times \vec{U})_z \rho (\vec{U} \cdot \vec{n}) dA$$

$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ r_x & r_y & r_z \\ U_x & U_y & U_z \end{vmatrix} \dot{m} \quad \text{OUT}$$

$$- \begin{vmatrix} \dot{m} & \text{IN} \\ & z \end{vmatrix}$$

$$= r \dot{m} [(U - U_c) \cos \theta + U_c - U]$$

$$= r \dot{m} (\cos \theta - 1) (U - U_c)$$

$$P = M_z \omega = M_z \frac{U_c}{r}$$

$$= [\dot{m} (\cos \theta - 1)] U_c (U - U_c)$$

$$\text{for } m = \frac{U_c}{U}$$

5.33 - CONTINUED

$$P = [\dots] m U^2 (1 - m)$$

$$\frac{dP}{dm} = [\dots] U^2 (1 - 2m) = 0$$

OR  $P_{\max}$  occurs when

$$m = \frac{U_c}{U} = \frac{1}{2}$$

## CHAPTER 6

6.1. For  $V = A + Br$

$$V(r_0) = 0 = A + Br_0$$

$$V(r_i) = \frac{\omega d}{2} = A + Br_i$$

$$A = -Br_0 = \frac{\omega d}{2} - Br_i$$

$$B(r_0 - r_i) = -\frac{\omega d}{2}$$

$$B = -\frac{1}{r_0 - r_i} \frac{\omega d}{2}$$

$$A = \frac{r_0}{r_0 - r_i} \frac{\omega d}{2}$$

$$V = \frac{r_0 - r}{r_0 - r_i} \frac{\omega d}{2}$$

6.2 Steady Flow:  $\frac{\delta Q}{dt} = \frac{\delta W}{dt} = 0$

$$-\frac{\delta W}{dt} = \iint_{CS} \left( e + \frac{p}{\rho} \right) \rho (\vec{v} \cdot \vec{n}) dA$$

$$-\frac{\delta W}{dt} = \dot{m} \left[ u_2 - u_1 + \frac{p_2 - p_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$

$$\dot{m} = \rho \dot{V} = 1025(21) = 21525 \text{ kg/s}$$

$$V_1 = \frac{\dot{V}}{A_1} = 4.278 \text{ m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = 11.573 \text{ m/s}$$

Since  $\Delta T = 0$   $u_2 - u_1 = 0$

$$z_2 - z_1 = 1.8 \text{ m}$$

$$p_2 = 175 \text{ kPa}$$

$$p_1 = -0.15 \text{ m Hg}$$

$$= -19.9 \text{ kPa}$$

6.2 - CONTINUOUS -

$$\frac{p_2 - p_1}{\rho} = \frac{175 + 19.9}{1025} = 190 \text{ m}^2/\text{s}^2$$

$$\frac{V_2^2 - V_1^2}{2} = \frac{(4.278)^2 - (11.57)^2}{2} = -57.7 \text{ m}^2/\text{s}^2$$

$$g(z_2 - z_1) = 9.81(1.8) = 17.7 \text{ m}^2/\text{s}^2$$

$$-\dot{W} = (190 - 57.7 + 17.7)(215.3)$$

$$= 32,295 \text{ W} = \underline{\underline{32.3 \text{ kW}}}$$

6.3  $\frac{\delta Q}{dt} - \frac{\delta W}{dt} - \frac{\delta W_{sh}}{dt} = \iint_{CS} \left( e + \frac{p}{\rho} \right) \rho (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{CV} e \rho dV$

$$-\dot{m} \left[ h_1 + \frac{V_1^2}{2} + g z_1 \right] + \frac{\partial}{\partial t} \left[ \dot{m} u \right] = 0$$

$$\delta V C_v \frac{dT}{dt} = \delta A V \left( \frac{V^2}{2} \right)$$

$$\frac{dT}{dt} = \frac{A V}{V C_v} \frac{V^2}{2}$$

$$= \frac{\frac{\pi}{4} \left( \frac{8}{12} \right)^2 \text{ft}^2 \left( \frac{110 \text{ ft}}{\text{s}} \right)^2}{2 \left( 10 \text{ ft}^3 \right) \left( 0.17 \frac{\text{Btu}}{\text{lbm} \cdot \text{F}} \right) \left( \frac{778 \text{ ft} \cdot \text{lb}}{\text{Btu}} \right) \left( \frac{32.2 \text{ lbm} \cdot \text{ft}}{\text{s}^2 \cdot \text{lb}} \right)}$$

$$= \underline{\underline{21.8 \text{ F/s}}}$$

6.4 Every Equation Reduces to

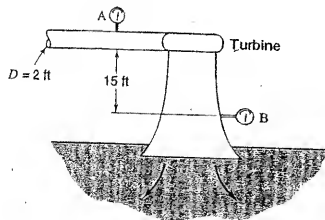
$$\iint_{C.S.} (\rho + \frac{P}{g}) \mathbf{g} \cdot \mathbf{n} dA = 0$$

$$u_2 - u_1 + \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) = 0$$

$$\Delta u = C \Delta T = \frac{P_1 - P_2}{\rho}$$

$$\Delta T = \frac{\Delta P}{C \rho} = \frac{10 (144)}{(1) (2.4) (718)} = \underline{0.0297 F}$$

6.5.



Between A & B:

$$-\frac{\delta W_s}{dt} = \iint_{C.S.} (\rho + \frac{P}{g}) \mathbf{g} \cdot \mathbf{n} dA$$

$$\frac{\delta W_s}{dt} = \frac{600 (550)}{0.82} = 402,000 \text{ ft} \cdot \text{lb}_f / \text{s}$$

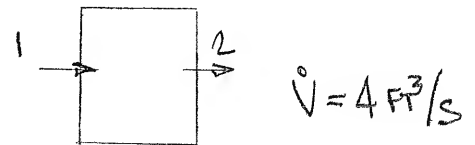
$$\iint_{C.S.} \rho \left[ \frac{V_2^2}{2} - \frac{V_1^2}{2} + \frac{P_2 - P_1}{\rho} + g(y_2 - y_1) + \Delta u \right] dA = 0$$

$$\frac{P_2 - P_1}{\rho} = \left\{ -\frac{\delta W_s}{dt} + g(y_2 - y_1) + \frac{V_1^2}{2} \right\}$$

$$P_B = P_A + \rho \left\{ \right\}$$

$$= \underline{46.5 \text{ PSIA}}$$

6.6.



$$P_1 = -6 \text{ PSIA}$$

$$P_2 = 40 \text{ PSIA}$$

$$V_1 = \frac{4}{\pi (1)^2} = 5.10 \text{ ft/s}$$

$$V_2 = \frac{4}{\pi (10/2)^2} = 7.38 \text{ ft/s}$$

$$y_2 - y_1 = 5 \text{ ft}$$

Energy Eqn. Reduces to:

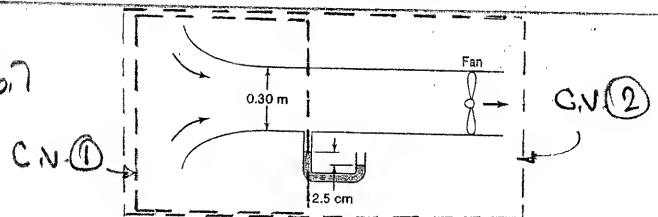
$$-\frac{\delta W_s}{dt} = \dot{m} \left[ \frac{\Delta P}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) \right]$$

$$= \dot{m} g \left[ \frac{\Delta P}{\rho g} + \frac{V_2^2 - V_1^2}{2g} + y_2 - y_1 \right]$$

$$= 27850 \text{ ft} \cdot \text{lb}_f / \text{s}$$

$$= \underline{50.6 \text{ Hp}}$$

6.7



for C.V. (1) - Energy Eqn. Reduces to

$$0 = \frac{V_2^2 - V_1^2}{2} + \frac{P_2 - P_1}{\rho} + g(z_2 - z_1)$$

$$\frac{V_2^2}{2} = \frac{P_1 - P_2}{\rho} = \frac{2.5 \text{ cm H}_2\text{O}}{\rho}$$

$$V_2 = \left[ 2 \frac{\Delta P}{\rho} \right]^{1/2} = 20 \text{ m/s}$$

$$\dot{V} = A V_2 = \frac{\pi}{4} (0.3)^2 (20) = \underline{1.47 \text{ m}^3/\text{s}}$$

# 6.7 - CONTINUED

for C.V. ② Energy Eqn is:

$$-\frac{\delta W_s}{dt} = \dot{m} \left[ A_1 u + \frac{V_2^2}{2} + \frac{P_1}{\rho} + g \Delta y \right]$$

$$= 8 \dot{V} \frac{V_2^2}{2}$$

$$= (1.22)(1.417)(20)^2/2$$

$$= \underline{\underline{346 \text{ W}}}$$

# 6.8 - CONTINUED

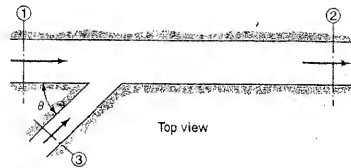
for  $u = c_v T$ ,  $T_1 = T_3$ ,  $P_1 = P_3$

LOTS OF ALGEBRA

$$c_v(T_2 - T_1) = \frac{V_1^2}{2} \left[ 1 + \left( \frac{A_3 V_3}{A_1 V_1} \right)^2 - 1 \right] \times$$

$$\times \left[ \frac{1 + 2 \frac{A_3 V_3}{A_1 V_1}}{1 + \frac{A_3 V_3}{A_1 V_1}} \right] + \frac{V_3^2}{2} \left[ \frac{\frac{A_3 V_3}{A_1 V_1}}{1 + \frac{A_3 V_3}{A_1 V_1}} - \frac{2 \frac{A_3 V_3}{A_1 V_1}}{1 + \frac{A_3 V_3}{A_1 V_1}} \right]$$

6.8.



Steady Flow Energy Eqn:

$$\dot{m}_1 \left( u_1 + \frac{V_1^2}{2} + \frac{P_1}{\rho} \right) + \dot{m}_3 \left( u_3 + \frac{V_3^2}{2} + \frac{P_3}{\rho} \right)$$

$$= \dot{m}_2 \left( u_2 + \frac{V_2^2}{2} + \frac{P_2}{\rho} \right)$$

Cons. of Mass:

$$V_1 A_1 + V_3 A_3 = V_2 A_2 \quad (1)$$

Energy Eqn can be written

$$A_1 V_1 \left[ c_v T_1 + \frac{V_1^2}{2} + \frac{P_1}{\rho} \right]$$

$$+ A_3 V_3 \left[ c_v T_3 + \frac{V_3^2}{2} + \frac{P_3}{\rho} \right]$$

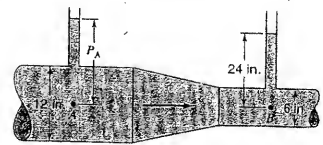
$$= A_2 V_2 \left[ c_v T_2 + \frac{V_2^2}{2} + \frac{P_2}{\rho} \right] \quad (2)$$

Momentum:

$$(P_1 - P_2) A_1 = \rho V_2^2 A_1 - \rho V_1^2 A_1$$

$$- \rho V_3^2 A_3 \quad (3)$$

6.9



$$\dot{V} = 3 \text{ ft}^3/\text{s}$$

Between A & B - Energy Eqn is:

$$\iint_{c.s.} \left( e + \frac{P}{\rho} \right) \rho (\vec{V} \cdot \vec{n}) dA = 0$$

$$\frac{V_B^2 - V_A^2}{2} + u_B - u_A + \frac{P_B - P_A}{\rho} = 0$$

$$V_A = \frac{3}{(\pi/4)(1)^2} = 3.82 \text{ ft/s}$$

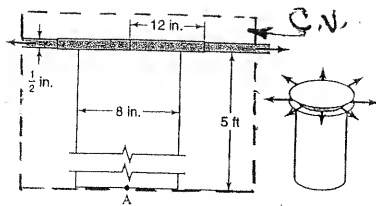
$$V_B = \frac{3}{\pi/4(1/2)^2} = 15.28 \text{ ft/s}$$

$$\frac{P_A - P_B}{\rho g} = \frac{(15.28)^2 - (3.82)^2}{2g} + 0.45$$

$$= 2.15 \text{ ft of } H_2O$$

$$P_A = 2.15 + 2 = 4.15 \text{ ft of } H_2O$$

6.10



$$P_A = 10 \text{ psia}$$

$$\sum F_y = \iint_{C.S.} \rho \mathbf{v} \cdot \mathbf{n} dA$$

Flow rate must be determined

Energy Eqn for C.V. shown:

$$\Delta \rho + \frac{\Delta P}{\rho} + \Delta \frac{v^2}{2} + g \Delta z = 0$$

$$\frac{\Delta P}{\rho} = \frac{10(44)(32.2)}{62.4} = 743 \text{ ft}^2/\text{s}^2$$

$$v_A = \frac{\dot{V}}{\frac{\pi}{4}(8)^2} = 2.865 \dot{V}$$

$$v_B = \frac{\dot{V}}{\pi(2)(0.5/2)} = 3.82 \dot{V}$$

$$\Delta \frac{v^2}{2} = \frac{(2.865^2 - 3.82^2) \dot{V}^2}{2} = -3.19 \dot{V}^2$$

$$g \Delta y = 32.2(-5) = -61 \text{ ft}^2/\text{s}^2$$

$$743 - 3.19 \dot{V}^2 - 61 = 0$$

$$\dot{V}^2 = \frac{682}{3.19} \quad \dot{V} = 14.6 \text{ ft}^3/\text{s}$$

$$v_A = 41.9 \text{ ft/s} \quad v_B = 55.9 \text{ ft/s}$$

$$F_y + P_A A_A - \rho g V = \dot{m}(-v_A)$$

$$P_A A_A = 10 \left( \frac{\pi}{4} \right) (8)^2 = 502 \text{ lbf}$$

$$\rho g V = \frac{(62.4)(32.2) \left( \frac{\pi}{4} \right) (8)^2 (5)}{32.2}$$

$$= 109 \text{ lbf}$$

6.10 - CONTINUED

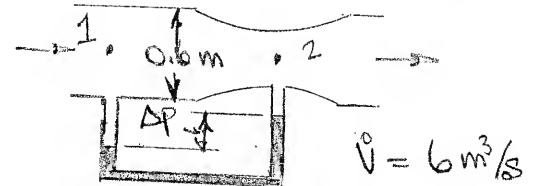
$$\dot{m}(-v_A) = \frac{-62.4(14.6)(41.9)}{32.2} = -1185 \text{ lbf}$$

$$F_y = -502 + 109 - 1185$$

$$= -1578 \text{ lbf}$$

Force on lid is 1578 lbf ↑

6.11



$$P_A = 0.10 \text{ m Alcoh. (S.G. = 0.8)}$$

$$= 0.08 \text{ m H}_2\text{O} = 785 \text{ Pa}$$

$$A_1 = \frac{\pi}{4} (0.6)^2 = 0.283 \text{ m}^2$$

$$v_1 = \frac{\dot{V}}{A_1} = \frac{6}{0.283} = 21.2 \text{ m/s}$$

Energy Eqn. reduces to:

$$\frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g \Delta y = 0$$

$$\frac{P_1 - P_2}{\rho} = \frac{v_2^2 - v_1^2}{2} = \frac{\dot{V}^2}{2} \left[ \frac{1}{A_2^2} - \frac{1}{A_1^2} \right]$$

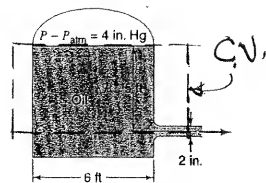
$$= \frac{v_1^2}{2} \left[ \left( \frac{A_1}{A_2} \right)^2 - 1 \right]$$

$$\frac{P_1 - P_2}{\rho} = \frac{785}{1.226} = 640 \text{ m}^2/\text{s}^2$$

$$640 = \frac{(21.2)^2}{2} \left[ \left( \frac{A_1}{A_2} \right)^2 - 1 \right]$$

$$A_2 = 0.144 \text{ m}^2 \quad D_2 = 0.428 \text{ m}$$

6.12



ENERGY EQN. REDUCES TO:

$$\Delta u + \frac{\Delta P}{\rho} + \Delta \frac{V^2}{2} + g\Delta y = 0$$

$$\text{FOR } \Delta u = V_1 = 0$$

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2}{2} + g(y_2 - y_1) = 0$$

$$V_2 = \left[ 2 \left( \frac{P_1 - P_2}{\rho} \right) + g\Delta y \right]^{1/2}$$

BY CONSERVATION OF MASS:

$$A_{TANK} \left( -\frac{dy}{dt} \right) = A_{JET} V_2$$

$$-\frac{A_t}{A_j} \frac{dy}{dt} = \left[ \right]^{1/2}$$

$$-\frac{A_t}{A_j} \int_{y_0}^{y_0-2} \frac{dy}{(K_1 + K_2 y)^{1/2}} = \int_0^t dt$$

$$K_1 = \frac{2\Delta P}{\rho} \quad K_2 = 2g$$

$$t = -\frac{A_t}{A_j} \left[ \frac{2}{K_2} (K_1 + K_2 y)^{1/2} \right]_{y_0}^{y_0-2}$$

$$K_1 = 344 \text{ FT}^2/\text{S}^2$$

$$[K_1 + K_2 (y_0 - 2)]^{1/2} = 23.2 \text{ FT/S}$$

$$[K_1 + K_2 (y_0)]^{1/2} = 25.8 "$$

$$t = 105 \text{ S}$$

6.13. ENERGY EQN. REDUCES TO

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$

$$P_1 = P_{ATM} = 29 \text{ "Hg} \left( \frac{14.7}{29.92} \right) = 14.25 \text{ PSI}$$

$$P_2 = ?$$

$$V_1^2 = \left[ \left( 85 \text{ mi/hr} \right) \left( \frac{5280}{3600} \right) \right]^2 = (124.7)^2$$

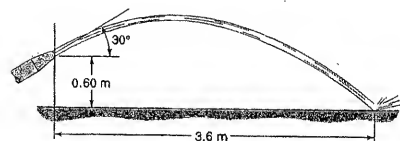
$$V_2^2 = 120^2$$

$$P_2 = 14.25 + \frac{\rho}{2} \left[ (124.7)^2 - (120)^2 \right]$$

$$\rho = P/\text{RT} = \frac{14.25(144)}{53.3(500)} = 0.0770 \text{ lbm/FT}^3$$

$$P_2 = 14.25 + 1.37 = 15.6 \text{ PSI}$$

6.14



$$\text{ENERGY EQN: } \frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) = 0$$

$$\text{IN X-DIRECTION: } V_0 \cos \theta = V_x = \frac{dx}{dt}$$

$$\text{IN Y-DIRECTION: } V_0 \sin \theta - gt = \frac{dy}{dt}$$

$$x = (V_0 \cos \theta) t$$

$$y = (V_0 \sin \theta) t - \frac{gt^2}{2}$$

$$\text{COMBINING: } y = x \tan \theta - \frac{g}{2} \frac{x^2}{(V_0 \cos \theta)^2}$$

6.14 - CONTINUED

$$y = 0.6 \text{ m} \quad \tan \theta = 0.577$$

$$x = 3.6 \text{ m} \quad \cot \theta = 0.866$$

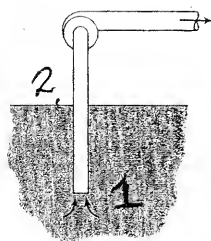
$$0.6 = 3.6(0.577) - \frac{9.81}{2} \frac{3.6^2}{(0.866)^2}$$

$$\underline{v_0 = 7.57 \text{ ft/s}}$$

$$\text{TOTAL HEAD} = 0.6 + \frac{v^2}{2g}$$

$$= \underline{3.52 \text{ m}}$$

6.15



$$\dot{V} = 550 \text{ g/m} = 1.225 \text{ ft}^3/\text{s}$$

$$v = \frac{\dot{V}}{A} = \frac{1.225}{\pi/4 (5.95/12)^2} = 6.35 \text{ ft/s}$$

ENERGY EQN: 2 IS AT H<sub>2</sub>O LEVEL  
OUTSIDE PIPE

$$\frac{P_1 - P_2}{\rho} + \frac{v_1^2 - v_2^2}{2} + g(y_1 - y_2) = 0$$

$$\frac{P_1}{\rho} = -\frac{v_1^2}{2} - g y_1$$

$$= -\frac{(6.35)^2}{2} - 32.2(6)$$

$$= -(20.16 + 193.2)$$

$$= -213.4 \text{ ft}^2/\text{s}^2$$

$$P_1 = -\frac{(62.4)(213.4)}{(144)32.2} = \underline{-2.87 \text{ psi}}$$

40

6.16 WITH REFERENCE TO PROB 16.15

BETWEEN H<sub>2</sub>O SURFACE & PUMP INLET  
ENERGY EQN IS

$$\frac{P_1 - P_2}{\rho g} + \frac{v_1^2 - v_2^2}{2g} + g(y_1 - y_2) = h_L$$

$$\frac{P_{\text{atm}} - P_v}{\rho g} - \frac{v_2^2}{2g} + y_1 - y_2 = h_L$$

$$\frac{v_2^2}{2g} = \frac{P_{\text{atm}} - P_v}{\rho g} - (y_2 - y_1) - h_L$$

$$= \frac{(14.7 - 0.247)(144)(32.2)}{62.4(32.2)} - 4 - 4$$

$$= 25.35 \text{ FT}$$

$$v_2 = [2(32.2)(25.35)]^{1/2} = 40.4 \text{ ft/s}$$

$$\dot{V} = A v_2 = \frac{\pi}{4} \left(\frac{5.95}{12}\right)^2 (40.4) = \underline{7.8 \text{ ft}^3/\text{s}}$$

6.17

FROM PROB 5.27

$$v_r = 10.22 \text{ ft/s}$$

$$v_{\theta} = 82.2 \text{ ft/s}$$

$$v_t = 10.22 \text{ ft/s}$$

$$\text{AT } r_2 - v_x = 82.2 - 10.22$$

$$v_y = 10.22$$

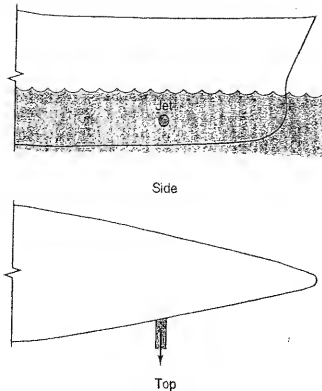
$$v = (v_x^2 + v_y^2)^{1/2} = 82.9 \text{ ft/s}$$

$$\text{HEAD} = \frac{v^2}{2g} = \underline{106.7 \text{ FT}}$$

$$\Delta P = \rho \frac{v^2}{2} = 62.4 (82.9)^2 / 32.2(2)$$

$$= \underline{6660 \text{ lb}_f/\text{ft}^2} = \underline{46.2 \text{ psi}}$$

6.18



FOR THE SITUATION SHOWN -

$$\text{THRUST} = F = \rho \dot{V} L$$

$$\text{POWER} = -\frac{\delta W}{\delta t} = \rho \dot{V} \frac{U^2}{2}$$

$$\frac{\text{POWER}}{\text{THRUST}} \sim \frac{\rho \dot{V} U^2 / 2}{\rho \dot{V} L} \approx U$$

$$\frac{\text{THRUST}}{\text{POWER}} \sim \frac{1}{U} \sim \frac{1}{h^{1/2}}$$

FAVORABLE CHOICE:  $\left\{ \begin{array}{l} \text{HIGH VOLUME} \\ \text{LOW PRESSURE} \end{array} \right.$

6.19 FROM PROB 5.7:

$$P_1 = 50 \text{ PSIA} \quad P_2 = 5 \text{ PSIA}$$

$$D_1 = 12 \text{ IN} \quad D_2 = 2.5 \text{ IN}$$

$$\dot{V} = 3 \text{ FT}^3/\text{S} \quad \text{S.G.} = 0.8$$

$$h_L = \frac{P_1 - P_2}{\rho g} + \frac{U_1^2 - U_2^2}{2g}$$

$$U_1 = \sqrt{\frac{3}{\pi} \left( \frac{1}{12} \right)^2} = 3.82 \text{ FT/S}$$

$$U_2 = \sqrt{\frac{3}{\pi} \left( \frac{2.5}{12} \right)^2} = 88 \text{ FT/S}$$

6.19 - CONTINUED

$$h_L = \frac{(50 - 5) 1.44}{0.8 (62.4)} + \frac{3.82^2 - 88^2}{2 (32.2)}$$

$$= \underline{\underline{9.79 \text{ FT}}}$$

6.20 FOR A C.V. ENCLOSED BY THE FLOW:

$$\Delta u + \frac{\Delta \rho^2}{\rho} + \frac{\Delta U^2}{2} + g \Delta y = 0$$

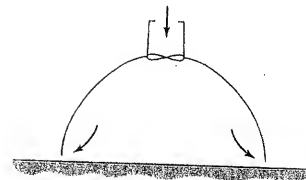
$$\Delta u = g \Delta y = 981 (165) = 1620 \text{ m}^2/\text{s}^2$$

$$= 1620 \text{ m}^2/\text{s}^2 \left( 1 \frac{\text{kg} \cdot \text{m}}{\text{kg} \cdot \text{m}} \right) = 1620 \text{ J/kg}$$

$$\text{FOR H}_2\text{O: } C_p = 4184 \text{ J/kg} \cdot \text{K}$$

$$\Delta T = \frac{\Delta u}{C_p} = \frac{1620}{4184} \approx \underline{\underline{0.39^\circ \text{C}}}$$

6.21



ASSUME VERTICAL FORCES DO NOT INCLUDE MOMENTUM OF INCOMING AIR -

$$(P - P_{\text{ATM}})A = Mg \quad \left\{ \begin{array}{l} \text{PRESSURE} \\ \text{FORCE} \end{array} \right\} = \text{WT}$$

ENERGY EQN BECOMES BERNOULLI EQN BETWEEN INSIDE & EXIT -

$$\frac{P - P_{\text{ATM}}}{\rho} = \frac{U^2}{2}$$

$$\text{OR } U^2 = 2 \frac{Mg}{\rho A}$$

6.21 CONTINUED

$$v^2 = 2 \frac{(8100 \text{ kg})(9.81 \text{ m/s}^2)}{(1205 \text{ kg/m}^3)(27 \text{ m}^2)}$$

$$= 4885 \text{ m}^2/\text{s}^2 \quad v = 69.9 \text{ m/s}$$

$$\dot{V} = 69.9(24)(0.03)$$

$$= 50.3 \text{ m}^3/\text{s}$$

$$\dot{m} = 60.6 \text{ kg/s}$$

ENERGY EQN:

$$-\frac{\delta W_s}{\delta t} = \dot{m} \left( \underbrace{A_1 v_1}_{\delta} + \underbrace{\frac{v_1^2}{2}}_{\delta} + \underbrace{\frac{P_1}{\rho}}_{\delta} + \underbrace{g y_1}_{\delta} \right)$$

$$= \dot{m} \frac{v^2}{2}$$

$$= 60.6 \frac{(69.9)^2}{2} = 148 \text{ kW}$$

6.22 FROM PROB 5.22

$$h_2 = \frac{h_1}{2} \left[ \left( 1 + \frac{8 v_1^2}{g h_1} \right)^{1/2} - 1 \right]$$



APPLIES TO  $\uparrow$

FOR BERNOULLI EQN. TO BE  
VALID -  $h_L = 0$

ENERGY EQN FOR THIS CASE IS

$$h_L = \frac{P_1 - P_2}{\rho g} + \frac{v_1^2 - v_2^2}{2g} + y_1 - y_2$$

$\frac{1}{2}$  SINCE  $P = P_{\text{atm}} + \rho g(h - y)$

6.22 - CONTINUED

$$h_L = \frac{v_1^2 - v_2^2}{2g} + h_1 - h_2$$

WRITING SOLN. TO PROB 5.22 AS

$$h_2 = \frac{h_1}{2} (\sqrt{1+B} - 1) \quad \left\{ B = \frac{8 v_1^2}{g h_1} \right\}$$

$\frac{1}{2}$  NOTE THAT - FOR  $h_2 > h_1$   $B > 8$

BERNOULLI EQN APPLIES FOR  $B = 8$

$\frac{1}{2}$  OBVIOUSLY  $h_L > 0$  FOR  $B > 8$

6.23

ENERGY EQN APPLIES IN FORM:

$$-\frac{\delta W_s}{\delta t} = \dot{m} \frac{P - P_{\text{atm}}}{\rho} = \dot{V} \Delta P$$

$$= \eta_P \eta_m (P_{\text{avail}})$$

$$\left\{ \begin{array}{l} \text{PER} \\ \text{PERSON} \end{array} \right\} \dot{V} = \frac{80}{(7.48)(24)(3600)} = 1.238 \times 10^{-4} \text{ ft}^3/\text{s}$$

$$P = \frac{\dot{V} \Delta P}{\eta_P \eta_m}$$

$$= \frac{(1.238 \times 10^{-4})(60)(144)}{1 \cdot 0.75(0.9)}$$

$$= 1.584 \frac{\text{ft} \cdot \text{lb}_f}{\text{s}}$$

$$= 2.148 \text{ W}$$

PER MONTH -

$$P = 2.148 \text{ W} (30)(24)$$

$$= 1547 \text{ Wh} = \underline{\underline{1.547 \text{ kWh}}}$$

6.24 BERNOULLI EQN

BETWEEN FREE STREAM &  
A REFERENCE POINT (1) ON WALL

$$\frac{P_{ATM}}{\rho g} + \frac{(W+U)^2}{2g} = \frac{P_1}{\rho g} + \frac{(U-W)^2}{2g}$$

$$\frac{P_1 - P_{ATM}}{\rho} = \frac{P_{1g}}{\rho g} = 2WU$$

$$\underline{P_{1g} = 2\rho WU}$$

6.25 ENERGY EQN IS

$$\frac{dQ}{dt} = \iint_{CS} \left( e + \frac{P}{\rho} \right) \rho (\vec{V} \cdot \vec{n}) dA$$

$$\dot{Q} = \dot{m} \left[ (u_2 - u_1) + \frac{P_2 - P_1}{\rho} + \frac{U_2^2 - U_1^2}{2} + g(y_2 - y_1) \right]$$

$$\Delta u = 200 \text{ kJ/kg}$$

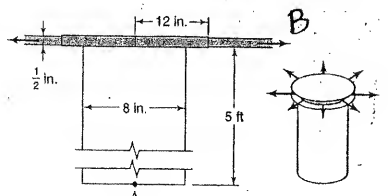
$$\frac{\Delta P}{\rho} = \frac{340 \times 10^3}{1001} = 340 \text{ kJ/kg}$$

$$\frac{\Delta U^2}{2} = 0$$

$$g \Delta y = 9.81 (15) = 0.147 \text{ kJ/kg}$$

$$\dot{Q} = 200 + 340 + 0.15 = 540 \text{ kJ/kg}$$

6.26



$$U = U_A \frac{\pi}{4} \left( \frac{8}{12} \right)^2 = U_B (2\pi) (1) \left( \frac{0.5}{12} \right)$$

6.26 - CONTINUED

$$U_A = 2.865 \text{ V}$$

$$U_A^2 = 8.22 \text{ V}^2$$

$$U_B = 3.82 \text{ V}$$

$$U_B^2 = 14.6 \text{ V}^2$$

FOR NEGLIGIBLE FRICTION -

BERNOULLI EQN APPLIES

$$\frac{P_A - P_B}{\rho} + \frac{U_A^2 - U_B^2}{2} + g(y_2 - y_1) = 0$$

$$\frac{U_B^2 - U_A^2}{2} = \text{V}^2 \frac{14.6 - 8.22}{2} = 3.32 \text{ V}^2$$

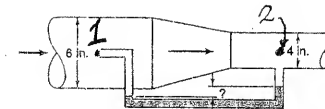
$$\frac{P_A - P_B}{\rho} = \frac{10(144)(32.2)}{62.4} = 743 \text{ FT}^2/\text{S}^2$$

$$g(y_A - y_B) = 32.2(-5) = -161$$

$$3.32 \text{ V}^2 = 743 - 161 = 582$$

$$\underline{V = 13.2 \text{ FT}^3/\text{S}}$$

6.27



BERNOULLI EQN BETWEEN 1 & 2:

$$\frac{P_2 - P_1}{\rho} + \frac{U_2^2 - U_1^2}{2} = 0$$

$$U_1 = \frac{1}{\pi} \left( \frac{1}{2} \right)^2 = 5.09 \text{ FT/S}$$

$$U_2 = \frac{1}{\pi} \left( \frac{4}{12} \right)^2 = 11.5 \text{ FT/S}$$

$$\frac{\Delta P}{\rho} = \frac{-[(11.5)^2 - (5.09)^2]}{2} = -1.65 \text{ FT H}_2\text{O}$$

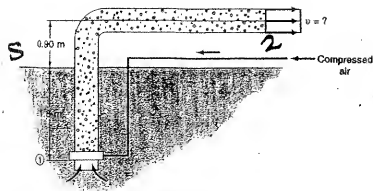
MANOMETER READ -

$$= -1.457 \text{ "Hg}$$

$$h \left[ 1 - \frac{1}{13.6} \right] = 1.457$$

$$\underline{h = 1.63 \text{ INCHES}}$$

6.28



FOR A CONTROL VOLUME BETWEEN 1 & 2  
(IN MIXTURE REGION)

$$\frac{P_2 - P_1}{\rho_m} + \frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) = 0$$

$$P_1 - P_{ATM} = \rho_m g \Delta y_1 \quad (1)$$

MASS BALANCE AROUND MIXING CHAMBER

$$\dot{m}_{AIR} + \dot{m}_W = \dot{m}_m$$

AS GIVEN:  $\rho_m = \rho_w / 2$

$$\therefore V_m = 2V_w + 2 \frac{\rho_w V_w}{\rho_m} \quad (2)$$

CONTROL VOLUME BETWEEN A20  
SURFACE & 1 (H2O ONLY)

$$\frac{P_{ATM} - P_1}{\rho_w} + \frac{0 - V_w^2}{2} + g \Delta y_2 = 0$$

$$P_1 - P_{ATM} = \rho_w g \Delta y_2 - \rho_w \frac{V_w^2}{2} \quad (3)$$

EQUATING (1) & (3):

$$\rho_m g \Delta y_1 = \rho_w \left( g \Delta y_2 - \frac{V_w^2}{2} \right)$$

$$\frac{V_w^2}{2} = g (\Delta y_2 - \Delta y_1 / 2)$$

$$V_w = [2g(0.45m)]^{1/2}$$

6.28 - CONTINUED

SUBSTITUTING EXPRESSION FOR  $V_w$   
INTO (2)

$$V_m = 2 \left[ g(0.9m) \right]^{1/2} + \frac{2(\rho_w V_w)}{\rho_m V_w}$$

SINCE  $\frac{V_w}{V_m} \gg 1$  2ND TERM IS SMALL

$$\therefore V_m = 2 \left[ 9.81(0.9) \right]^{1/2} = \underline{\underline{5.94 \text{ m/s}}}$$

6.29 FOR CONDITIONS OF PROB 6.28

CONTROL VOLUME AROUND MIXING CHAMBER

$$\sum F_y = \iint_{CS} V_y \rho (\vec{V} \cdot \vec{n}) dA$$

$$\Delta P A = \rho_m A V_m \left[ V_m - \frac{\rho_m}{\rho_w} V_w \right]$$

THIS NEGLECTS MOMENTUM OF AIR

FROM PROB 6.28

ABOVE MIXER -  $P = P_{ATM} + \rho_m g \Delta y_1$

BELOW "  $P = P_{ATM} + \rho_w g \Delta y_2 - \rho_w \frac{V_w^2}{2}$

$$\Delta P = \rho_w g (y_2 - y_1) - \rho_w \frac{V_w^2}{2}$$

EQUATING WITH MOMENTUM EXPRESSION

$$\Delta P = \rho_m V_m^2 \left( 1 - \frac{\rho_m}{\rho_w} \right)$$

$$= \rho_w \left[ g(y_2 - y_1) - \frac{V_w^2}{2} \right]$$

$$\frac{V_m^2}{4} = g(y_2 - y_1) - \frac{V_w^2}{2}$$

FOR  $V_w/V_m = 2$

$$\underline{\underline{V_m = 4.6 \text{ m/s}}}$$

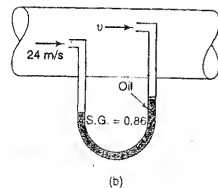
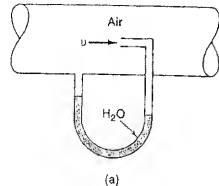
6.29 - CONTINUED

$$\Delta p = \rho_m v_m^2 (1 - 1/2)$$

$$= \frac{1}{2} (4.6)^2 (1000/2)$$

$$\approx \underline{5.3 \text{ kPa}}$$

6.30



IN BOTH CASES - BERNOULLI EQN. IS

$$\frac{v_2^2 - v_1^2}{2} + \frac{p_2 - p_1}{\rho} = 0$$

(a)  $\Delta p = \frac{v^2}{2g} = \frac{15^2}{2(9.81)}$

$$= 11.47 \text{ m Air}$$

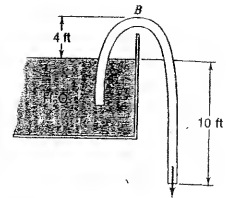
$$= \underline{1.39 \text{ cm H}_2\text{O}}$$

(b)  $\Delta p = \frac{24^2 - 15^2}{2(9.81)}$

$$= 17.9 \text{ m Air}$$

$$= \underline{2.52 \text{ cm Oil}}$$

6.31



BETWEEN LIQUID SURFACE & EXIT:

BERNOULLI EQN:

$$\frac{v^2}{2} = g \Delta y$$

$$v = (2g \Delta y)^{1/2}$$

$$= 2(32.2)(10) = 25.35 \text{ ft/s}$$

$$\dot{V} = \frac{\pi}{4} \left( \frac{1}{12} \right)^2 (25.35) = \underline{1.66 \text{ ft}^3/\text{s}}$$

BETWEEN POINT B & EXIT:

$$\frac{p_B - p_{atm}}{\rho} + g(y_B - y_{exit}) = 0$$

$$p_B = p_{atm} - \rho g \Delta y$$

$$= 14.7 \text{ psi} - \frac{62.4(32.2)(14)}{32.2(144)}$$

$$= \underline{8.63 \text{ psi}}$$

By CONTINUITY -

$$\dot{V}_{TANK} = A_{TANK} \left( -\frac{dy}{dt} \right)$$

$$= A_{PIPE} \sqrt{2g y}$$

$$-\frac{dy}{dt} = \frac{A_{PIPE}}{A_{TANK}} \sqrt{2g y}^{1/2}$$

6.31 CONTINUED

$$-\int_{10}^7 y^{-1/2} dy = \frac{A_p}{A_t} \sqrt{2g} \int_0^t dt$$

$$2y^{1/2} \Big|_7^{10} = \frac{A_p}{A_t} \sqrt{2g} t$$

$$2 \left[ 10^{1/2} - 7^{1/2} \right] = \frac{(1/12)^2}{10^2} \sqrt{2(32.2)} t$$

$$t = 1854 \text{ s} = 0.515 \text{ h}$$

6.32 ENERGY EQN FOR THIS CASE:

$$\frac{U_2^2 - U_1^2}{2} + g(y_2 - y_1) + U_2 - U_1 = 0$$

$$U_1 = 0$$

$$y_2 - y_1 = -10 \text{ FT}$$

$$U_2 - U_1 = 3.2 \frac{U_2^2}{g}$$

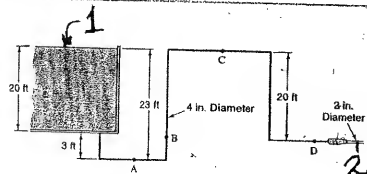
$$\frac{U_2^2}{2g} + (-10) + 3.2 \frac{U_2^2}{g} = 0$$

$$\frac{3.2 U_2^2}{g} = 10$$

$$U = 10.03 \text{ FT/s}$$

$$\dot{V} = \frac{\pi}{4} \left( \frac{1}{12} \right)^2 (10.03) = 0.0547 \text{ FT}^3/\text{s}$$

6.33



BETWEEN 1 & 2

$$\frac{U_2^2 - 0}{2} + g(y_2 - y_1) = 0$$

6.33 CONTINUED

$$U_2 = \left[ 2(32.2)(20) \right]^{1/2} = 35.9 \text{ FT/s}$$

$$\dot{V} = A U = \frac{\pi}{4} \left( \frac{2}{12} \right)^2 (35.9) = 0.783 \text{ FT}^3/\text{s}$$

$$\text{IN 4" LINE} - U = \frac{U_2}{4} = 8.975 \text{ FT/s}$$

$$U^2 = 80.55 \text{ FT}^2/\text{s}^2$$

BETWEEN 1 & A:

$$\frac{P_A - P_1}{\rho} + \frac{U_A^2 - U_1^2}{2} + g(y_A - y_1) = 0$$

$$P_A = P_{\text{ATM}} + \rho \left( -\frac{U_A^2}{2} \right) + \rho g(y_1 - y_A)$$

$$= P_{\text{ATM}} + \frac{62.4}{32.2} \left( -\frac{80.55}{2} \right) + 62.4(23)$$

$$= P_{\text{ATM}} + 1356 \text{ LB/FT}^2 = \underline{3475 \text{ PSF}} \quad (24.12 \text{ PSI})$$

$$U_A = \underline{8.975 \text{ FT/s}}$$

BETWEEN A & B:

$$\frac{P_A - P_B}{\rho} + \frac{U_A^2 - U_B^2}{2} + g(y_A - y_B) = 0$$

$$P_B = P_A + \rho g(-3) = \underline{3290 \text{ PSF}} \quad (22.83 \text{ PSI})$$

$$U_B = \underline{8.975 \text{ FT/s}}$$

CONDITIONS AT D & B ARE EQUAL

$$\therefore P_D = \underline{3290 \text{ PSF}}$$

$$U_D = \underline{8.975 \text{ FT/s}}$$

6.33 CONTINUED

BETWEEN B & C:

$$\frac{P_B - P_C}{\rho} + \frac{V_B^2 - V_C^2}{2} + g(y_B - y_C) = 0$$

$$P_C = P_B + \rho g(y_B - y_C)$$

$$= P_B + 62.4(-20)$$

$$= \underline{2042 \text{ PSF}} \quad (14.18 \text{ PSI})$$

$$\underline{V_C = 8.975 \text{ FT/S}}$$

6.34 BETWEEN WATER LEVEL (1) & EXIT (2)

$$\frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) = 0$$

$$V_2 = \sqrt{2g\Delta y}$$

$$\dot{V} = \frac{\pi}{4} D_{\text{EXIT}}^2 [2gy]^{1/2}$$

H<sub>2</sub>O IN TANK:

$$\dot{V} = \frac{\pi}{4} D_{\text{TANK}}^2 \left(-\frac{dy}{dt}\right)$$

$$\frac{dy}{dt} = \left(\frac{D_{\text{EXIT}}}{D_{\text{TANK}}}\right)^2 \sqrt{2g} y^{1/2}$$

$$\int y^{-1/2} dy = \left(\frac{D_{\text{EXIT}}}{D_{\text{TANK}}}\right)^2 \sqrt{2g} \int dt$$

$$2y^{1/2} \Big|_4^{28} = \left(\frac{D_{\text{EXIT}}}{D_{\text{TANK}}}\right)^2 \sqrt{2g} t$$

6.34 - CONTINUED

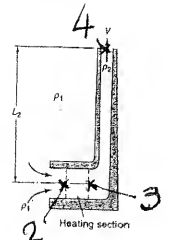
$$t = \frac{2(28^{1/2} - 4^{1/2})}{\left(\frac{2/12}{15}\right)^2 [2(32.2)]^{1/2}}$$

$$= \underline{6644 \text{ S} = 1.846 \text{ HOURS}}$$

6.35

From 1 TO 2

$$\frac{P_1 - P_2}{\rho_1} + \frac{V_1^2 - V_2^2}{2} + g(y_1 - y_2) = 0 \quad 1$$



$$\frac{P_1 - P_2}{\rho_1} + \frac{V_2^2}{2} = 0 \quad (1)$$

From 3 TO 4:

$$\frac{P_3 - P_4}{\rho_2} + \frac{V_3^2 - V_4^2}{2} + g(y_3 - y_4) = 0$$

$$\frac{V_4^2}{2} = \frac{V_3^2}{2} + \frac{P_3 - P_4}{\rho_2} - gL \quad (2)$$

NOTE THAT  $P_4 + \rho_1 gL = P_1$ , GIVING

$$\frac{V_4^2}{2} = \frac{V_3^2}{2} + \frac{P_3 - P_1 + \rho_1 gL}{\rho_2} \quad (3)$$

From 2 TO 3

$$\frac{P_2}{\rho_1} - \frac{P_3}{\rho_2} + \frac{V_2^2 - V_3^2}{2} = 0$$

FOR  $V_2 \approx V_3$  NEGLIGIBLE.

$$P_2 = P_3$$

From (2)

$$\underline{\underline{\frac{V^2}{2} = -gL + \frac{\rho_1}{\rho_2} gL = gL \left(\frac{\rho_1}{\rho_2} - 1\right)}}$$

6.36 From Prob 6.28:

$$\frac{P_1 - P_2}{\rho_1} + \frac{U_2^2}{2} = 0$$

$$\frac{P_3 - P_1}{\rho_2} = \frac{U_2^2}{2} + gL \left( \frac{\rho_2 - \rho_1}{\rho_2} \right) - \frac{U_3^2}{2}$$

CONS. OF MASS:

$$\rho_1 U_2 = \rho_2 U_3 = \rho_2 U/R$$

$$\frac{A_{ATR}}{A_{STK}} = R$$

$$\therefore U_2^2 = \left( \frac{\rho_2}{\rho_1} \right)^2 \left( \frac{U}{R} \right)^2 \quad U_3^2 = \frac{U^2}{R^2}$$

GIVING  $P_2 - P_1 = -\frac{\rho_1}{2} \left( \frac{\rho_2}{\rho_1} \right)^2 \frac{U^2}{R^2}$

$$P_3 - P_1 = \rho_2 \frac{U^2}{2} + gL \left( \frac{\rho_2 - \rho_1}{\rho_2} \right) - \frac{\rho_2 U^2}{2R^2}$$

From MOMENTUM THEOREM -

$$\begin{matrix} U_2 & \left[ \begin{matrix} \text{---} \\ \text{ATR} \end{matrix} \right] & U_3 \\ P_2 & & P_3 \end{matrix}$$

$$F_x = \iint_{CS} U_x \rho (\vec{U} \cdot \vec{n}) dA$$

$$(P_2 - P_3)A = \rho_1 U_2 A (U_3 - U_2)$$

$$P_2 - P_3 = \frac{\rho_1 U^2}{R^2} \left( 1 - \frac{\rho_2}{\rho_1} \right)$$

BERNOULLI EQN:

$$\frac{U^2}{R^2} \left( 1 - \frac{\rho_2}{\rho_1} \right) + \frac{\rho_1}{2\rho_2} \left( \frac{\rho_2}{\rho_1} \right)^2 \frac{U^2}{R^2} + \frac{U^2}{2} + gL \left( 1 - \frac{\rho_1}{\rho_2} \right) - \frac{U^2}{2R^2} = 0$$

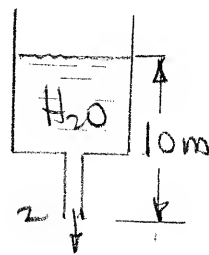
6.36 CONTINUED

DOING THE ALGEBRA:

$$U_2 = \frac{2gL \left( \frac{\rho_1}{\rho_2} - 1 \right)}{1 + \frac{1 - \rho_2/\rho_1}{R^2}}$$

6.37

FRICTIONLESS FLOW:



From BERNOULLI

$$\frac{P_2 - P_1}{\rho g} + \frac{U_2^2 - U_1^2}{2g} + y_2 - y_1 = 0$$

$$U_2^2 = 2g(y_1 - y_2)$$

$$U = [2(9.81)(10)]^{1/2} = 14 \text{ m/s}$$

$$\dot{m} = (1000) \left( \frac{\pi}{4} \right) (0.04)^2 (14) = 17.6 \text{ kg/s}$$

WITH NOZZLE -  $U = 14 \text{ m/s}$  {still}

$$\dot{m} = (1000) \left( \frac{\pi}{4} \right) (0.01)^2 (14) = 1.10 \text{ kg/s}$$

WITH  $U_2 - U_1 = 3U^2$

ENERGY EQN Reduces to

$$\frac{U_2^2}{2g} + \frac{3U^2}{g} = 2(y_1 - y_2)$$

$$U_2 = \left[ \frac{4}{7} (9.81) 10 \right]^{1/2} = 7.49 \text{ m/s}$$

PIPE:  $\dot{m} = 9.42 \text{ kg/s}$

NOZZLE:  $\dot{m} = 0.589 \text{ "}$

6.38 SAME TANK AS IN  
PROB 6.37 BUT 2 EXIT  
PIPES —

PIPE 1 :  $D = 0.04 \text{ m}$   
 $\Delta y = 10 \text{ m}$

PIPE 2  $D = 0.04 \text{ m}$   
 $\Delta y = 20 \text{ m}$

FRictionless FLOW:

PIPE 1 —

AS IN PROB 6.37

$$U = \sqrt{2g\Delta y} = 14 \text{ m/s}$$

$$\dot{m} = \underline{\underline{17.6 \text{ kg/s}}}$$

PIPE 2: ALSO —  $U = \sqrt{2g\Delta y}$

$$U = [2(9.81)(20)]^{1/2}$$

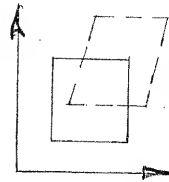
$$= \underline{\underline{19.81 \text{ kg/s}}}$$

$$\dot{m} = (1000) \left( \frac{\pi}{4} \right) (0.04)^2 (19.81)$$

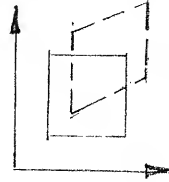
$$= \underline{\underline{24.9 \text{ kg/s}}}$$

# CHAPTER 7

7.1

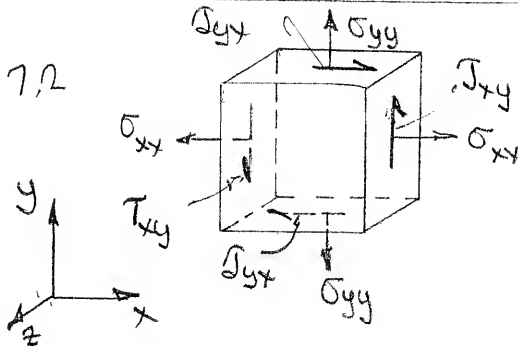


$$(a) \frac{\partial v_x}{\partial y} \gg \frac{\partial v_y}{\partial x}$$



$$(b) \frac{\partial v_y}{\partial x} \gg \frac{\partial v_x}{\partial y}$$

7.2



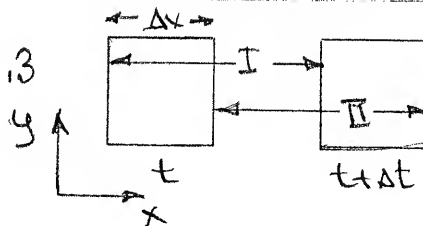
for 2-D flow - in x, y

$$v_z = 0 \quad \sigma_{zz} = 0$$

$$\frac{\partial v_z}{\partial x} = 0 \quad \tau_{zx} = \tau_{xz} = 0$$

$$\frac{\partial v_z}{\partial y} = 0 \quad \tau_{zy} = \tau_{yz} = 0$$

7.3



$$I = v_x(x) \Delta t$$

$$II = v_x(x + \Delta x) \Delta t$$

## 7.3 - CONTINUED

AXIAL STRAIN RATE:

$$= \lim_{\Delta x \rightarrow 0, \Delta t \rightarrow 0} \frac{v_x(x + \Delta x) \Delta t - v_x(x) \Delta t}{\Delta x \Delta t} = \frac{\partial v_x}{\partial x}$$

RATE OF VOLUME CHANGE:

$$= \lim_{\Delta x \rightarrow 0, \Delta t \rightarrow 0} \frac{A \Delta x|_{t+\Delta t} - A \Delta x|_t}{A \Delta x \Delta t}$$

$$= \lim_{\Delta x \rightarrow 0, \Delta t \rightarrow 0} \frac{v_x(x + \Delta x) \Delta t - v_x(x) \Delta t}{\Delta x \Delta t} = \frac{\partial v_x}{\partial x}$$

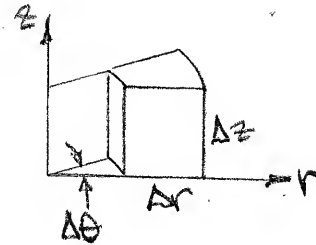
IN 3 DIMENSIONS

BOTH AXIAL STRAIN RATE AND

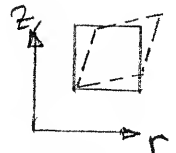
VOLUME CHANGE RATE ARE GIVEN BY

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

7.4



IN r-z PLANE:

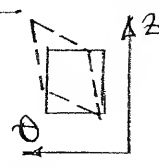


$$\begin{aligned} -\frac{d\delta}{dt} &= -\lim_{\Delta t \rightarrow 0} \frac{\delta|_{t+\Delta t} - \delta|_t}{\Delta t} \\ &= -\lim_{\Delta t \rightarrow 0} \left[ \frac{\pi/2 - \tan^{-1}\left(\frac{v_r|_{z+\Delta z} - v_r|_z}{\Delta z}\right) \Delta t}{\Delta t} \right. \\ &\quad \left. - \tan^{-1} \frac{v_z|_{r+\Delta r} - v_z|_r \Delta t}{\Delta r} + \pi/2 \right] \end{aligned}$$

# 7.4 - CONTINUED

$$= \lim_{\substack{\Delta z \rightarrow 0 \\ \Delta r \rightarrow 0}} \left[ \frac{U_r|_{z+\Delta z} - U_r|_z}{\Delta z} + \frac{U_z|_{r+\Delta r} - U_z|_r}{\Delta r} \right]$$

$$\therefore \tau_{rz} = \tau_{zr} = \mu \left[ \frac{\partial U_r}{\partial z} + \frac{\partial U_z}{\partial r} \right]$$

IN THE  $\theta$ - $z$  PLANE — 

} SAME PROCEDURE

$$\lim_{\substack{\Delta z \rightarrow 0 \\ \Delta \theta \rightarrow 0}} \left[ \frac{U_\theta|_{z+\Delta z} - U_\theta|_z}{\Delta z} + \frac{1}{r} \frac{U_z|_{\theta+\Delta \theta} - U_z|_\theta}{\Delta \theta} \right]$$

$$\tau_{\theta z} = \tau_{z\theta} = \mu \left[ \frac{\partial U_\theta}{\partial z} + \frac{1}{r} \frac{\partial U_z}{\partial \theta} \right]$$

$\frac{1}{r}$  IN  $r$ - $\theta$  PLANE



$$\lim_{\substack{\Delta r \rightarrow 0 \\ \Delta \theta \rightarrow 0}} \left[ \frac{U_r|_{\theta+\Delta \theta} - U_r|_\theta}{r \Delta \theta} + r \left( \frac{U_\theta|_{r+\Delta r} - U_\theta|_r}{\Delta r} \right) \right]$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu \left[ \frac{1}{r} \frac{\partial U_r}{\partial \theta} + r \frac{\partial}{\partial r} \left( \frac{U_\theta}{r} \right) \right]$$

# 7.5 NITROGEN 175 K

$$\mu = 2.6693 \times 10^{-6} \frac{\sqrt{MT}}{\sigma^2 \Omega_\mu}$$

$$T = 175 \text{ K} \quad \sigma = 3.681 \text{ \AA}$$

$$M = 28 \quad \Omega_\mu = 1.1942$$

$$\epsilon_A / K = 91.5$$

$$KT / \epsilon = 1.91$$

$$\mu \approx 11.55 \times 10^{-6} \text{ Pa.s.}$$

# 7.6 OXYGEN @ 350 K

$$\text{EQU. 7.10} \quad \mu = 2.6693 \times 10^{-6} \frac{\sqrt{MT}}{\sigma^2 \Omega_\mu}$$

$$\frac{KT}{\epsilon} = \frac{T}{\epsilon / K} = \frac{350}{113} = 3.097$$

$$\text{YIELDING } \Omega_\mu = 1.03$$

$$M = 32 \quad \sigma = 3.433$$

$$\mu = 2.327 \times 10^{-5} \text{ Pa.s.}$$

$$\text{TABLE VALUE: } \mu = 2.318 \text{ Pa.s.}$$

# 7.7 For H<sub>2</sub>O

$$\mu|_{60^\circ} = 0.76 \times 10^{-3} \text{ kgm/s.ft.}$$

$$\mu|_{120^\circ} = 0.375 \times 10^{-3} \text{ "}$$

$$\text{PERCENT CHANGE} = \frac{0.76 - 0.375}{0.76}$$

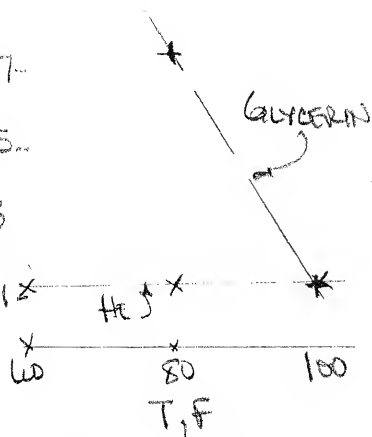
$$= 0.51 \text{ OR } 51\%$$

7.8 PROPERTIES OF HELIUM,  
GLYCERIN FROM APPENDIX

T, F	$\nu$ , He	$\nu$ , GLYCERIN
60	0.00125	0.017
80	0.00132	0.00762
100	0.00141	0.00128

$\nu$  ft<sup>2</sup>/s

0.007  
0.005  
0.003  
0.001



INTERSECTION IS VERY  
CLOSE TO 100 F

7.9 FOR H<sub>2</sub>O  $\dot{V} \sim 1/\mu$

@ 120 F  $\mu_w = 0.391 \times 10^{-3}$  lbm/s·ft  
@ 32 F  $= 1.2 \times 10^{-3}$  "

$$\frac{\dot{V}_{140}}{\dot{V}_{32}} = \frac{1.2 \times 10^{-3}}{0.391 \times 10^{-3}} = 3.07$$

PER CENT CHANGE

$$= \frac{1.2 - 0.391}{0.391}$$

$$= 3.07 - 1 = 2.07$$

OR 207%

7.10 FOR AIR:

@ 140 F  $\mu = 1.34 \times 10^{-5}$  lbm/s·ft  
32 F  $\mu = 1.15 \times 10^{-5}$  "

FOR  $\dot{V} \sim 1/\mu$

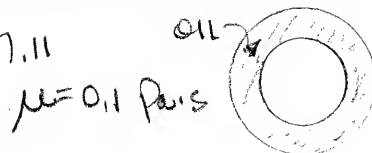
$$\frac{\dot{V}_{140}}{\dot{V}_{32}} = \frac{1.15}{1.34} = 0.852$$

PER CENT CHANGE =  $\frac{1.15 - 1.34}{1.34}$

$$= 0.852 - 1 = -0.148$$

= -14.8%

7.11



$R_2 = 3.175$  cm

$R_1 = 3.183$  "

1<sup>ST</sup> LAW:  $\frac{\partial Q}{\partial t} - \frac{\partial W}{\partial t} - \frac{\partial W_u}{\partial t} = 0$

$\dot{Q} = \dot{W}_{\text{VISCOUS}}$

{ NO FLOW IN OR OUT }

=  $\tau(A)U$  - AT MOVING BOUNDARY

$\tau_i = \mu \frac{\partial v}{\partial r} \approx \mu \frac{rw}{t}$  { t = GAP WIDTH }

$$\dot{Q} = \left( \mu \frac{rw}{t} \right) (\pi DL) (rw)$$

$$= \frac{\mu (rw)^2 \pi DL}{t}$$

$\omega = 1700 \left( \frac{2\pi}{60} \right) = 178$  RAD/s

$$\dot{Q} = \frac{(0.01) \left( \frac{0.03175}{2} \right)^2 (178)^2 (\pi (0.03175) (0.028))}{4 \times 10^{-5}}$$

= 5.58 W

7.12 REFER TO PROB. 7.13

FOR  $\omega_2 = 2\omega_1$

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(\omega_2)^2}{\omega_1^2} = 4$$

PERCENT INCREASE

$$= \frac{\dot{Q}_2 - \dot{Q}_1}{\dot{Q}_1} = 4 - 1 = 3$$

$$= \underline{\underline{300\%}}$$

7.13 SHIP 1  $\rightarrow V_1 = 4 \text{ m/s}$

SHIP 2  $\rightarrow V_2 = 3.1 \text{ m/s}$

CHOOSE CONTROL VOLUME  
ATTACHED TO SHIP 1

$$\begin{aligned}\sum F_x &= \iint_{CS} V_x \rho (\vec{V} \cdot \vec{n}) dA \\ &= \dot{m} V_{x1} - \dot{m} V_{x2}\end{aligned}$$

RELATIVE TO MOVING SHIP

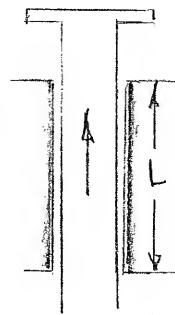
$$V_{x1} = 0 \quad V_{x2} = -0.9 \text{ m/s}$$

$$\begin{aligned}F_x &= +\dot{m} V_{x2} \\ &= 100 \text{ kg/s} (0.9 \text{ m/s}) \\ &= 90 \text{ N}\end{aligned}$$

THIS IS FORCE APPLIED TO  
MAINTAIN STATED CONDITIONS

FORCE EXERTED BY FLOW  
TRANSFER  $= \underline{\underline{-90 \text{ N}}}$

7.14



$$\begin{aligned}t = \text{GAP} &= \frac{D_{\text{OUTSIDE}} - D_{\text{INSIDE}}}{2} \\ &= \frac{36.04 - 36.02}{2} = 0.01 \text{ cm}\end{aligned}$$

$$F = \tau A = \tau \pi D L$$

$$\tau = \mu \frac{du}{dy} = \mu \frac{\Delta u}{\Delta y} \quad \left\{ \begin{array}{l} \text{ASSUMES} \\ \text{LINEAR} \\ \text{PROFILE} \end{array} \right\}$$

$$\tau = \mu \frac{V}{t}$$

$$\begin{aligned}F &= \frac{\mu V}{t} \pi D L = \rho \nu \frac{V \pi D L}{t} \\ &= \frac{[0.85(1000)(3.7 \times 10^{-4})(0.15) \times \\ &\quad \times \pi (0.3602)(3.14)]}{1 \times 10^{-4}}\end{aligned}$$

$$= \underline{\underline{1676 \text{ N}}}$$

7.15 REFER TO CONDITIONS OF PROB 7.14

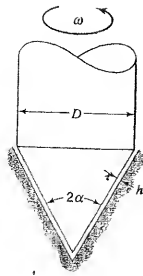
LOAD ON RAM = 680 kg,  $L = 2.44 \text{ m}$

$$F = mg = \frac{\rho \nu V \pi D L}{t}$$

$$V = \frac{mgt}{\rho \nu \pi D L}$$

$$\begin{aligned}&= \frac{680(9.81)(1 \times 10^{-4})}{0.85(1000)(3.7 \times 10^{-4})\pi(0.3602)(2.44)} \\ &= \underline{\underline{0.768 \text{ m/s}}}\end{aligned}$$

7.16



$$M = \int r df$$

$$df = \tau dA$$

$dA$  IS ON THE CONICAL SURFACE

$$= 2\pi r dL$$

{  $dL$  IS ALONG SLANTED SURF }

$$dL = dr / \sin \alpha$$

SO:  $df = \tau dA$

$$= \mu \frac{r\omega}{h} 2\pi r \frac{dr}{\sin \alpha}$$

$$\int_0^M dm = r df$$

$$= \frac{2\pi \mu \omega}{h \sin \alpha} \int_0^{D/2} r^3 dr$$

$$M = \frac{\pi \mu \omega D^4}{32 h \sin \alpha}$$

7.17  $u_x = u_{max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$

$$= 2u_{avg} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

$$\tau = \mu \left. \frac{du}{dr} \right|_{r=R}$$

$$\frac{du}{dr} = 2u_{avg} \left[ -\frac{2r}{R^2} \right]$$

7.17 - CONTINUED

At  $r=R$   $\frac{du}{dr} = -4 \frac{u_{avg}}{R}$

$$\tau = -\frac{4\mu u_{avg}}{R}$$

$\mu_w = 0.76 \times 10^{-3} \text{ lbm/s} \cdot \text{ft} @ 60^\circ \text{F}$

$$\tau = \frac{-4(0.76 \times 10^{-3})(2)}{(0.05/12)(32.2)}$$

$$= -0.0453 \text{ lbf/ft}^2$$

7.17 FOR CONDITIONS OF PROB 7.16

$$\tau = -\frac{4\mu u_{avg}}{R}$$

$$F = \tau A = \tau \pi D L$$

$$= -\frac{4\mu u_{avg}}{R} (\pi D L)$$

$$= (-0.0453)(\pi)(\frac{0.1}{2})(1)$$

$$= \underline{0.00119 \text{ lbf}}$$

$$AP = \frac{F}{\pi D^2/4}$$

$$= 0.00119 \text{ lbf}$$

$$\pi (0.1/2)^2/4$$

$$= \underline{21.75 \text{ PSF}}$$

7.19 SHEAR WORK RATE =  $\dot{S}U$

$$\dot{S}U = \mu U \frac{dU}{dr}$$

For PARABOLIC PROFILE -

$$U = U_{\max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

$$\dot{S}U = \mu U_{\max}^2 \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \left[ - \frac{2r}{R^2} \right]$$

$$= \mu U_{\max}^2 \left[ - \frac{2r}{R^2} + 2 \frac{r^3}{R^4} \right]$$

$$\frac{d}{dr}(\dot{S}U) = \mu U_{\max}^2 \left[ - \frac{2}{R^2} + 6 \frac{r^2}{R^4} \right]$$

$$\text{For } \frac{d}{dr}(\dot{S}U) = 0$$

$$6 \frac{r^2}{R^4} = \frac{2}{R^2}$$

$$\underline{\underline{\frac{r}{R} = \frac{1}{\sqrt{3}}}}}$$

## CHAPTER 8

### 8.1 HAGEN-POISEUILLE EQN.

$$\frac{dP}{dx} = \frac{32\mu V_{avg}}{D^2}$$

$$= \frac{32\mu \dot{V}}{D^2 A} = \frac{32\mu \dot{V}}{\pi/4 D^4}$$

$$\text{For } D = D_0 \quad \dot{V}_0 = \left[ \left( -\frac{dP}{dx} \right) \frac{\pi/4}{32\mu} \right] D_0^4$$

$$\text{For } D_1 = 2D_0 \quad \dot{V}_1 = [ ] (2D_0)^4$$

$$\Rightarrow \dot{V}_1 = 16 \dot{V}_0$$

PER CENT CHANGE

$$= \frac{\dot{V}_1 - \dot{V}_0}{\dot{V}_0} = \frac{\dot{V}_1}{\dot{V}_0} - 1 = 15$$

$$= \underline{1500\%}$$

### 8.2 FOR SINGLE PIPE:

$$\Delta P_0 = \left[ \frac{32\mu}{D^2 A} \right] \dot{V}_0 (40)$$

FOR SINGLE-PARALLEL COMBINATION

$$\Delta P_1 = [ ] \dot{V}_1 (22) \quad \left\{ \begin{array}{l} \text{SINGLE} \\ \text{BRANCH} \end{array} \right\}$$

$$\Delta P_2 = [ ] \dot{V}_2 (18) \quad \left\{ \begin{array}{l} \text{PARALLEL} \\ \text{BRANCH} \end{array} \right\}$$

$$\Delta P_1 + \Delta P_2 = \Delta P_0 = 3.45 \times 10^6 \text{ Pa}$$

$$\dot{V}_1 = 2 \dot{V}_2$$

$$\text{CASE 2: } \Delta P_1 + \Delta P_2 = [ ] (22+9) \dot{V}_1$$

$$\dot{V}_1 = \frac{40}{31} \dot{V}_0 = \frac{40}{31} (4000)$$

$$= \underline{5161 \text{ BBL/DAY}}$$

8.3 Pres  $D = 0.635 \text{ cm}$   
 $= 207 \text{ kPa}$   $\xrightarrow{8 \text{ m}}$

FOR 1 - RESERVOIR

2 - PIPE ENTRANCE

3 - " EXIT

BETWEEN 1 & 2:  $\frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(y_1 - y_2) = 0$

$$\frac{P_1}{\rho} = \frac{P_2}{\rho} + \frac{V^2}{2}$$

BETWEEN 2 & 3:

$$\frac{P_2 - P_3}{\rho} + \frac{V_2^2 - V_3^2}{2} + g(y_2 - y_3) - \Delta u = 0$$

$$\frac{P_2}{\rho} = \frac{P_{ATM}}{\rho} + \Delta u$$

FOR INVISCID FLOW -  $\Delta u = 0$

FOR LAMINAR, VISCOUS FLOW

$$\Delta u = \frac{\Delta P}{\rho} \Big|_{\text{FRICTION}} = \frac{32\mu}{\rho D^2} V$$

INVISCID CASE:

$$\frac{V^2}{2} = \frac{P_1 - P_{ATM}}{\rho} = \frac{P_{IG}}{\rho}$$

ASSUMING FLUID IS HYDRAULIC FLUID

@ 60 F - 15.9 K

$$\rho = 849 \text{ kg/m}^3 \quad \mu = 0.0165 \text{ Pa.s}$$

$$V = \left[ \frac{2(207000)}{849} \right]^{1/2} = 22.08 \text{ m/s}$$

$$\dot{V} = AV = \frac{\pi}{4} (0.00635^2) (22.08)$$

$$\approx \underline{7 \times 10^{-4} \text{ m}^3/\text{s}}$$

### 8.3 - CONTINUED

VISCOUS CASE:

$$\frac{P_1 - P_{ATM}}{\rho} = \frac{V^2}{2} + \Delta u$$

$$\frac{P_{1g}}{\rho} = \frac{V^2}{2} + \frac{32\mu V}{\rho D^2}$$

$$V^2 + \frac{64\mu V}{\rho D^2} - 2 \frac{P_{1g}}{\rho} = 0$$

$$\frac{64\mu}{\rho D^2} = \frac{64(0.0165)}{849(0.00635)^2}$$

$$= 30.85 \text{ m}^2/\text{s}^2$$

$$2 \frac{P_{1g}}{\rho} = 487.6$$

$$V^2 + 30.85 V - 487.6 = 0$$

$$V = \frac{-30.85 \pm \left[ (30.85)^2 + 4(487.6) \right]^{1/2}}{2}$$

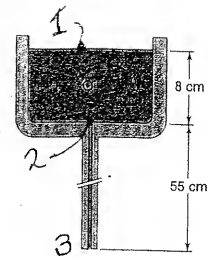
$$= 11.51 \text{ m/s}$$

$$\dot{V} = \frac{\pi}{4} (0.00635)^2 (11.51)$$

$$= \underline{\underline{3.645 \times 10^{-4} \text{ m}^3/\text{s}}}$$

$$\frac{\dot{V}_{\text{INVISCID}}}{\dot{V}_{\text{VISCOUS}}} = \frac{7}{3.645} = \underline{\underline{1.92}}$$

### 8.4



FROM 1 TO 2 (BERNOULLI)

$$\frac{P_2 - P_{ATM}}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(y_2 - y_1) = 0$$

$$\frac{V_2^2}{2} = \frac{P_{ATM} - P_2}{\rho} + g(y_1 - y_2)$$

FROM 2 TO 3

$$\frac{P_2 - P_{ATM}}{\rho} + \frac{V_2^2 - V_3^2}{2} + g(y_2 - y_3) + \Delta u = 0$$

$$\Delta u = \frac{32\mu V L}{\rho D^2} = \frac{32 V L}{D^2} \nu$$

COMBINING EXPRESSIONS:

$$\frac{32 V L}{D^2} \nu = g(y_1 - y_3) - \frac{V^2}{2}$$

$$V = \frac{4 \dot{V}}{\pi D^2}$$

$$\nu = g A y \frac{\pi D^4}{128 L \dot{V}} - \frac{\dot{V}}{16 \pi L}$$

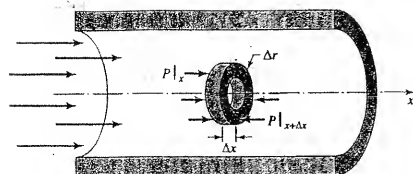
$$\frac{9.81(0.63) \pi (0.0018)^4}{128(0.155)(0.273 \times 10^{-6})} = 1.0605 \times 10^{-5}$$

$$\frac{0.273 \times 10^{-6}}{16(\pi)(0.155)} = 0.000987 \times 10^{-5}$$

$$\nu = (1.0605 - 0.001) \times 10^{-5}$$

$$= \underline{\underline{1.0595 \times 10^{-5} \text{ m}^2/\text{s}}}$$

8.5



USING THE SAME DEVELOPMENT AS IN SECTION 8.1:

$$\frac{d}{dr}(r\tau) = r \frac{\Delta P}{\Delta x}$$

FOR AN ELEMENT OF LENGTH, L

$$\frac{d}{dr}(r\tau) \int_0^L \Delta x = r \int_{P_1}^{P_2} \Delta P$$

WHICH BECOMES

$$\frac{d}{dr}(r\tau) = r \frac{\Delta P}{L}$$

INTEGRATING:

$$r\tau = \frac{\Delta P}{L} \frac{r^2}{2} + C_1$$

$$\tau = \frac{\Delta P}{L} \frac{r}{2} + C_1/r$$

FOR LAMINAR FLOW, NEWTONIAN

$$\tau = \mu \frac{dv}{dr}$$

$$\text{SO } \mu \frac{dv}{dr} = \frac{\Delta P}{L} \frac{r}{2} + \frac{C_1}{r}$$

$$dv = \frac{\Delta P}{2\mu L} r dr + \frac{C_1}{\mu} \frac{dr}{r}$$

INTEGRATING:

$$v = \frac{\Delta P}{4\mu L} r^2 + \frac{C_1}{\mu} \ln r + C_2$$

BOUNDARY CONDITIONS:

$$v(r=R) = 0$$

$$v(r=kR) = 0 \quad k < 1$$

8.5 - CONTINUED -

CONSIDERABLE ALGEBRA YIELDS

$$C_1 = -\frac{\Delta P}{4\mu L} \frac{R^2(1-k^2)}{\ln 1/k}$$

$$C_2 = -\frac{\Delta P}{4\mu L} R^2 \left[ 1 - (1-k^2) \frac{\ln R}{\ln 1/k} \right]$$

WITH SUBSTITUTION & SIMPLIFICATION:

$$v = -\frac{\Delta P R^2}{4\mu L} \left[ 1 - \frac{r^2}{R^2} - \frac{1-k^2}{\ln 1/k} \frac{\ln R}{r} \right]$$

8.6 THIS IS SAME CONFIGURATION AS SHOWN IN PROB 8.5

$$\therefore \frac{d}{dr}(r\tau) - \frac{\Delta P}{\Delta x} r = 0$$

$$\text{INTEGRATING: } r\tau - \frac{\Delta P}{\Delta x} \frac{r^2}{2} = \frac{C_1}{r}$$

FOR LAMINAR FLOW, NEWTONIAN FLUID:

$$\tau = \mu \frac{dv}{dr}$$

$$\frac{dv}{dr} - \frac{r}{2\mu} \frac{\Delta P}{\Delta x} = \frac{C_1}{\mu r}$$

INTEGRATING:

$$v - \frac{1}{4\mu} \frac{\Delta P}{\Delta x} r^2 = \frac{C_1}{\mu} \ln r + C_2$$

BOUNDARY CONDITIONS:

$$v(r=R/2) = 0$$

$$v(r=d/2) = v$$

MORE ALGEBRA:

$$C_1 = -\frac{\mu}{\ln d/2} \left[ v + \frac{1}{16\mu} \frac{\Delta P}{\Delta x} (d^2 - d^2) \right]$$

8.6 - CONTINUED

$$C_2 = -\frac{1}{4\mu} \frac{dP}{dx} \frac{D^2}{4} - \frac{C_1}{\mu} \ln \frac{D}{2}$$

DRAW FORCE PER UNIT LENGTH

$$F = \tau A = \tau (\pi d)(1) \\ = \mu \left. \frac{d\tau}{dr} \right|_{r=d/2} (\pi d)$$

GIVEN:

$$F = \pi d \mu \left[ \frac{C_1}{\mu r} + \frac{r}{2\mu} \frac{dP}{dx} \right]_{r=d/2} \\ = \pi d \mu \left[ \frac{2C_1}{\mu d} + \frac{d}{4\mu} \frac{dP}{dx} \right]$$

FOR THE CASE WITH  $\frac{dP}{dx} = 0$

$$F = -\frac{2\pi\mu V}{\ln D/d}$$

8.7



IN  $\theta$ -DIRECTION:

$$\sum F_\theta = -r \Delta\theta \Delta z \tau_{re} \Big|_r + r \Delta\theta \Delta z \tau_{ro} \Big|_{r+dr} \\ + \underbrace{\Delta r \Delta z \tau_{\theta r} \Delta\theta}_{\theta \text{ component of force on } (+\theta) \text{ face}}$$

DIVIDE BY  $r \Delta\theta \Delta z$  & TAKE LIMIT AS  $\Delta r \rightarrow 0$ :

$$\frac{d}{dr} (r \tau_{re}) + \tau_{\theta r} = 0 \\ r \frac{d\tau}{dr} + 2\tau = 0$$

8.7 - CONTINUED

$$\frac{d\tau}{\tau} + 2 \frac{dr}{r} = 0$$

$$\ln \tau + 2 \ln r = \ln(\text{CONSTANT})$$

$$\underline{r^2 \tau = \text{CONSTANT}}$$

$$\tau = \mu r \frac{d}{dr} \left( \frac{V_\theta}{r} \right)$$

$$r^2 \tau = \mu r^3 \frac{d}{dr} \left( \frac{V_\theta}{r} \right) = \text{CONSTANT (c)}$$

$$d\left(\frac{V_\theta}{r}\right) = \frac{c}{\mu} \frac{dr}{r^3}$$

$$\frac{V_\theta}{r} = c_1 \left( -\frac{1}{r^2} \right) + c_2$$

$$V_\theta = -\frac{c_1}{r} + r c_2$$

BOUNDARY CONDITIONS:

$$V_\theta(R) = 0 \quad \rightarrow \quad 0 = \frac{c_1}{R} + R c_2$$

$$V_\theta(KR) = V \quad \rightarrow \quad V = -\frac{c_1}{KR} + K R c_2$$

ALGEBRA

$$\underline{V_\theta = \frac{VR}{K-1/K} \left( \frac{r}{R^2} - \frac{1}{r} \right)}$$

IF PROFILE IS LINEAR:

$$V_\theta = ar + b$$

$$\underline{V_\theta = \frac{V}{K-1} \left( \frac{r}{R} - 1 \right)}$$

# 8.7 CONTINUED

$$\text{PERCENT ERROR} = \frac{\Delta U}{U}$$

$$\Delta U = U_{\text{ACTUAL}} - U_{\text{LINEAR}}$$

$$= \frac{VRk}{k^2-1} \left( \frac{r}{R^2} - \frac{1}{r} \right) - \frac{V}{R(k-1)} (r-R)$$

$$\begin{aligned} \frac{d}{dr} \Delta U &= \frac{VRk}{k^2-1} \left( \frac{1}{R^2} + \frac{1}{r^2} \right) - \frac{V}{R(k-1)} \\ &= \frac{V}{k-1} \left[ \frac{Rk}{k+1} \left( \frac{1}{R^2} + \frac{1}{r^2} \right) - \frac{1}{R} \right] \\ &= 0 \end{aligned}$$

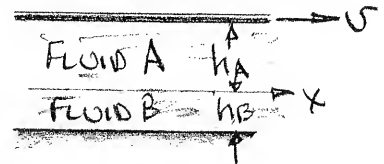
$$\Delta U_{\text{MAX}} \text{ OCCURS AT } \frac{r}{R} = \sqrt{k}$$

$$\begin{aligned} \left. \frac{\Delta U}{V} \right|_{\text{MAX}} &= 1 - \frac{(\sqrt{k}-1)(k+1)\sqrt{k}}{k(k-1)} \\ &= 0.01 \end{aligned}$$

RESULTING IN

$$\begin{aligned} \frac{(\sqrt{k}-1)(k+1)}{\sqrt{k}(k-1)} &= 0.99 \\ \} \\ \underline{\underline{k = 0.96}} \end{aligned}$$

# 8.8 FOR FLOW BETWEEN 2 HORIZONTAL PLATES -



GOVERNING D.E. -

$$\frac{d}{dy} (\tau_{yx}) - \frac{dp}{dx} = 0$$

LAMINAR, STEADY NEWTONIAN

$$\tau_{yx} = \mu \frac{du_x}{dy}$$

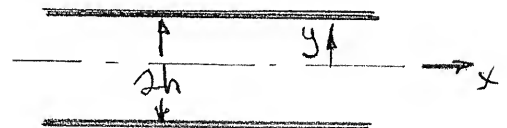
B.C. - AT INTERFACE (y=0)

$$(1) \quad u_{xA} = u_{xB}$$

$$(2) \quad \tau_{yxA} = \tau_{yxB}$$

$$(3) \quad u_x(-h_B) = u_x(h_A) = 0$$

# 8.9



FULLY DEVELOPED, STEADY, LAMINAR FLOW; NEWTONIAN FLUID -

$$\frac{d}{dy} \tau_{yx} - \frac{dp}{dx} = 0$$

$$\tau_{yx} = \mu \frac{du_x}{dy}$$

B.C.  $u_x = 0$  for  $y = \pm h$

$$\mu \frac{d^2 u}{dy^2} = \frac{dp}{dx}$$

$$\frac{du}{dy} = \frac{1}{\mu} \frac{dp}{dx} y + C_1$$

$$u = \frac{1}{2\mu} \frac{dp}{dx} y^2 + C_1 y + C_2$$

8.9 - CONTINUED

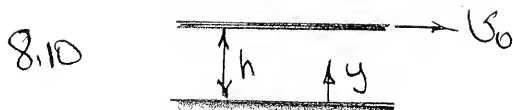
APPLYING BOUNDARY CONDITIONS

$$C_1 = 0$$

$$C_2 = -\frac{1}{2\mu} \frac{dp}{dx} \frac{h^2}{2}$$

GIVEN:

$$u_x = \frac{1}{2\mu} \frac{dp}{dx} (y^2 - h^2)$$



GOVERNING D.E. IS

$$\frac{d}{dy} \tau_{yx} - \frac{dp}{dx} = 0$$

INTEGRATING:  $\tau_{yx} - \frac{dp}{dx} y = C_1$

LAMINAR FLOW, NEWTONIAN FLUID:

$$\tau_{yx} = \mu \frac{du_x}{dy}$$

$$\mu \frac{du_x}{dy} - \frac{dp}{dx} y = C_1$$

FOR  $\tau_{yx}(0) = 0$   $C_1 = 0$

$$\frac{du_x}{dy} - \frac{1}{\mu} \frac{dp}{dx} y = 0$$

$$u_x - \frac{1}{\mu} \frac{dp}{dx} \frac{y^2}{2} = C_2$$

$$u_x @ y=h = u_0$$

$$C_2 = u_0 - \frac{1}{\mu} \frac{dp}{dx} \frac{h^2}{2}$$

8.10 - CONTINUED -

Also -  $u_x @ y=0 = 0 \therefore C_2 = 0$

GIVEN  $\frac{dp}{dx} = \frac{2\mu u_0}{h^2}$

8.11 FOR HORIZONTAL PIPE FLOW:

DEVELOPMENT IN SECTION 8.1  
RESULTS IN

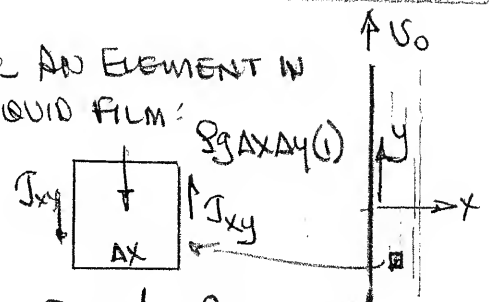
$$u_x = \left( \frac{dp}{dx} \right) \frac{r^2}{4\mu} + C_2$$

FOR  $\mu = 0$   $\frac{dp}{dx} \frac{r^2}{4}$  MUST  
= 0 FOR ALL  $r$

$\therefore u_x = \text{CONSTANT} = V$

8.12

FOR AN ELEMENT IN  
LIQUID FILM:



$$\tau_{xy} \Delta y |_{x+\Delta x} - \tau_{xy} \Delta y |_x - \rho g \Delta x \Delta y = 0$$

$$\frac{\tau_{xy} |_{x+\Delta x} - \tau_{xy} |_x}{\Delta x} - \rho g = 0$$

IN LIMIT AS  $\Delta x \rightarrow 0$

$$\frac{d}{dx} \tau_{xy} - \rho g = 0$$

8.12- CONTINUED

$$\tau_{xy} = \mu \frac{du}{dy}$$

$$\frac{dU_y^2}{dy^2} - \frac{8g}{\mu} = 0$$

$$\frac{dU}{dy} - \frac{8g}{\mu} y = C_1$$

$$U - \frac{8g}{2\mu} y^2 = C_1 y + C_2$$

BOUNDARY CONDITIONS:

$$U(0) = U_0 \quad U(h) = 0$$

$$C_1 = -\frac{8g}{\mu} h \quad C_2 = U_0$$

GIVING

$$U = U_0 - \frac{8g}{2\mu} (2hy - y^2)$$

$$= U_0 - \frac{8gh^2}{2\mu} \left[ 2\frac{y}{h} - \left(\frac{y}{h}\right)^2 \right]$$

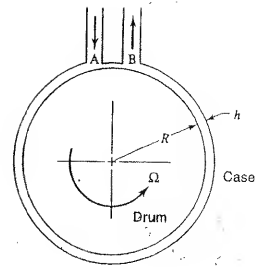
$$\dot{V} = \int_0^h U dy$$

$$= \int_0^h \left[ U_0 - \frac{8gh^2}{2\mu} \left( 2\frac{y}{h} - \left(\frac{y}{h}\right)^2 \right) \right] dy$$

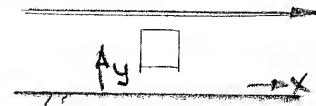
$$= U_0 h - \frac{8g}{2\mu} (h^3 - h^3/3)$$

$$= U_0 h - \frac{8gh^3}{3\mu}$$

8.13



TREAT FLUID LAYER AS A THIN LINEAR LAYER:



IN THE USUAL WAY:

$$\frac{d\tau}{dy} - \frac{dp}{dx} = 0$$

TREAT  $\frac{dp}{dx}$  CONSTANT  $\sim \frac{dp}{dx} = \frac{\Delta p}{L}$

$$\tau = \mu \frac{du}{dy}$$

$$\text{GIVING } U = \frac{\Delta p}{L} \frac{y^2}{2\mu} + C_1 y + C_2$$

BOUNDARY CONDITIONS:

$$U(0) = R\Omega$$

$$U(h) = 0$$

$$\text{GIVING } R\Omega = C_2$$

$$0 = \frac{\Delta p}{L} \frac{h^2}{2\mu} + C_1 h + R\Omega$$

$$C_1 = -\frac{\Delta p h}{L 2\mu} - \frac{R\Omega}{h}$$

$$\therefore U = R\Omega \left( 1 - \frac{y}{h} \right) - \frac{\Delta p h^2}{2\mu L} \left[ \left(\frac{y}{h}\right) - \left(\frac{y}{h}\right)^2 \right]$$

$$\text{Flow Rate} = \int_0^h U dy$$

8.13 - CONTINUED

$$\dot{V} = \int_0^h \left\{ \text{EXPRESSION FOR } v \right\} dy$$

$$= \frac{R \Omega h}{2} - \frac{\Delta p h^3}{12 \mu L}$$

GIVING:  $\Delta p = \frac{12 \mu L}{h^3} \left[ \frac{R \Omega h}{2} - \dot{V} \right]$

$$\text{EFFICIENCY} = \frac{\text{POWER OUT}}{\text{POWER IN}}$$

$$= \frac{\dot{m} \Delta p / 8}{R \Omega \dot{V}}$$

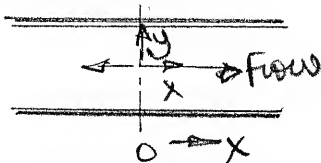
$\dot{V}$  EVALUATED AT  $R(y=0)$

$$\dot{V} = \mu \frac{R \Omega}{h} + \frac{\Delta p}{L} \frac{h}{2}$$

AFTER DOING THE ALGEBRA:

$$\eta = \frac{12 \dot{V}}{R \Omega h} \frac{R \Omega h / 2 - \dot{V}}{4 R \Omega h - 6 \dot{V}}$$

8.14



FLUID ENTERS AT  $x=0$  &  
FLOWS EQUALLY IN  $+x$  &  $-x$   
DIRECTIONS, EXITING AT  $x=L/2$   
WHERE  $p = p_{\text{atm}}$ .

LOOKING WITH THE R.H. FLOW  
(IN  $+x$  DIRECTION)

THE APPLICABLE DE. IS

$$\frac{d\dot{V}}{dy} = \frac{dp}{dx}$$

8.14 - CONTINUED

& AS USUAL -  $\dot{V} = \mu \frac{dv}{dy}$

GIVEN  $\frac{d^2 v}{dy^2} = \frac{1}{\mu} \frac{dp}{dx}$

INTEGRATING:  $\frac{dv}{dy} = \frac{1}{\mu} \frac{dp}{dx} y + C_1$

BOUNDARY COND:  $\frac{dv}{dy}(0) = 0 \therefore C_1 = 0$

AGAIN  $v = \frac{1}{2\mu} \frac{dp}{dx} y^2 + C_2$

BOUNDARY COND:  $v(b/2) = 0$

SO  $C_2 = \left( -\frac{dp}{dx} \right) \frac{b^2}{8\mu}$

VELOCITY EXPRESSION IS:

$$v = \frac{1}{2\mu} \left( -\frac{dp}{dx} \right) \left( \frac{b^2}{4} - y^2 \right)$$

$$\dot{V} = 2 \int_0^{b/2} v dy$$

$$= \frac{1}{\mu} \left( -\frac{dp}{dx} \right) \int_0^{b/2} \left( \frac{b^2}{4} - y^2 \right) dy$$

$$= \frac{1}{\mu} \left( -\frac{dp}{dx} \right) \frac{b^3}{12}$$

SO THE EXPRESSION FOR  $-\frac{dp}{dx}$  IS:

$$-\frac{dp}{dx} = \frac{12 \mu \dot{V}}{b^3}$$

$$\int_{p_0}^{p_{\text{atm}}} dp = \frac{12 \mu \dot{V}}{b^3} \int_0^{L/2} dx$$

$$p_0 - p_{\text{atm}} = \frac{6 \mu \dot{V} L}{b^3}$$

& FOR THE PLATE OF TOTAL LENGTH,  $L$ ,

$$F_y = (p_0 - p_{\text{atm}}) 2L = \frac{12 \mu \dot{V} L^2}{b^3}$$

8.15 LIQUID FLOWING DOWN  
THE OUTSIDE OF A CYLINDER:

GOVERNING D.E. IS

$$\frac{1}{r} \frac{d}{dr}(r\tau) + \rho g = 0$$

$\tau$ , AS USUAL  $\tau = \mu \frac{dv}{dr}$

$$\frac{\mu}{r} \frac{d}{dr} \left( r \frac{dv}{dr} \right) + \rho g = 0$$

$$r \frac{dv}{dr} + \frac{\rho g}{\mu} \frac{r^2}{2} = C_1$$

B.C.  $\frac{dv}{dr} = 0$  @  $r = R + h$

$$r \frac{dv}{dr} = \frac{\rho g}{2\mu} \left[ (R+h)^2 - r^2 \right]$$

AND AGAIN:

$$v = \frac{\rho g}{2\mu} \left[ (R+h)^2 \ln r - \frac{r^2}{2} \right] + C_2$$

B.C.  $v(R) = 0$

GIVING:

$$v = \frac{\rho g}{2\mu} (R+h)^2 \ln \frac{r}{R} + \frac{\rho g R^2}{4\mu} \left( 1 - \frac{r^2}{R^2} \right)$$

8.16 FOR RESULT OF PROB. 8.15

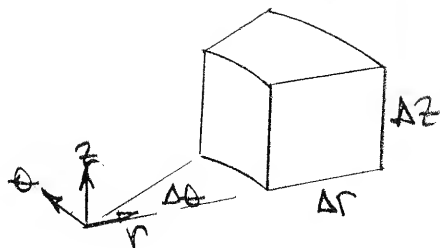
$$v_{\max} \text{ OCCURS WHERE } \frac{dv}{dr} = 0$$

$$\text{WHICH IS AT } r = R + h$$

$$v_{\max} = \frac{\rho g R^2}{4\mu} \left[ 2 \left( 1 + \frac{h}{R} \right) \ln \left( 1 + \frac{h}{R} \right) - \frac{h^2}{R^2} - \frac{2h}{R} \right]$$

# CHAPTER 9

9.1



$$\oint_{C.S.} \rho(\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{C.V.} \rho dV = 0$$

$$\begin{aligned} \oint_{C.S.} \rho(\vec{v} \cdot \vec{n}) dA &= \rho v_r \Delta z \Delta \theta \big|_{r+\Delta r} \\ &\quad - \rho v_r \Delta z \Delta \theta \big|_r + \rho v_\theta \Delta r \Delta z \big|_{\theta+\Delta \theta} \\ &\quad - \rho v_\theta \Delta r \Delta z \big|_\theta + \rho v_z \Delta r \Delta \theta \big|_{z+\Delta z} \\ &\quad - \rho v_z \Delta r \Delta \theta \big|_z \end{aligned}$$

$$\frac{\partial}{\partial t} \iiint_{C.V.} \rho dV = \frac{\partial}{\partial t} \rho \Delta r \Delta \theta \Delta z$$

SUBSTITUTING INTO C.V. RELATIONSHIP  
& EVALUATING IN LIMIT AS  $\Delta r, \Delta \theta, \Delta z \rightarrow 0$

$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

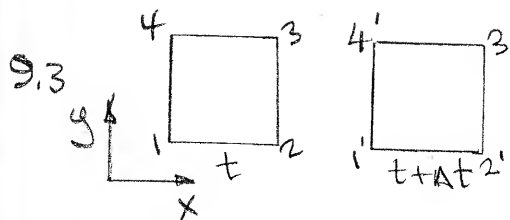
9.2  $\vec{v} = v_x \vec{e}_x + v_y \vec{e}_y + v_z \vec{e}_z$

$$\nabla = \frac{\partial}{\partial x} \vec{e}_x + \frac{\partial}{\partial y} \vec{e}_y + \frac{\partial}{\partial z} \vec{e}_z$$

$$\begin{aligned} \vec{v} \cdot \nabla &= v_x \frac{\partial}{\partial x} (\vec{e}_x \cdot \vec{e}_x) + v_y \frac{\partial}{\partial y} (\vec{e}_y \cdot \vec{e}_y) \\ &\quad + v_z \frac{\partial}{\partial z} (\vec{e}_z \cdot \vec{e}_z) \end{aligned}$$

NOTE:  $\vec{e}_i \cdot \vec{e}_j = 1$  for  $j=i$   
 $= 0$  for  $j \neq i$

$$\therefore \vec{v} \cdot \nabla = v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$$



FOR 2-DIMENSIONAL FLOW

$$\text{VOLUME CHANGE} = (\bar{1}'\bar{2}')(\bar{3}'\bar{2}') - (\bar{1}\bar{2})(\bar{3}\bar{2})$$

$$\bar{1}\bar{2} = \Delta x \quad \bar{3}\bar{2} = \Delta y$$

$$\bar{1}'\bar{2}' = \Delta x + [v_x(x+\Delta x, y) - v_x(x, y)] \Delta t$$

$$\begin{aligned} \bar{3}'\bar{2}' &= \Delta y + [v_y(x+\Delta x, y+\Delta y) \\ &\quad - v_y(x+\Delta x, y)] \Delta t \end{aligned}$$

$$(\bar{1}\bar{2})(\bar{3}\bar{2}) = \Delta x \Delta y$$

$$\begin{aligned} (\bar{1}'\bar{2}')(\bar{3}'\bar{2}') &= \Delta x \Delta y + [v_y(x+\Delta x, y+\Delta y) \\ &\quad - v_y(x+\Delta x, y)] \Delta x \Delta t \\ &\quad + [v_x(x+\Delta x, y) - v_x(x, y)] \Delta y \Delta t \\ &\quad + [\quad] \Delta t^2 \end{aligned}$$

DIVIDING BY  $\Delta x \Delta y \Delta t$  &  
EVALUATING IN LIMIT AS  $\Delta x, \Delta y, \Delta t \rightarrow 0$

$$\begin{aligned} \text{VOLUME CHANGE} &= \frac{\partial v_y}{\partial y} + \frac{\partial v_x}{\partial x} \\ &= \nabla \cdot \vec{v} \end{aligned}$$

BUT, FROM CONTINUITY  $\nabla \cdot \vec{v} = 0$

9.4  $\vec{v} = v_r \vec{e}_r + v_\theta \vec{e}_\theta$

$$d\vec{v} = \frac{\partial \vec{v}}{\partial r} dr + \frac{\partial \vec{v}}{\partial \theta} d\theta + \frac{\partial \vec{v}}{\partial t} dt$$

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial r} \frac{dr}{dt} + \frac{\partial \vec{v}}{\partial \theta} \frac{d\theta}{dt} + \frac{\partial \vec{v}}{\partial t}$$

#### 9.4 CONTINUED -

$$\frac{\partial \vec{v}}{\partial r} = \frac{\partial v_r}{\partial r} \vec{e}_r + \frac{\partial v_\theta}{\partial \theta} \vec{e}_\theta + v_r \frac{\partial \vec{e}_r}{\partial r} + v_\theta \frac{\partial \vec{e}_\theta}{\partial \theta}$$

$$\frac{\partial \vec{v}}{\partial \theta} = \frac{\partial v_r}{\partial \theta} \vec{e}_r + \frac{\partial v_\theta}{\partial r} \vec{e}_\theta + v_r \frac{\partial \vec{e}_r}{\partial \theta} + v_\theta \frac{\partial \vec{e}_\theta}{\partial \theta}$$

$$\vec{e}_r = \vec{e}_x \cos \theta + \vec{e}_y \sin \theta$$

$$\vec{e}_\theta = -\vec{e}_x \sin \theta + \vec{e}_y \cos \theta$$

$$\frac{\partial \vec{e}_r}{\partial r} = \frac{\partial \vec{e}_r}{\partial \theta} \frac{\partial \theta}{\partial r} = \vec{e}_\theta \frac{\partial \theta}{\partial r} = 0$$

$$\frac{\partial \vec{e}_r}{\partial \theta} = -\vec{e}_x \sin \theta + \vec{e}_y \cos \theta = \vec{e}_\theta$$

IN SIMILAR FASHION

$$\frac{\partial \vec{e}_\theta}{\partial r} = 0 \quad \frac{\partial \vec{e}_\theta}{\partial \theta} = -\vec{e}_r$$

GIVEN:

$$\frac{\partial \vec{v}}{\partial r} = \frac{\partial v_r}{\partial r} \vec{e}_r + \frac{\partial v_\theta}{\partial \theta} \vec{e}_\theta$$

$$\frac{\partial \vec{v}}{\partial \theta} = \left( \frac{\partial v_r}{\partial \theta} - v_\theta \right) \vec{e}_r + \left( \frac{\partial v_\theta}{\partial \theta} + v_r \right) \vec{e}_\theta$$

For  $\frac{d\vec{v}}{dt}$  TO BE  $\frac{D\vec{v}}{Dt}$   $\frac{dr}{dt} = v_r$

$$\frac{1}{r} \frac{d\theta}{dt} = \omega = \frac{v_\theta}{r}$$

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \left( v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} \right) \vec{e}_r + \left( v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} \right) \vec{e}_\theta$$

#### 9.5 NAVIER-STOKES Eqn - INCOMPRESSIBLE FORM:

$$\frac{D\vec{v}}{Dt} = \vec{g} - \frac{\nabla P}{\rho} + \nu \nabla^2 \vec{v}$$

a) FOR  $\vec{v}$  SMALL - ALL TERMS INVOLVING  $\vec{v}$  ( $\sim \frac{\partial \vec{v}}{\partial t} \frac{1}{\rho} D \nabla^2 \vec{v}$ ) ARE SMALL RELATIVE TO OTHER TERMS.

b) FOR  $D$  SMALL &  $\vec{v}$  LARGE THE PRODUCT  $\nu \vec{v}$  CANNOT BE CONSIDERED SMALL RELATIVE TO OTHER TERMS



9.6

INCOMPRESSIBLE N.S. Eqn. IN X DIRECTION

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = g_x - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \nabla^2 v_x$$

$$\Rightarrow \nabla^2 v_x = \frac{d^2 v_x}{dy^2} = \frac{1}{\mu} \frac{dP}{dx}$$

$$\frac{dv_x}{dy} = \frac{1}{\mu} \frac{dP}{dx} y + C_1$$

$$v_x = \frac{1}{2\mu} \frac{dP}{dx} y^2 + C_1 y + C_2$$

B.C.  $v_x = 0$  @  $y = \pm L$

$$C_1 = 0 \quad C_2 = -\frac{1}{2\mu} \frac{dP}{dx} L^2$$

$$v_x = \frac{1}{2\mu} \frac{dP}{dx} (y^2 - L^2)$$

$$9.7 \quad \vec{v} = \omega R^2 \hat{e}_\theta$$

$$\nabla \cdot \vec{v} = \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$\begin{aligned} \nabla \cdot \vec{v} &= \frac{1}{r} \frac{\partial}{\partial \theta} (\omega R^2) \\ &= \frac{\omega R^2}{r} \frac{\partial}{\partial \theta} \left( \frac{1}{r} \right) = 0 \end{aligned}$$

AND CONTINUITY IS SATISFIED

$$\begin{aligned} 9.8 \quad \frac{D\rho}{Dt} &= \frac{\partial \rho}{\partial t} + v_y \frac{\partial \rho}{\partial y} = -v \frac{\partial \rho}{\partial y} \\ &= -v \frac{\partial}{\partial y} \rho_0 e^{-y/\beta} = \frac{\rho_0 v}{\beta} e^{-y/\beta} \end{aligned}$$

$$\text{At } y = 100,000 \text{ FT } v = 20,000 \text{ FT/S}$$

$$\begin{aligned} \frac{D\rho}{Dt} &= \frac{20,000}{22,000} \rho_0 e^{-4.545} \\ &= 0.0096 \rho_0 \text{ s}^{-1} \end{aligned}$$

$$\begin{aligned} 9.9 \quad \nabla p &= \rho \left( \vec{g} - \frac{D\vec{v}}{Dt} \right) \\ \vec{v} &= 400 \left[ \left( \frac{y}{L} \right)^2 \hat{e}_x + \left( \frac{x}{L} \right)^2 \hat{e}_y \right] \end{aligned}$$

$$\begin{aligned} \frac{D\vec{v}}{Dt} &= \frac{\partial \vec{v}}{\partial t} + v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} \\ &= 400 \left( \frac{y}{L} \right)^2 800 \frac{x}{L^2} \hat{e}_y \\ &\quad + 400 \left( \frac{x}{L} \right)^2 800 \frac{y}{L^2} \hat{e}_x \\ &= \frac{32 \times 10^4}{L^4} \left[ x^2 y \hat{e}_x + x y^2 \hat{e}_y \right] \end{aligned}$$

9.9 CONTINUED -

EVALUATED AT  $(L, 2L)$  WE GET

$$\begin{aligned} \nabla p &= - \frac{128 \times 10^4}{L} \hat{e}_x \\ &\quad - \left[ 2g + \frac{256 \times 10^4}{L} \right] \hat{e}_y \quad \frac{\text{LB/FT}^2}{\text{FT}} \end{aligned}$$

9.10 IN X-DIRECTION:

$$\begin{aligned} \rho \frac{Dv_x}{Dt} &= \rho g_x - \frac{\partial p}{\partial x} - \frac{\partial}{\partial x} \left( \frac{2}{3} \mu \nabla \cdot \vec{v} \right) \\ &\quad + \nabla \cdot \left( \mu \frac{\partial \vec{v}}{\partial x} \right) + \nabla \cdot (\mu \nabla x) \\ &= \rho g_x - \frac{\partial p}{\partial x} - \frac{\partial}{\partial x} \left[ \frac{2}{3} \mu \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) \right] \\ &\quad + \frac{\partial}{\partial x} \left( \mu \frac{\partial v_x}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial v_y}{\partial x} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial v_z}{\partial x} \right) \\ &\quad + \frac{\partial}{\partial x} \left( \mu \frac{\partial v_x}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial v_y}{\partial x} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial v_z}{\partial x} \right) \end{aligned}$$

SIMILARLY IN  $y \hat{e}_y$  -

A TOTAL OF 45 TERMS!

$$9.11 \quad \text{FOR } \vec{v} = \vec{v}_0 + \vec{v}_r$$

0 - OF COORDINATE ORIGIN

r - RELATIVE TO COORD. ORIGIN

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}_0}{\partial t} + \frac{D\vec{v}_r}{Dt} = \vec{a}$$

1. N.S. EQN REDUCES TO

$$\rho \vec{a} = \rho \vec{g} - \nabla p$$

$$\nabla p = \rho (\vec{g} - \vec{a})$$

9.12 GIVEN THAT

$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = 0$$

a) for  $v_\theta = 0$   $\frac{\partial}{\partial r} (r v_r) = 0$

$$\frac{1}{r} r v_r(\theta) = f(\theta)$$

OR  $v_r = f(\theta)/r$

b) for  $v_r = 0$   $\frac{\partial v_\theta}{\partial \theta} = 0$

$v_\theta = f(r)$

9.13 N.S. FOR INCOMP, LAM. FLOW

$$\frac{D\vec{U}}{Dt} = \vec{g} - \frac{\nabla P}{\rho} + \nu \nabla^2 \vec{U}$$

FOR  $\vec{g}$  NEGLIGIBLE

a) VECTOR PROPERTIES  $\sim \vec{U} \nabla P$   
ARE INDEPENDENT BY THEMSELVES  
BUT IN SAME RELATIONSHIP  
MUST LIE IN SAME PLANE.

b) IF VISCOUS FORCES ARE NEGLIGIBLE

$$\frac{D\vec{U}}{Dt} = - \frac{\nabla P}{\rho}$$

$\frac{D\vec{U}}{Dt}$  IS DETERMINED BY  $-\frac{\nabla P}{\rho}$

$\frac{1}{\rho}$  IS POSITIVE IN DIRECTION  
OF DECREASING PRESSURE.

c) IN SIMILAR FASHION, ANY  
FLUID - EITHER MOVING OR  
STATIC - WILL MOVE OR  
TEND TO MOVE IN  
DIRECTION OF DECREASING  $P$ .

9.14, FOR 1-D STEADY FLOW:

$$v_x = v_x(x) \quad v_y = v_z = 0$$

$$\rho v_x \frac{\partial v_x}{\partial x} = - \frac{dP}{dx} + \frac{d}{dx} \left[ -\frac{2}{3} \left( \mu \frac{\partial v_x}{\partial x} \right) + \mu \frac{\partial v_x}{\partial x} \right]$$

$$\rho v_x \frac{\partial v_x}{\partial x} = - \frac{dP}{dx} + \frac{4}{3} \left( \mu \frac{\partial v_x}{\partial x} \right)$$

$\frac{d}{dx} (\rho v_x) = 0$

9.15 CONTINUITY:  $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) = 0$

MOMENTUM:  $\rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} \right) = - \frac{\partial P}{\partial x}$

9.16 TAKING  $z$  AS POSITIVE DOWN

WITH  $v_r = v_\theta = 0 \quad \& \quad v_z = f(r)$

EQN. E.6. YIELDS

$z$  direction

$$\rho \left( \cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r \frac{\partial v_z}{\partial r}} + \cancel{v_\theta \frac{\partial v_z}{\partial \theta}} + v_z \frac{\partial v_z}{\partial z} \right)$$

$$= - \cancel{\frac{\partial P}{\partial z}} + \rho g_z + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right]$$

$\& \text{ SINCE } g_z = -g$

$$\frac{g}{\omega} = \frac{1}{r} \frac{d}{dr} \left( r \frac{dv_z}{dr} \right)$$

PROCEED AS WAS DONE IN  
SOLNS TO PROBS 8.17 & 8.18

9.17 FOR INCOMPRESSIBLE,  
STEADY FLOW, WITH  $U_\theta = U_z = 0$   
EQN (E-4) HAS THE FORM

r direction

$$\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = -\frac{\partial P}{\partial r} + \rho g_r + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right]$$

0 DUE TO CONTINUITY

§ BECOMES

$$\frac{\partial}{\partial r} \left( P + \rho \frac{U_r^2}{2} \right) = \rho g_r$$

9.18 GOVERNING EQNS. ARE

r direction

$$\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = -\frac{\partial P}{\partial r} + \rho g_r + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right]$$

$\theta$  direction

$$\rho \left( \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = -\frac{1}{r} \frac{\partial P}{\partial \theta} + \rho g_\theta + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right]$$

z direction

$$\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial P}{\partial z} + \rho g_z + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right]$$

WHEN  $U_\theta = f(r)$  §  $U_r = U_z = 0$   
THE ONLY NON-ZERO TERM ON  
THE LEFT-HAND SIDE OF ALL  
COMPONENT EQNS.  
IS  $-U_\theta^2/r$

$$\therefore \frac{D\vec{U}}{Dt} = \frac{d\vec{U}}{dt} = -\frac{U_\theta^2}{r} \vec{e}_r$$

Q.E.D.

9.19 EQN (E-5) IS SIMPLIFIED  
FOR THIS CASE AS

$\theta$  direction

$$\rho \left( \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = -\frac{1}{r} \frac{\partial P}{\partial \theta} + \rho g_\theta + \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right]$$

§ IN THE ABSENCE OF GRAVITY  
WE HAVE

$$\frac{\partial v_\theta}{\partial t} = \nu \frac{\partial}{\partial r} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right]$$

9.20 - FROM PROB 9.19 §  
STEADY FLOW

$$\frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} (r v_\theta) \right] = 0$$

$$\text{GIVING } \frac{1}{r} \frac{d}{dr} (r v_\theta) = C_1$$

INTEGRATING AGAIN

$$r v_\theta = C_1 \ln r + C_2$$

$$\text{B.C. } v_\theta(R_1) = R_1 \Omega_1$$

$$v_\theta(R_2) = R_2 \Omega_2$$

$$v_\theta = \frac{1}{r} \left[ R_1^2 \Omega_1 + \frac{(R_2^2 \Omega_2 - R_1^2 \Omega_1) \ln r / R_1}{\ln R_2 / R_1} \right]$$

# CHAPTER 10

$$10.1 \quad \nabla \times \vec{U} = \left( \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} \right) \times (v_r \vec{e}_r + v_\theta \vec{e}_\theta)$$

$$= (\vec{e}_r \times \vec{e}_r) \frac{\partial v_r}{\partial r} + v_r \vec{e}_r \times \frac{\partial \vec{e}_r}{\partial r} + (\vec{e}_r \times \vec{e}_\theta) \frac{\partial v_\theta}{\partial r} + v_\theta \vec{e}_r \times \frac{\partial \vec{e}_\theta}{\partial r} + (\vec{e}_\theta \times \vec{e}_r) \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_r}{r} \vec{e}_\theta \times \frac{\partial \vec{e}_r}{\partial \theta} - \vec{e}_\theta \times \vec{e}_\theta \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta}{r} \vec{e}_\theta \times \frac{\partial \vec{e}_\theta}{\partial \theta}$$

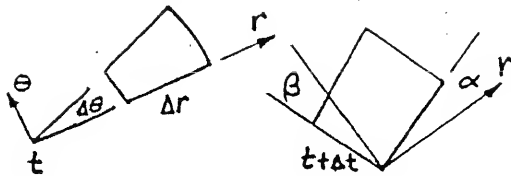
For reference - See Prob 9.4

$$\frac{\partial \vec{e}_r}{\partial r} = 0, \frac{\partial \vec{e}_\theta}{\partial r} = 0, \frac{\partial \vec{e}_r}{\partial \theta} = \vec{e}_\theta, \frac{\partial \vec{e}_\theta}{\partial \theta} = -\vec{e}_r$$

All remaining (non-zero) terms give:

$$\nabla \times \vec{U} = \left[ \frac{\partial v_\theta}{\partial r} + \frac{1}{r} (v_\theta - v_r \frac{\partial v_r}{\partial \theta}) \right] \vec{e}_z$$

10.2



$$\omega_z = \frac{d}{dt} \left( \frac{\alpha + \beta}{2} \right)$$

$$= \lim_{\Delta t \rightarrow 0} \frac{1}{2} \left[ \frac{\tan^{-1} \left( \frac{r v_\theta|_{t+\Delta t} - r v_\theta|_t}{r \Delta r} \right) \Delta t}{\Delta t} + \frac{\tan^{-1} \left( \frac{v_r|_{t+\Delta\theta} - v_r|_t}{r \Delta \theta} \right) \Delta t}{\Delta t} \right]$$

10.2 CONTINUED -

IN THE LIMIT: {NOTE THAT  $\tan z \rightarrow z$ }

$$\omega_z = \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0 \\ \Delta z \rightarrow 0}} \left[ \frac{r v_\theta|_{t+\Delta r} - r v_\theta|_t}{r \Delta r} - \frac{v_r|_{t+\Delta\theta} - v_r|_t}{r \Delta \theta} \right]$$

$$= \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) - \frac{1}{r} \frac{\partial v_r}{\partial \theta}$$

$$= \frac{\partial v_\theta}{\partial r} + \frac{1}{r} (v_\theta - v_r \frac{\partial v_r}{\partial \theta}) \quad \text{Q.E.D.}$$

$$10.3 \quad d\psi = -v_y dx + v_x dy$$

$$= (-v_y \sin \alpha) dx + (v_x \cos \alpha) dy$$

$$\psi = -v_y (\sin \alpha) x + v_x (\cos \alpha) y + \psi_0$$

$$10.4 \quad \nabla \cdot \vec{U} = \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = 0$$

$$\text{Let } r v_r = \frac{\partial \psi(r, \theta)}{\partial \theta}$$

$$\text{Then } \nabla \cdot \vec{U} = \frac{1}{r} \left[ \frac{\partial}{\partial r} \frac{\partial \psi}{\partial \theta} + \frac{\partial v_\theta}{\partial \theta} \right] = 0$$

$$\frac{\partial}{\partial \theta} \left( \frac{\partial \psi}{\partial r} + v_\theta \right) = 0 \quad \therefore v_\theta = -\frac{\partial \psi}{\partial r}$$

$$\therefore v_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad \therefore v_\theta = -\frac{\partial \psi}{\partial r}$$

Q.E.D.

$$10.5 \quad \phi = \frac{5}{3} x^3 - 5xy^2$$

$$\text{SINCE } \vec{v} = \nabla \phi$$

CONTINUITY CAN BE EXPRESSED  
AS  $\nabla \cdot \vec{v} = 0$  OR  $\nabla^2 \phi = 0$

$$\text{USING } \nabla^2 \phi = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$10x - 10x = 0$$

$$v_x = \frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial y} = 5x^2$$

$$\text{OR } \psi = 5x^2 y$$

$$- \frac{\partial \phi}{\partial y} = v_y = - \frac{\partial \psi}{\partial x} = \text{CHECK}$$

10.6 IN THE CORE: EULER'S EON.

$$\frac{D\vec{v}}{dt} = \vec{a} = -\frac{\nabla P}{\rho}$$

$$= -\frac{v^2}{r} \vec{e}_r = \frac{\nabla P}{\rho} = \frac{\partial P}{\partial r} \vec{e}_r$$

$$\frac{dP}{\rho} = -\frac{v^2}{r} dr$$

SINCE VELOCITY VARIATION IS LINEAR

$$v = v_{\max} r/R$$

$$\int_{P_0}^{P_R} dP = -\frac{\rho v_{\max}^2}{R^2} \int_0^R r dr$$

$$P_R - P_0 = \frac{\rho v_{\max}^2}{2} \quad (1)$$

OUTSIDE THE CENTRAL CORE -  
BERNOULLI EON. APPLIES

10.6 - CONTINUED

$$\frac{P_{\infty}}{\rho} = \frac{P}{\rho} + \frac{v^2}{2}$$

$v$  VARIES INVERSELY WITH  $r$ :

$$v = v_{\max} \frac{R}{r}$$

$$\text{AT } r = R \quad P_{\infty} - P_R = \rho v_{\max}^2 / 2 \quad (2)$$

ADDING (1) & (2)

$$P_{\infty} - P_0 = \rho v_{\max}^2$$

$$v_{\max} = \left[ \frac{(38)(32.2)}{0.0766} \right]^{1/2} = \underline{126 \text{ ft/s}} \quad (a)$$

FOR  $P = -10 \text{ PSF}$

$$P_{\infty} - P = \rho v^2 / 2$$

$$v = \left[ \frac{(10)(32.2)}{0.0766} \right]^{1/2} = 91.7 \text{ ft/s}$$

$$r = \frac{v_{\max} R}{v} = \frac{126 (100)}{91.7} = 138 \text{ ft}$$

PRESSURE WILL FALL FROM -10 TO -38 PSF  
IN A DISTANCE OF 138 FT

AT 60 MPH = 88 FT/S

$$\text{TIME} = 138 / 88 = \underline{1.57 \text{ SECONDS}} \quad (b)$$

PRESSURE AT TORNADO CENTER = -38 PSF

$$\text{AT EDGE OF CORE: } = -\frac{\rho v_{\max}^2}{2} + P_{\infty}$$

FAR FROM CENTER -  $P = P_{\text{ATM}}$

$$\text{TOTAL } \Delta P = \underline{38 \text{ PSF}} \quad (c)$$

$$10.7 \quad U_r = U_\infty \cos \theta (1 - a^2/r^2)$$

ALONG THE STAGNATION STREAMLINE

$$\theta = \pi$$

$$U_r = -U_\infty (1 - a^2/r^2) \quad (a)$$

$$\frac{\partial U_r}{\partial r} = -\frac{2U_\infty a^2}{r^3}$$

$$\left. \frac{\partial U_r}{\partial r} \right|_{r=a} = -\frac{2U_\infty}{a} \quad (b)$$

10.8 FROM CONTINUITY

$$\frac{\partial}{\partial r}(rU_r) = -\frac{\partial U_\theta}{\partial \theta}$$

$$\therefore \left. \frac{\partial U_\theta}{\partial \theta} \right|_{r=a, \theta=\pi} = -\left. \frac{\partial}{\partial r}(rU_r) \right|_{r=a, \theta=\pi}$$

$$10.9 \quad P + \frac{\rho U^2}{2} = \text{CONSTANT}$$

$$\text{for } P = P_\infty \quad U^2 = U_\infty^2$$

$$\therefore |U_\infty| = |U_\theta| = 2U_\infty \sin \theta$$

$$\sin \theta = 0.5$$

$$\therefore \theta = \pm 30^\circ, \pm 150^\circ$$

$$10.10 (a) \quad \phi = U_\infty L \left[ \left( \frac{x}{L} \right)^3 - 3 \frac{xy^2}{L^2} \right]$$

$$U = \nabla \phi = U_x \vec{e}_x + U_y \vec{e}_y$$

$$U_x = \frac{\partial \phi}{\partial x} = \frac{3U_\infty}{L^2} (x^2 - y^2) = \frac{\partial \phi}{\partial y}$$

$$U_y = \frac{\partial \phi}{\partial y} = -\frac{6U_\infty xy}{L^2} = -\frac{\partial \phi}{\partial x}$$

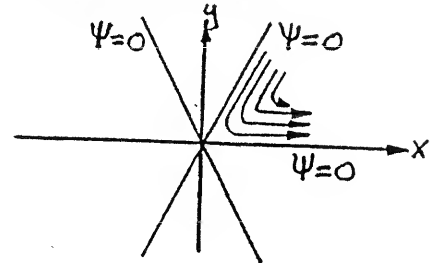
10.10 CONTINUED.

$$\phi = \frac{3U_\infty}{L^2} \left( x^2 y - \frac{y^3}{3} \right) + f(x)$$

$$= \frac{3U_\infty x^2 y}{L^2} + g(y)$$

$$\text{So } \phi = \frac{U_\infty x^2 y}{L^2} (6x^2 - y^2)$$

Flow configuration is:



when  $\phi = 0$   $y = 0$  or  $\pm \sqrt{6}x$

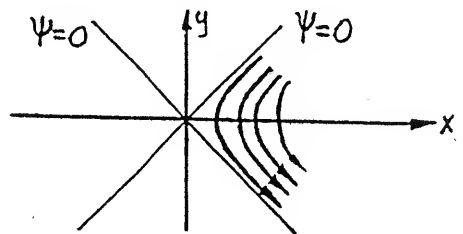
$$b) \quad \phi = U_\infty \frac{xy}{L}$$

$$U_x = \frac{\partial \phi}{\partial x} = \frac{U_\infty y}{L} = \frac{\partial \phi}{\partial y}$$

$$U_y = \frac{\partial \phi}{\partial y} = \frac{U_\infty x}{L} = -\frac{\partial \phi}{\partial x}$$

$$\phi = \frac{U_\infty}{2L} y^2 + f(x); \quad \phi = -\frac{U_\infty}{2L} x^2 + g(y)$$

$$\phi = \frac{U_\infty}{2L} (y^2 - x^2)$$



when  $\phi = 0$   $y = \pm x$

10.10 CONTINUED

c)  $\phi = \frac{U_\infty L}{2} \ln(x^2 + y^2)$

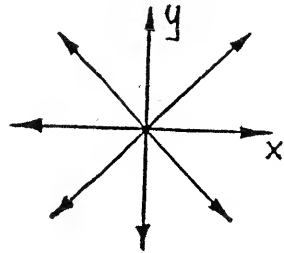
$$u_x = \frac{\partial \phi}{\partial x} = \frac{U_\infty L}{2} \frac{2x}{x^2 + y^2} = \frac{\partial \psi}{\partial y}$$

$$u_y = \frac{\partial \phi}{\partial y} = \frac{U_\infty L}{2} \frac{2y}{x^2 + y^2} = -\frac{\partial \psi}{\partial x}$$

$$\psi = \frac{U_\infty L}{2} x \tan^{-1}(y/x) + f(x)$$

$$\psi = \frac{U_\infty L}{2} x \tan^{-1}(x/y) + g(y)$$

$$\psi = U_\infty L \left[ \tan^{-1}(x/y) - \tan^{-1}(y/x) \right]$$



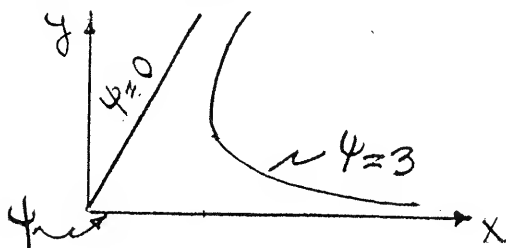
when  $\phi=0$   $y=x$

10.11  $\phi = 2r^3 \sin 3\theta$  For  $\theta = 0, \pi/3$

IN FIGURE - FOR  $\phi=0$  - DRAWING NO. - PICK 3 -

$$r = \left[ \frac{3}{2 \sin 3\theta} \right]^{1/3}$$

CHOOSE  $\theta$  - SOLVE FOR  $r$  -  
PLOT LOOKS LIKE:

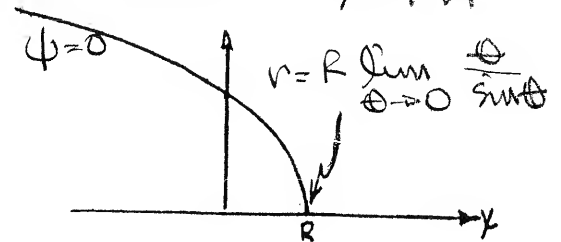


10.12  $\phi=0 = U_\infty r \sin \theta + \frac{Q}{2\pi} \theta$

SINCE  $r > 0$  WHEN  $U_\infty > 0$ ,  $\phi=0$   
GIVES  $\theta=0 \sim$  THE +X AXIS.

WHEN  $U_\infty < 0$  (FLOW IN -X DIRECTION)

$$y = R\theta - \text{WHERE } R = Q/2\pi|U_\infty|$$



10.13 FOR SOURCE AT ORIGIN  $\phi = \frac{\dot{m}\theta}{2\pi S}$

$\dot{m}$  = SOURCE STRENGTH

FREE STREAM:  $\phi = U_\infty y$

ADDING:  $\phi = U_\infty y + \frac{\dot{m}\theta}{2\pi S}$

$$u_r = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = U_\infty \sin \theta + \frac{\dot{m}}{2\pi S r}$$

$$y = r \sin \theta$$

$$u_r = 0 @ \theta = \pi$$

AT  $\theta = \pi$   $r = \frac{\dot{m}}{2\pi S U_\infty} = \frac{Q}{2\pi U_\infty}$

10.14  $\nabla P = \rho \frac{D\vec{U}}{Dt}$   
 $= \rho \left[ \frac{\partial \vec{U}}{\partial t} + \nabla \left( \frac{U^2}{2} \right) - U_x (\nabla \times \vec{U}) \right]$

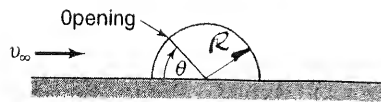
FOR STEADY, IRROTATIONAL FLOW

$$\nabla P = \rho \nabla \left( \frac{U^2}{2} \right)$$

@ STAGNATION POINT - WHERE  $U=0$

$$\nabla P = 0$$

10.15



LIFT FORCE:

$$dF_y = dF \sin \theta$$

$$= (P_{in} - P_{out}) R \sin \theta d\theta$$

$$F_y = \int_0^\pi \Delta P R \sin \theta d\theta$$

From Bernoulli Eqn:

$$P + \rho U^2/2 = \text{const.}$$

$$P_{\infty} + \rho \frac{U_{\infty}^2}{2} = P + \rho \frac{U^2}{2}$$

$$\text{ON HWT } U = 2U_{\infty} \sin \theta$$

$$\therefore P = P_{\infty} + \frac{\rho U_{\infty}^2}{2} [1 - 4 \sin^2 \theta]$$

SUBST. INTO EXPRESSION FOR  $F_y$ 

$$F_y = \int_0^\pi \frac{\rho U_{\infty}^2}{2} [1 - 4 \sin^2 \theta - 1 + 4 \sin^2 \theta] R \sin \theta d\theta$$

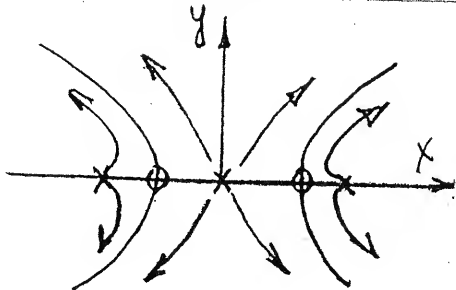
$$= 2 \rho U_{\infty}^2 R \int_0^\pi [\sin^3 \theta - \sin \theta \sin^2 \theta] d\theta$$

$$= 2 \rho U_{\infty}^2 R \left[ \frac{4}{3} - 2 \sin^2 \theta_0 \right]$$

$$\text{for } F_y = 0 \quad \sin^2 \theta_0 = 2/3$$

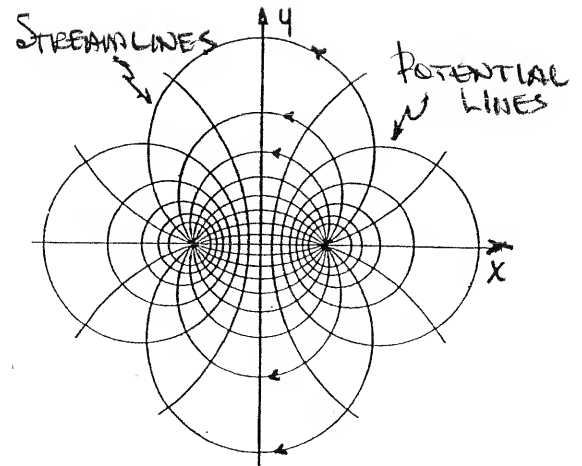
$$\theta_0 = 54.7^\circ$$

10.16

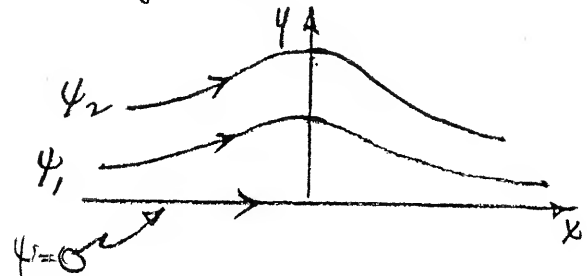
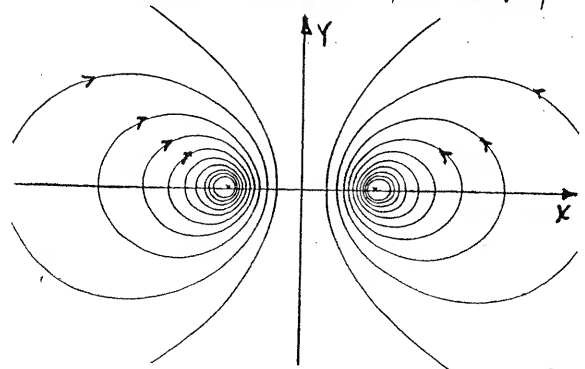


STAGNATION PTS AT CIRCLES

10.17

10.18 FOR THIS CASE -  $\nabla^2 \psi \neq 0$ 

$$\psi = \frac{\phi}{1+3x^2}$$

10.19  $\psi = -\frac{K}{2\pi} \ln r$ ,  $U_\theta = \frac{K}{2\pi r}$   
WHEN ORIGIN IS AT VORTEX

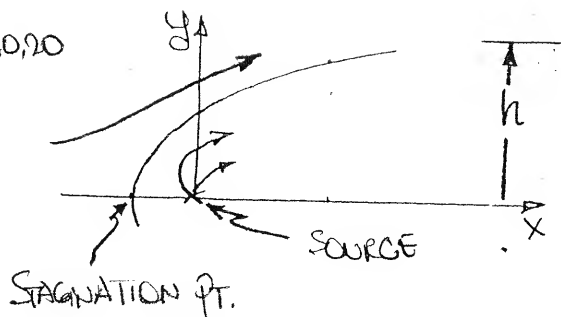
$$U_\theta(-a, 0) = \frac{K}{2\pi(2a)} = \frac{K}{4\pi a}$$

$$\vec{U}_\theta(-a, 0) = -K/4\pi a \vec{e}_y$$

$$\vec{U}(a, 0) = -K/4\pi a \vec{e}_y$$

$$\text{SINCE } \psi = -\frac{K}{2\pi} \ln r$$

10/20



STAGNATION PT.

$$\psi = U_0 r \sin \theta + \frac{Q}{2\pi} \theta$$

a) STAGNATION POINT

$$\vec{V} = 0 \text{ REQUIRES } V_r = V_\theta = 0$$

$$V_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} = \frac{1}{r} \left( \frac{Q}{2\pi} + U_0 r \cos \theta \right)$$

$$V_\theta = -\frac{\partial \psi}{\partial r} = -U_0 \sin \theta$$

$$\therefore \theta = \pi, r \cos \pi = x = -\frac{Q}{2\pi U_0}$$

AT STAGNATION PT.

$$x = -\frac{Q}{2\pi U_0} = -\frac{1.5}{2\pi(9)} = -0.0265 \text{ m}$$

$$y = 0$$

b) BODY HEIGHT

STAGNATION STREAMLINE

$$\psi = U_0 r \sin \pi + \frac{Q\pi}{2\pi} = \frac{Q}{2}$$

THUS

$$\frac{Q}{2} = U_0 r \sin \theta + \frac{Q\theta}{2\pi}$$

SO WHEN  $\theta = \pi/2$ 

$$r \sin \theta = y = \frac{1}{U_0} \left( \frac{Q}{2} - \frac{Q\pi}{2\pi(2)} \right)$$

$$= \frac{Q}{4U_0} = 0.0417 \text{ m}$$

10/20 CONTINUED

c) FOR  $x$  LARGE - ALL FLOW IS AT  $U_0$ 

$$\therefore Q = U_0 (2h)$$

$$h = \frac{Q}{2U_0} = \frac{1.5}{2(9)} = 0.0833 \text{ m}$$

d) MAXIMUM SURFACE VELOCITY

$$V^2 = V_r^2 + V_\theta^2 \text{ IS ON S.L. } \psi = Q/2$$

 $V_r \neq U_0$  DETERMINED IN PART (a)

$$V^2 = \frac{Q^2}{4\pi^2 r^2} + \frac{Q U_0 \cos \theta}{\pi r} + U_0^2$$

$$\text{ON SURFACE } \psi = \frac{Q}{2} = U_0 r \sin \theta + \frac{Q\theta}{2\pi}$$

$$\text{THUS } \frac{Q^2}{4\pi^2 r^2} = \frac{Q^2 4 U_0^2 \sin^2 \theta}{4\pi^2 Q^2 (1 - \theta/\pi)^2}$$

$$\frac{Q U_0 \cos \theta}{\pi r} = \frac{U_0^2 Q \cos \theta \sin \theta}{\pi Q/2 (1 - \theta/\pi)}$$

RESULTS IN

$$\frac{V^2}{U_0^2} = \frac{\sin^2 \theta}{\pi^2 (1 - \theta/\pi)^2} + \frac{2}{\pi} \frac{\sin \theta \cos \theta}{(1 - \theta/\pi)} + 1$$

$$\frac{V^2}{U_0^2} \text{ IS MAX AT } \theta \approx 63^\circ$$

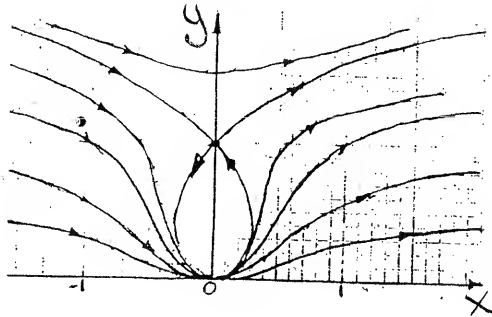
$$\text{SO } \frac{V_{\text{MAX}}}{U_0} \approx 1.26$$

10/21 IN THIS CASE -

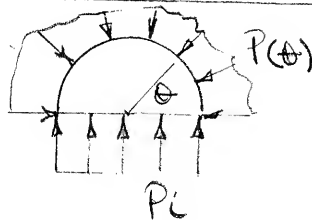
$$\psi = U_\infty r \sin\theta \left(1 + \frac{a^2}{r^2}\right)$$

STREAMLINES CAN BE  
PLOTTED FOR  $U_\infty = 1$ ,  $a = 1$

- IN UPPER HALF PLANE  
THEY APPEAR AS



10/22



$$\sum F_y = 0$$

$$P_i D - 12T - \frac{D}{2} \int_0^\pi P \sin\theta d\theta = 0$$

$$P = P_{atm} + \frac{\rho}{2} (U_\infty^2 - U^2)$$

$$U = -2U_\infty \sin\theta$$

$$12T = P_i D - P_{atm} + \frac{\rho}{2} U_\infty^3$$

$$+ 2\rho U_\infty^2 \frac{D}{2} \int_0^\pi \sin^3\theta d\theta$$

$$12T = (P_i - P_{atm})D + \frac{5}{6} \rho U_\infty^2 D$$

$$T = 10.1 \text{ kN}$$

PER BELT

# CHAPTER 11

## 11.1 VARIABLE DIMENSIONS

D	L
P	M/L <sup>3</sup>
H	L
g	L/t <sup>2</sup>
ω	1/t
Q	L <sup>3</sup> /t
η	M L <sup>2</sup> /t <sup>3</sup>

$$i = 8 - 3 = 5$$

CHOOSE CORE AS S, D, ω

$$\pi_1 = \eta \quad \text{--- ALREADY DIMENSIONLESS}$$

$$\pi_2 = S^a D^b \omega^c H \quad \sim = H/D$$

$$\pi_3 = S^d D^e \omega^f g \quad \sim = g/D\omega^2$$

$$\pi_4 = S^g D^h \omega^i Q \quad \sim = Q/D^3\omega$$

$$\pi_5 = S^j D^k \omega^l P \quad \sim = P/S D^5 \omega^3$$

## 11.2 VARIABLE DIMENSIONS

U	L/t
D	L
g	M/L <sup>3</sup>
μ	M/Lt
e	L

$$i = 5 - 3 = 2$$

CHOOSE CORE AS D, U, g

$$\pi_1 = D^a U^b g^c \mu \quad \sim = \mu/D U g = \frac{1}{Re}$$

$$\pi_2 = D^d U^e g^f e \quad \sim = e/D$$

$$f = f(Re, e/D)$$

## 11.3 VARIABLE DIMENSIONS

ΔP	M/Lt <sup>2</sup>
S	M/L <sup>3</sup>
ω	1/t
D	L
Q	L <sup>3</sup> /t
μ	M/Lt

$$i = 6 - 3 = 3$$

CHOOSE CORE AS S, D, ω

$$\pi_1 = S^a D^b \omega^c \Delta P \quad \sim = \Delta P/S D^3 \omega^2$$

$$\pi_2 = S^d D^e \omega^f Q \quad \sim = Q/D^3 \omega$$

$$\pi_3 = S^g D^h \omega^i \mu \quad \sim = \mu/S D^2 \omega$$

## 11.4 VARIABLE DIMENSIONS

C <sub>max</sub>	M L <sup>2</sup> /t <sup>2</sup>
α	---
β	---
M	M
L	L
S	M/L <sup>3</sup>
g	L/t <sup>2</sup>
R	L

$$i = 8 - 3 = 5$$

$$\pi_1 = \alpha \quad \text{--- ALREADY DIMENSIONLESS}$$

$$\pi_2 = \beta \quad \text{--- " "}$$

CHOOSE CORE AS M, L, g

$$\pi_3 = M^a L^b g^c C_{max} \quad \sim = C_{max}/M L g$$

$$\pi_4 = M^d L^e g^f S \quad \sim = S L^3/M$$

$$\pi_5 = M^g L^h g^i R \quad \sim = R/L$$

11.5	VARIABLE	DIMENSIONS
	$k$	$L/t$
	$D$	$L^2/t$
	$d$	$L$
	$\omega$	$1/t$
	$\rho$	$m/L^3$
	$\mu$	$m/Lt$

$$i = 6 - 3 = 3$$

CHOOSE CORE AS  $d, \omega, \rho$

$$\pi_1 = d^a \omega^b \rho^c k \sim = k/d\omega$$

$$\pi_2 = d^a \omega^b \rho^c D \sim = D/d^2 \omega$$

$$\pi_3 = d^a \omega^b \rho^c \mu \sim = \mu/\rho d^2 \omega$$

PLOT  $\pi_1$  VS.  $\pi_3$

OVER A RANGE IN VALUES OF  $\pi_2$

11.6	VARIABLE	DIMENSIONS
	$Q$	$L^3/t$
	$d$	$L$
	$\omega$	$1/t$
	$\mu$	$m/Lt$
	$\sigma$	$M/t^2$
	$\rho$	$m/L^3$

$$i = 6 - 3 = 3$$

CHOOSE CORE AS  $d, \omega, \rho$

$$\pi_1 = d^a \omega^b \rho^c Q \sim = Q/\omega d^3$$

$$\pi_2 = d^a \omega^b \rho^c \mu \sim = d^2 \omega \rho / \mu$$

$$\pi_3 = d^a \omega^b \rho^c \sigma \sim = \sigma / \rho n^2 d^3$$

11.7	VARIABLE	DIMENSIONS
	$M$	$m$
	$d$	$L$
	$\rho$	$m/L^3$
	$g$	$L/t^2$
	$\sigma$	$M/t^2$

$$i = 5 - 3 = 2$$

CHOOSE CORE AS  $d, \rho, g$

$$\pi_1 = d^a \rho^b g^c M \sim = M/d^3 \rho$$

$$\pi_2 = d^a \rho^b g^c \sigma \sim = \sigma \rho d^2 / g$$

11.8	VARIABLE	DIMENSIONS
------	----------	------------

$$n$$

$$1/t$$

$$L$$

$$L$$

$$d$$

$$L$$

$$\rho$$

$$m/L^3$$

$$T$$

$$ML/t^2$$

$$i = 5 - 3 = 2$$

CHOOSE CORE AS  $n, d, \rho$

$$\pi_1 = n^a d^b \rho^c L \sim = L/d$$

$$\pi_2 = n^a d^b \rho^c T \sim = T/n^2 L^4 \rho$$

11.9	VARIABLE	DIMENSIONS
------	----------	------------

$$p$$

$$mL^2/t^3$$

$$d$$

$$L$$

$$\omega$$

$$1/t$$

$$Q$$

$$L^3/t$$

$$\rho$$

$$m/L^3$$

$$\mu$$

$$m/Lt$$

$$i = 6 - 3 = 3$$

11.9- CONTINUED

CHOOSE CORE AS  $d, \rho, \omega$

$$\pi_1 = d^a \rho^b \omega^c \omega \sim = d^3 \omega / \omega$$

$$\pi_2 = d^a \rho^b \omega^c \mu \sim = d \mu / \rho \omega$$

$$\pi_3 = d^a \rho^b \omega^c \rho \sim = d^4 \rho / \rho \omega^3$$

$$\pi_3 = f(\pi_1, \pi_2)$$

11.10 VARIABLE DIMENSIONS

$r$	$L$
$t$	$t$
$\rho$	$M/L^3$
$E$	$M L^2 / t^2$

$$L = 4 - 3 = 1$$

$$\pi_1 = t^a \rho^b E^c r = \frac{\rho^{1/5} r}{t^{2/5} E^{1/5}}$$

$$\text{OR } \pi_1 = \frac{\rho r^5}{t^2 E}$$

$$\text{SO: } r^5 = C_1 \frac{E t^2}{\rho} \quad (1)$$

$$\text{SPEED OF WAVE FRONT} = \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{2}{5} C_2 \frac{t}{r^4}$$

$$\text{FROM (1)} \quad t = C_3 r^{5/2}$$

$$\therefore \frac{dr}{dt} = C^4 / r^{3/2}$$

$$\sim \frac{dr}{dt} \text{ DECREASES AS } r \text{ INCREASES}$$

11.11

VARIABLE

DIMENSIONS

$d$	$L$
$D$	$L$
$\omega$	$L/t$
$\rho$	$M/L^3$
$\mu$	$M/Lt$
$\sigma$	$M/t^2$

$$L = 6 - 3 = 3$$

CHOOSE CORE AS  $D, \rho, \omega$

$$\pi_1 = D^a \rho^b \omega^c d \sim = d/D$$

$$\pi_2 = D^a \rho^b \omega^c \mu \sim = \frac{\mu}{D \rho \omega} = 1/Re$$

$$\pi_3 = D^a \rho^b \omega^c \sigma \sim = \frac{D \sigma}{\rho \omega^2}$$

$$\pi_1 = f(\pi_2, \pi_3)$$

11.12

VARIABLE

DIMENSIONS

$\Delta P$	$M/Lt^2$
$Q$	$L^3/t$
$h$	$L$
$\Delta$	$1/t$
$\mu$	$M/Lt$
$L$	$L$
$R$	$L$

$$L = 7 - 3 = 4$$

CHOOSE CORE AS  $h, \Delta, \mu$

$$\pi_1 = h^a \Delta^b \mu^c L \sim = L/h$$

$$\pi_2 = h^a \Delta^b \mu^c R \sim = R/h$$

$$\pi_3 = h^a \Delta^b \mu^c \Delta P \sim = \Delta P / \Delta \mu$$

$$\pi_4 = h^a \Delta^b \mu^c Q \sim = Q / h^3 \Delta$$

$$11.13 \quad Re = \frac{LV}{\nu} \quad \text{for Air @ } 20^\circ\text{C} \\ (293\text{K}) \\ \nu = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$$

a) BASED ON  $L = 58 \text{ m}$

$$Re = \frac{(58)(22.2)}{1.505 \times 10^{-5}} = 8.55 \times 10^6$$

b) BASED ON  $D = 0.0064 \text{ m}$

$$Re = \frac{(0.0064)(22.2)}{1.505 \times 10^{-5}} = 9440$$

$$11.14 \quad \rho \frac{D\vec{U}}{dt} = -\nabla p + \mu \nabla^2 \vec{U} + \rho \vec{g} \left( \frac{T_A}{T_0} - 1 \right)$$

TO PUT P.E. INTO DIMENSIONLESS FORM - USE TEXT PROCEDURE -

$L$  = REFERENCE LENGTH

$V_0$  = " VELOCITY

$$\text{THEN } x^* = \frac{x}{L}, y^* = \frac{y}{L}, t^* = \frac{t V_0}{L}$$

$$\nabla^* = L \nabla \quad \nabla^{*2} = L^2 \nabla^2$$

$$\rho \frac{D\vec{U}}{dt} = \rho \frac{D\vec{U}^*}{dt^*} \left( \frac{\partial \vec{U}}{\partial x^*} \right) \left( \frac{\partial t^*}{\partial t} \right) \\ = \frac{\rho V_0^2}{L} \frac{D\vec{U}^*}{dt^*}$$

WHERE  $\frac{\rho V_0^2}{L}$  IS INERTIAL FORCE

WE COULD DO ALL OTHER TERMS IN A LIKE MANNER BUT PROBLEM STATEMENT ONLY ASKS FOR RATIO OF GRAVITY FORCES TO INERTIAL FORCES

11.14 - CONTINUED -

THE GRAVITATIONAL (BOUYANCY) FORCE IS  $\rho \vec{g} \left( \frac{T_A}{T_0} - 1 \right)$

SO RATIO ASKED FOR IS

$$\frac{L g \left[ \frac{T_A}{T_0} - 1 \right]}{V_0^2} \quad \text{Q.E.D.}$$

11.15	VARIABLE	MODEL	PROTOTYPE
	$D$	$D$	6D
	$U$	$U$	20 KT
	$\rho$	$\rho$	$\rho$
	$\mu$	$\mu$	$\mu$
	$F$	10 lbf	$F_p$
	$A$	$D^2$	$(6D)^2$

DYNAMIC SIMILARITY REQUIRES:

$$Re_m = Re_p$$

$$\frac{D U \rho}{\mu} \bigg|_m = \frac{D U \rho}{\mu} \bigg|_p$$

$$U_m = U_p \left[ \frac{D_p}{D_m} \frac{\rho_p}{\rho_m} \frac{\mu_m}{\mu_p} \right] = 6 U_p \\ = 120 \text{ KT}$$

$$\text{§ ALSO THAT } Eu_m = Eu_p$$

$$\frac{F/A}{\rho U^2} \bigg|_m = \frac{F/A}{\rho U^2} \bigg|_p$$

$$F_p = F_m \left[ \frac{A_p}{A_m} \left( \frac{\rho_p}{\rho_m} \right) \left( \frac{U_p}{U_m} \right)^2 \right] \\ = 10 \text{ lbf}$$

# 11.16 SIMILARITY REQUIRES

$$Fr|_m = Fr|_p$$

$$U^2/gL|_m = U^2/gL|_p$$

MODEL PROTOTYPE

$U$	$U_m$	$U_p$
$L$	$L/10$	$L$

$$\left(\frac{U_m}{U_p}\right)^2 = \frac{L_m}{L_p} = 0.1$$

$$\frac{U_m}{U_p} = 0.316$$

11.17

MODEL PROTOTYPE

$L=3m$	$4L$
$U_m$	$16 \text{ m/s}$
$\rho_A$	$\rho_w$
$\nu_A$	$\nu_w$
$F_m$	$F_p$
$A_m$	$16A_m$

For Air At  $20^\circ\text{C}$  (293K)

$$\nu = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$$

At 6 ATM  $\nu = 0.251 \times 10^{-5} \text{ "}$

For H<sub>2</sub>O @  $20^\circ\text{C}$   $\nu = 0.995 \text{ "}$

$$Re_m = Re_p$$

$$\frac{LU}{\nu}|_m = \frac{LU}{\nu}|_p$$

$$U_m = U_p \left( \frac{L_p}{L_m} \frac{\nu_m}{\nu_p} \right)$$

# 11.17 - CONTINUED

$$U_m = 16 \left[ (4) \frac{0.251 \times 10^{-5}}{0.995 \times 10^{-5}} (6) \right]$$

$$= 96.9 \text{ m/s}$$

$$Eu_m = Eu_p$$

$$\frac{F/A}{\rho U^2}|_m = \frac{F/A}{\rho U^2}|_p$$

$$\frac{F_m}{F_p} = \left( \frac{A_m}{A_p} \right) \left( \frac{\rho_m}{\rho_p} \right) \left( \frac{U_m}{U_p} \right)^2$$

$$= \frac{1}{16} \left( \frac{7.229}{998.2} \right) \left( \frac{96.9}{16} \right)^2$$

At  $20^\circ\text{C}$   $\rho_w = 998.2 \text{ kg/m}^3$

At  $20^\circ\text{C}$ , 6 ATM  $\rho_A = 7.229 \text{ "}$

$$\frac{F_m}{F_p} = 0.0166$$

{ RESULT IS  
QUITE  
TEMPERATURE  
SENSITIVE }

# 11.18 PROPERTIES -

Air:  $\rho = 5 \times 10^{-3} \frac{\text{slug}}{\text{ft}^3}$   $\nu = 8 \times 10^{-5} \text{ ft}^2/\text{s}$

H<sub>2</sub>O:  $\rho = 1.94 \text{ "}$   $\nu = 1 \times 10^{-5} \text{ "}$

$$Re_m = Re_p$$

$$U_m = U_p \left[ \frac{L_p}{L_m} \frac{\nu_m}{\nu_p} \right]$$

$$= 60 \left[ \left( \frac{1}{2} \right) \left( \frac{8 \times 10^{-5}}{1 \times 10^{-5}} \right) \right]$$

$$= 240 \text{ MPH}$$

11.19

VARIABLES DIMENSIONS

L	L
a	L
U	LA
$\gamma$	t
g	L/t <sup>2</sup>

$$L = 5 - 2 = 3$$

{ NOTE -  $r = 2$  - NOT THE  
NO. OF FUNDAMENTAL DIM-  
ENSIONS - NO M }

CHOOSE CORE AS L, g

$$\pi_1 = L^a g^b a \sim = \frac{a}{L}$$

$$\pi_2 = L^c g^d U \sim = \frac{U}{(Lg)^{1/2}}$$

$$\pi_3 = L^E g^F \gamma \sim = \gamma \sqrt{\frac{g}{L}}$$

a) FOR GEOMETRIC SIMILARITY

$$\pi_1|_m = \pi_1|_p$$

$$\begin{aligned} a_m &= a_p \frac{L_m}{L_p} \\ &= 2m \left( \frac{1}{360} \right) \\ &= 0.0056m = \underline{\underline{5.6mm}} \end{aligned}$$

DYNAMIC SIMILARITY DICTATES-

$$\pi_2|_m = \pi_2|_p$$

$$\begin{aligned} U_m &= U_p \frac{(Lg)_m^{1/2}}{(Lg_p)^{1/2}} \\ &= 8m/s \left( \frac{1}{360} \right)^{1/2} \\ &= \underline{\underline{0.421 m/s}} \end{aligned}$$

11.19 - CONTINUED

KINEMATIC SIMILARITY DICTATES

$$\pi_3|_m = \pi_3|_p$$

$$\gamma_m = \gamma_p \left[ \sqrt{\frac{g}{L}} \right]_p \left[ \sqrt{\frac{L}{g}} \right]_m$$

$$= 12AR \left( \frac{1}{360} \right)^{1/2} = 0.632 AR = \underline{\underline{37.9 MIN.}}$$

11.20 FOR EQUAL REYNOLDS NOS.:

$$Re|_m = Re|_p$$

$$\frac{L_p U_p}{\mu} = \frac{L_m U_m}{\mu}$$

$$S_m = S_p \left( \frac{L_p}{L_m} \right) \left( \frac{U_p}{U_m} \right) \left( \frac{\mu_m}{\mu_p} \right)$$

FOR IDEAL GAS BEHAVIOR -  $S = \frac{P}{RT}$

$$P_m = P_p \left( \frac{T_m}{T_p} \right) \left( \frac{L_p}{L_m} \right) \left( \frac{U_p}{U_m} \right) \left( \frac{\mu_m}{\mu_p} \right)$$

$$P_p = 287 \text{ Pa}$$

$$T_p = 250.4 \text{ K} \quad T_m = 294 \text{ K}$$

$$U_m \approx 340.3 \text{ m/s} \quad U_p = 317.2 \text{ m/s}$$

$$\mu_m = 1.22 \times 10^{-5} \text{ kg/m}\cdot\text{s}$$

$$\mu_p = 9.53 \times 10^{-6} \text{ kg/m}\cdot\text{s}$$

$$\begin{aligned} P_m &= 287 \left( \frac{294}{250.4} \right) \left( \frac{1}{0.4} \right) \left( \frac{317.2}{340.3} \right) \left( \frac{1.22 \times 10^{-5}}{9.53 \times 10^{-6}} \right) \\ &= 1000 \text{ Pa} \sim \underline{\underline{1 \text{ kPa}}} \end{aligned}$$

11.20 - CONTINUED

DIMENSIONLESS TIME SCALE:

$$t^* = \frac{tU}{L}$$

$$\therefore \frac{tU}{L}|_m = \frac{tU}{L}|_p$$

$$\begin{aligned} \frac{t_m}{t_p} &= \left( \frac{U_p}{U_m} \right) \left( \frac{L_m}{L_p} \right) \\ &= \left( \frac{317.2}{340.3} \right) \left( \frac{0.4}{1} \right) \\ &= \underline{\underline{0.373}} \end{aligned}$$

11.21  $Fr = \frac{U^2}{gL}$  SPEED RATIO =  $\frac{U}{N\Delta}$

	MODEL	PROTOTYPE
L	0.41	2.45
U	2.58	U

EQUATING FROUDE NUMBERS

$$\begin{aligned} \frac{U^2}{gL}|_m &= \frac{U^2}{gL}|_p \\ U_p &= 2.58 \left( \frac{2.45}{0.41} \right)^{1/2} \\ &= \underline{\underline{6.31 \text{ m/s}}} \end{aligned}$$

EQUATING  $U/N\Delta$

$$\frac{U}{N\Delta}|_m = \frac{U}{N\Delta}|_p$$

11.21 CONTINUED

$$N\Delta|_p = N\Delta|_m \frac{U_p}{U_m}$$

$$\begin{aligned} N_p &= N_m \left( \frac{\Delta_m}{\Delta_p} \right) \left( \frac{U_p}{U_m} \right) \\ &= 450 \left( \frac{0.41}{2.45} \right) \left( \frac{6.31}{2.58} \right) \\ &= \underline{\underline{184 \text{ RPM}}} \end{aligned}$$

THRUST FORCE INVOLVES EULER NO.

$$Eu|_m = Eu|_p$$

$$\begin{aligned} \frac{F/A}{\rho U^2/2}|_m &= \frac{F/A}{\rho U^2/2}|_p \\ F_p &= F_m \left( \frac{U_p}{U_m} \right)^2 \left( \frac{\rho_p}{\rho_m} \right) \left( \frac{L_p}{L_m} \right)^2 \\ &= 245 \left( \frac{6.31}{2.58} \right)^2 \left( \frac{2.45}{0.41} \right)^2 \\ &= \underline{\underline{52.3 \text{ kN}}} \end{aligned}$$

$$\text{TORQUE} = FL$$

-FROM EULER #

$$\begin{aligned} \frac{T/L}{\rho U^2/2}|_m &= \frac{T/L}{\rho U^2/2}|_p \\ T_p &= T_m \left( \frac{L_p}{L_m} \right) \left( \frac{U_p}{U_m} \right)^2 \\ &= 20 \left( \frac{2.45}{0.41} \right) \left( \frac{6.31}{2.58} \right)^2 \\ &= \underline{\underline{715 \text{ N.m}}} \end{aligned}$$

## CHAPTER 12

12.1 AT TRANSITION  $Re_0 = 2300$

$$Re = \frac{D U}{\nu} = 2300$$

$$\text{H}_2\text{O @ } 20^\circ\text{C}: \nu = 0.995 \times 10^{-6} \text{ m}^2/\text{s}$$

$$U = \frac{2300 (0.995 \times 10^{-6})}{0.038 \text{ m}}$$

$$= 0.060 \text{ m/s} \sim 6 \text{ cm/s}$$

12.2  $F_D = C_D A \rho U^2 / 2$

$$\text{For } 35000 \text{ FT} - \rho = 0.0237 \text{ lbm/ft}^3$$

$$\text{S.L. } \rho = 0.0766 \text{ "}$$

a) @ 35,000 FT  $500 \text{ MPH} = 733 \text{ FT/s}$

$$F_{Df} = \frac{0.011 (2400) (0.0237) (733)^2}{2 (32.2)}$$

$$= 5220 \text{ lbf}$$

b) @ SEA LEVEL  $700 \text{ MPH} = 293 \text{ FT/s}$

$$F_D = \frac{0.011 (2400) (0.0766) (293)^2}{2 (32.2)}$$

$$= 2700 \text{ lbf}$$

12.3 AT TRANSITION  $Re_x = 2 \times 10^5$

$$\text{For AIR @ } 20^\circ\text{C}, \nu = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Re_x = x U / \nu$$

$$x = (2 \times 10^5) (1.505 \times 10^{-5}) / 30 \text{ m/s}$$

$$= 0.100 \text{ m}$$

$$12.4 \quad \frac{u_x}{U_\infty} = C_1 + C_2 y + C_3 y^2 + C_4 y^3$$

BOUNDARY CONDITIONS:

$$(1) \quad u_x(0) = 0$$

$$(2) \quad u_x(\delta) = U_\infty$$

$$(3) \quad \frac{\partial u_x}{\partial y}(\delta) = 0$$

ONE MORE B.C. IS NEEDED -

$$\text{IF } \frac{dp}{dx} = 0 \text{ THE OTHER ONE IS } \frac{\partial^2 u_x}{\partial y^2}(0) = 0$$

BUT THIS ISN'T THE CASE CONSIDERED

THE GOVERNING EON OF MOTION IS

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = -\frac{dp}{dx} + \mu \frac{\partial^2 u_x}{\partial y^2}$$

$$\text{AT } y=0 - u_x = u_y = 0$$

$\frac{dp}{dx}$  CAN BE RELATED TO  $U_\infty$  BY THE BERNOLLI EON:  $\frac{p}{\rho} + \frac{U_\infty^2}{2} = \text{CONST}$

$$\text{SO } \frac{dp}{dx} = -\rho U_\infty \frac{dU_\infty}{dx}$$

∴ @  $y=0$  THE EON OF MOTION GIVES

$$(4) \quad \frac{\partial^2 u_x}{\partial y^2} = -\frac{1}{\nu} U_\infty \frac{dU_\infty}{dx}$$

THIS IS THE 4th B.C.

FROM (1):  $C_1 = 0$  THE REMAINING EXPRESSION FOR  $\frac{u_x}{U_\infty}$  WILL BE

$$\frac{u_x}{U_\infty} = C_2 \frac{y}{\delta} + C_3 \left(\frac{y}{\delta}\right)^2 + C_4 \left(\frac{y}{\delta}\right)^3$$

12.4 CONTINUED -

From (2)  $1 = C_2 + C_3 + C_4$

(3)  $0 = C_2 + 2C_3 + 3C_4$

(4)  $-\frac{\delta^2}{\nu} \frac{dU_\delta}{dy} = 2C_3$

SUBSTITUTION YIELDS:

$$\frac{U_x}{U_\delta} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3$$

$$+ \frac{\delta^2}{4\nu} \frac{dU_\delta}{dy} \left[ \frac{y}{\delta} - 2 \left( \frac{y}{\delta} \right)^2 + \left( \frac{y}{\delta} \right)^3 \right]$$

12.5 GIVEN  $U_x = X \sin \beta y$

FOR A LAMINAR B.L.  $\frac{d\rho}{dx} = 0$

B.C. (1)  $U_x(0) = 0$

(2)  $U_x(\delta) = U_\infty$

(3)  $\frac{dU_x}{dy}(\delta) = 0$

From (1)  $0 = 0$  — NO HELP

(2)  $U_\infty = X \sin \beta \delta$

(3)  $0 = X \beta \cos \beta \delta$

GIVING  $\beta \delta = \pi/2 \quad \beta = \pi/2\delta$

$X = U_\infty$

So PROFILE IS  $U_x = U_\infty \sin\left(\frac{\pi y}{2\delta}\right)$

VON KARMAN INTEGRAL EQN FOR B.L

$$\frac{\delta}{\rho} = \frac{d}{dx} \int_0^\delta U_x (U_\infty - U_x) dy$$

$$\frac{\delta}{\rho} = \frac{\mu}{\rho} \frac{dU_x}{dy} \Big|_0 = \frac{\mu}{\rho} U_\infty \frac{\pi}{2\delta} \cos \frac{\pi y}{2\delta} \Big|_0$$

12.5 CONTINUED -

EQUATING

$$\int_0^\delta U_x (U_\infty - U_x) dy$$

$$= U_\infty^2 \int_0^\delta \frac{U_x}{U_\infty} \left(1 - \frac{U_x}{U_\infty}\right) dy$$

$$= U_\infty^2 \int_0^\delta \left[ \sin^2 \frac{\pi y}{2\delta} - \sin^4 \frac{\pi y}{2\delta} \right] dy$$

$$= U_\infty^2 \left[ -\frac{2\delta}{\pi} \frac{\cos \pi y}{2\delta} - \frac{y}{2} + \frac{\delta}{2\pi} \frac{\sin \pi y}{\delta} \right]_0^\delta$$

$$= U_\infty^2 \left[ -\frac{2\delta}{\pi} - \frac{\delta}{2} \right] = U_\infty^2 \delta \left[ \frac{2}{\pi} - \frac{1}{2} \right]$$

NOW:  $\frac{d}{dx} \left[ \int_0^\delta \sim \right] = \frac{d}{dx} \left\{ U_\infty^2 \delta \left[ \frac{2}{\pi} - \frac{1}{2} \right] \right\}$

$$= \left( \frac{2}{\pi} - \frac{1}{2} \right) U_\infty^2 \frac{d\delta}{dx}$$

EQUATING BOTH PARTS:

$$\frac{\nu U_\infty \pi}{2\delta} = \left( \frac{2}{\pi} - \frac{1}{2} \right) U_\infty^2 \frac{d\delta}{dx}$$

$$\delta d\delta = \frac{\nu \pi^2}{U_\infty (4-\pi)} \int_0^x dx$$

$$\delta = \left[ \frac{\nu x \pi^2}{U_\infty (4-\pi)} \right]^{1/2}$$

$$\delta = 4.81 \sqrt{\frac{\nu x}{U_\infty}} = \frac{4.81 x}{\sqrt{Re_x}}$$

$$C_{fx} = \frac{\tau}{\rho U_\infty^2 / 2} = \frac{\mu U_\infty \pi / 2\delta}{\rho U_\infty^2 / 2} = \frac{\nu \pi}{U_\infty \delta}$$

PUTTING IN OUR EXPRESSION FOR  $\delta$   
WE HAVE

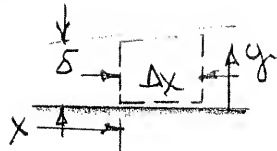
$$C_{fx} = 0.653 Re_x^{-1/2}$$

12.5 CONTINUED -

$$\begin{aligned}
 C_{FL} &= \frac{1}{L} \int_0^L C_{fx} dx \\
 &= \frac{0.653}{L} \sqrt{\frac{D}{U_\infty}} \int_0^L x^{-1/2} dx \\
 &= \frac{0.653}{L} \sqrt{\frac{D}{U_\infty}} (2x^{1/2})_0^L \\
 &= \underline{1.306 Re_L^{-1/2}}
 \end{aligned}$$

Comparison	APPROXIMATE	EXACT
$\delta$	$4.81 x Re_x^{-1/2}$	$5.0 x Re_x^{-1/2}$
$C_{fx}$	$0.653 Re_x^{-1/2}$	$0.644 Re_x^{-1/2}$
$C_{fL}$	$1.305 Re_L^{-1/2}$	$1.328 Re_L^{-1/2}$

12.6



MOMENTUM THEOREM:

$$\sum F_x = \iint_{CS} \rho (\vec{v} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \int_V \rho \vec{v} dV$$

$$\begin{aligned}
 \sum F_x &= P|_x - P|_{x+\Delta x} \\
 &+ P|_x + P|_{x+\Delta x} (\delta|_{x+\Delta x} - \delta|_x) \\
 &- T_0 \Delta x
 \end{aligned}$$

$$\begin{aligned}
 \iint_{CS} \rho u_x (\vec{v} \cdot \vec{n}) dA &= \int_0^\delta \rho u_x^2 dy|_{x+\Delta x} \\
 &- \int_0^\delta \rho u_x^2 dy|_x - \rho u_\infty \left[ \int_0^\delta u_x dy|_{x+\Delta x} \right. \\
 &\left. - \int_0^\delta u_x dy|_x - u_{y0} \Delta x \right]
 \end{aligned}$$

12.6 - CONTINUED -

REARRANGING, DIVIDING BY  $\Delta x$ , &  
EVALUATING IN THE LIMIT AS  $\Delta x \rightarrow 0$ :

$$\begin{aligned}
 -\rho \frac{dP}{dx} &= T_0 + \rho u_\infty u_{y0} + \frac{d}{dx} \int_0^\delta \rho u_x^2 dy \\
 &- u_\infty \frac{d}{dx} \int_0^\delta \rho u_x dy
 \end{aligned}$$

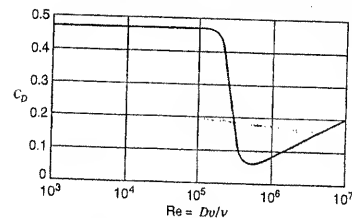
NOTING THAT BERNOULLI'S EQN APPLIES OUTSIDE THE B.L., WE CAN WRITE (SEE CHAPTER)

$$\rho \frac{dP}{dx} = \frac{d}{dx} (\rho u_\infty^2) - \rho u_\infty \frac{d}{dx} (\rho u_\infty)$$

& THE FINAL RESULT BECOMES:

$$\begin{aligned}
 \frac{T_0}{\rho} + \frac{u_\infty u_{y0}}{\rho} &= \frac{d u_\infty}{dx} \int_0^\delta (u_\infty - u_x) dy \\
 &+ \frac{d}{dx} \int_0^\delta u_x (u_\infty - u_x) dy
 \end{aligned}$$

12.7



FOR A SMOOTH SPHERE - FIG. ABOVE

$$Re_{cr} \approx 2 \times 10^5$$

$$\text{FOR AIR @ } 20^\circ\text{C } \nu = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\begin{aligned}
 u_{cr} &= \frac{\nu}{D} Re_{cr} \\
 &= \frac{(1.505 \times 10^{-5}) (2 \times 10^5)}{0.042} \\
 &= \underline{71.7 \text{ m/s}}
 \end{aligned}$$

FOR SUCH A SPHERE (GOLF BALL SEE)  
A VELOCITY GREATER THAN THIS  
WILL REDUCE DRAG & BALL WILL  
TRAVEL FURTHER

12.8 For Air @ 80°F

$$\nu = 0.169 \times 10^{-3} \text{ ft}^2/\text{s}$$

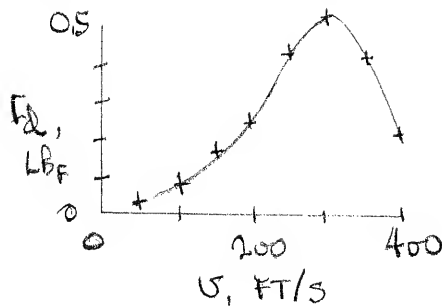
$$\rho = 0.0735 \text{ lbm/ft}^3$$

$$Re = \frac{D U}{\nu} = \frac{(1.65/12) U}{0.169 \times 10^{-3}}$$

$$F_D = A \rho \frac{U^2}{2} C_D$$

$$= \frac{\pi}{4} \left( \frac{1.65}{12} \right)^2 \left( \frac{0.0735}{32.2} \right) \frac{U^2}{2} C_D$$

$U, \text{ft/s}$	$C_D$	$Re$	$F_D, \text{Lbf}$
50	0.47	$4.07 \times 10^4$	0.0199
100	0.47	8.14	0.0797
150	0.46	12.2	0.175
200	0.45	16.3	0.305
250	0.41	20.3	0.434
300	0.35	24.4	0.534
350	0.20	28.5	0.415
400	0.08	32.5	0.217



12.9 IN THE UNSTEADY WAKE REGION

$$1 < Re < 10^3$$

@ 20°C  $\nu = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$

$$Re = D U / \nu$$

@  $Re = 1 = \frac{0.0127 U}{1.505 \times 10^{-5}}$

$$U = 0.00119 \text{ m/s}$$

@  $Re = 10^3$   $U = 1.185 \text{ m/s}$

THESE ARE THE LOWER & UPPER BOUNDS FOR  $U$

12.10 For Air @ 80°F

$$\nu = 0.169 \times 10^{-3} \text{ ft}^2/\text{s}$$

$$\rho = 0.0735 \text{ lbm/ft}^3$$

$$Re = \frac{(0.2/12)(88)}{0.169 \times 10^{-3}} = 8680$$

From Fig 12.2  $C_D \approx 1.2$

$$F_D = C_D A \rho \frac{U^2}{2}$$

$$= 1.2 \left( \frac{0.2}{12} \right) \left( \frac{3}{32.2} \right) \left( \frac{0.0735}{32.2} \right) \left( \frac{88^2}{2} \right)$$

$$= 0.530 \text{ lbf}$$

12.11  $F_D = C_D A \rho \frac{U^2}{2}$

Air @ 20°C  $\rho = 1.2048 \text{ kg/m}^3$   
 $\nu = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$

$$F_D = 0.26(2.33)(1.2048)(30)^2$$

$$= 657 \text{ N}$$

$$\text{Power} = F_D U = (657)(30) = 19.7 \text{ kW}$$

WITH A HEADWIND OF 6 m/s

$$F_D = 0.26(2.33)(1.2048)(36)^2 = 28.4 \text{ kW}$$

& A TAILWIND OF 6 m/s

$$F_D = (24)^2 = 12.6 \text{ kW}$$

FOR STILL AIR  $P = 15.9 \text{ hp}$

WITH HEADWIND  $P = 37.4 \text{ "}$

WITH TAILWIND  $P = 16.9 \text{ "}$

$$12.12 \quad 100 \text{ mi/hr} = 44.7 \text{ m/s}$$

$$F_L = C_L A S U^2 / 2$$

$$= \frac{0.21 (2.33) (1.2048) (44.7)^2}{2}$$

$$= \underline{\underline{589 \text{ N}}}$$

$$12.13 \quad \text{IF } C_L = 1$$

$$F_L = 589 \left( \frac{1}{0.21} \right) = \underline{\underline{2,805 \text{ kN}}}$$

$$12.14 \quad F_D = C_D A S U^2 / 2$$

IN SAME ENVIRONMENT AT SAME SPEED

$$C_D A |_{\text{CAR}} = C_D A |_{\text{PLATE}}$$

$$C_D A |_{\text{CAR}} = 0.26 (2.33)$$

$$C_D A |_{\text{PLATE}} = 1.1 \frac{\pi D^2}{4}$$

$$D = \left[ \frac{0.26 (2.33) (4)}{(1.1) (\pi)} \right]^{1/2}$$

$$= \underline{\underline{0.837 \text{ m}}}$$

$$12.15 - \text{CIRCULAR SIGN} - D = 8 \text{ FT}$$

$$U = 120 \text{ MPH} \quad (176 \text{ FT/S})$$

$$F_D = C_D A S U^2 / 2$$

$$\text{AT } 80^\circ \text{F} - \rho = 0.0735 \text{ lb}_m / \text{ft}^3$$

$$F_D = \frac{1.1 \left( \frac{\pi}{4} \right) (8)^2 (0.0735) (176)^2}{32.2 (2)}$$

$$= \underline{\underline{1955 \text{ lbf}}}$$

$$12.16 \quad \text{FOR AIR @ } 100^\circ \text{F} - \rho = 0.0710 \text{ lb}_m / \text{ft}^3$$

$$0^\circ \text{F} - \rho = 0.0862 "$$

$$C_D = 0.18 \quad A = 2.4 \text{ m}^2 = 25.83 \text{ ft}^2$$

$$P = F_D U = C_D A S U^3 / 2$$

$$= \frac{0.18 (25.83) (0.0710) (102.7)^3}{2 (32.2) (550)}$$

$$= \underline{\underline{15.7 \text{ hp}}} \quad \text{AT } 100^\circ \text{F}$$

$$= 15.7 \left( \frac{0.0862}{0.0710} \right) = \underline{\underline{19.1 \text{ hp}}} \quad \text{AT } 0^\circ \text{F}$$

$$12.17 \quad \text{SPHERE} - D = 9.25 / \pi = 2.94 \text{ IN}$$

$$\text{WT} = 5.25 \text{ OUNCES}$$

$$\text{AT } U = 95 \text{ MPH} \quad (139.3 \text{ FT/S})$$

$$\text{AT } 80^\circ \text{F} \quad \rho_{\text{AIR}} = 0.0735 \text{ lb}_m / \text{ft}^3$$

$$\mu_{\text{AIR}} = 0.169 \times 10^{-3} \text{ FT}^2 / \text{S}$$

$$a) \quad Re = \frac{DU}{\mu} = \frac{(2.94/12) (139.3)}{0.169 \times 10^{-3}} = \underline{\underline{2.02 \times 10^5}}$$

$$F_D = C_D A S U^2 / 2$$

$$b) \quad \text{AT } Re = 2.02 \times 10^5 \quad C_D \approx 0.4$$

$$F_D = \frac{0.4 \left( \frac{\pi}{4} \right) \left( \frac{2.94}{12} \right)^2 (0.0735) (139.3)^2}{2 (32.2)}$$

$$= \underline{\underline{0.418 \text{ lbf}}}$$

c) FLOW IS NEAR TRANSITION - BUT STILL IN LAMINAR RANGE

12.18

Re · 10 <sup>-4</sup>	7.5	10	15	20	25
C <sub>D</sub>	0.48	0.38	0.22	0.12	0.10

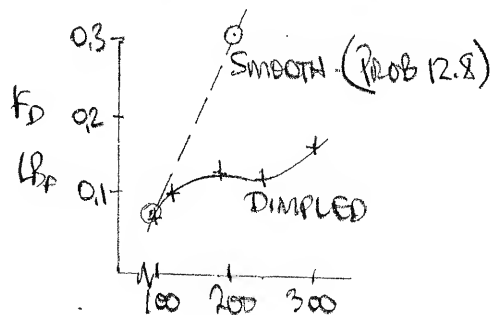
At 80°F -  $\nu = 0.169 \times 10^{-3} \text{ ft}^2/\text{s}$   
 $\rho = 0.0735 \text{ lbm/ft}^3$

For  $Re = 7.5 \times 10^4 = Dv/\nu$   
 $v = \frac{(7.5 \times 10^4)(0.169 \times 10^{-3})}{1.65/12}$   
 $= 92.18 \text{ ft/s}$

$F_D = C_D A \rho v^2 / 2$   
 $= \frac{0.48 \left( \frac{\pi}{4} \right) \left( \frac{1.65}{12} \right)^2 (0.0735) (92.18)^2}{2 (32.2)}$   
 $= 0.069 \text{ lbf}$

Doing this calculation for all given conditions we generate the following:

$Re \times 10^4$	$v$	$C_D$	$F_D, \text{lbf}$
7.5	92.2	0.48	0.069
10	122.9	0.48	0.100
15	184.4	0.47	0.129
20	245.8	0.44	0.125
25	307.3	0.10	0.164



12.19  $W_T = 5.25 \text{ ounces} = 0.328 \text{ lbf}$

$F_L = W_T = C_L A \rho v^2 / 2$

$A = \frac{\pi}{4} \left( \frac{2.94}{12} \right)^2 = 0.04714 \text{ ft}^2$

$C_L = 0.224$

From PROBLEM STATEMENT

$C_L \approx 0.24 \frac{R \cdot \Omega}{v} - 0.05$

So FOR THIS CASE

$\frac{R \cdot \Omega}{v} = \frac{0.224 + 0.05}{0.24} = 1.142$

$v = 110 \text{ mph} = 161.3 \text{ ft/s}$

$\Omega = \frac{1.142 (161.3)}{0.385} = 478 \text{ rad/s}$   
 $= 76.1 \text{ rev/s}$

To TRAVEL 60.5 ft  $t = \frac{60.5}{161.3} = 0.375 \text{ s}$

No of REVOLUTIONS =  $76.1 (0.375) = \underline{\underline{28.5}}$

12.20 BLASIUS EQN FOR LAMINAR BOUNDARY LAYER FLOW IS

$\rho \frac{D u_x}{D t} = \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u_x}{\partial y^2}$

OR, WRITTEN AS

$u_x \frac{\partial u_x}{\partial x} + v_y \frac{\partial u_x}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left[ \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} \right]$

AT  $y=0$  -  $u_x=0$  BUT, IN THIS CASE,  $v_y \neq 0$   
 THE RESULTING FORM IS

$v_y \frac{\partial u_x}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u_x}{\partial y^2}$

↑  
 THIS TERM IS NOT PRESENT FOR  $v_y(0)=0$  (EQN (12-33) CASE

### 12.21 TURBULENCE INTENSITY

$$I = \left[ \frac{(\overline{U_x^2} + \overline{U_y^2} + \overline{U_z^2})/3}{U_p^2} \right]^{1/2}$$

$$\text{KINETIC ENERGY} = \frac{U_p^2 + \overline{U_x^2} + \overline{U_y^2} + \overline{U_z^2}}{2} = \frac{U_p^2 (1 + 3I^2)}{2}$$

$$\text{FOR } I = 0.1 \quad \text{K.E.}_{\text{TOTAL}} = \frac{U_p^2}{2} (1.03)$$

$$\text{WHILE} \quad \text{K.E.}_{\text{TURB}} = \frac{U_p^2}{2} (0.03)$$

FRACTION DUE TO TURBULENCE

$$= \frac{0.03}{1.03} = 2.91\%$$

$$12.22 \quad \dot{V} = 2 \text{ gpm} = 0.446 \times 10^{-2} \text{ ft}^3/\text{s}$$

$$V = \frac{\dot{V}}{A} = \frac{0.446 \times 10^{-2}}{\pi/4 (0.75/12)^2} = 1.45 \text{ ft/s}$$

$$\text{FOR } H_2O @ 120^\circ F \quad \nu = 0.62 \times 10^{-5} \text{ ft}^2/\text{s}$$

$$@ 45^\circ F \quad \nu = 1.57 \times 10^{-5} \text{ "}$$

$$@ 120^\circ F \quad Re = \frac{(0.75/12)(1.45)}{0.62 \times 10^{-5}} = 14,600 \quad (a)$$

$$@ 45^\circ F \quad Re = \frac{(0.75/12)(1.45)}{1.57 \times 10^{-5}} = 5770 \quad (b)$$

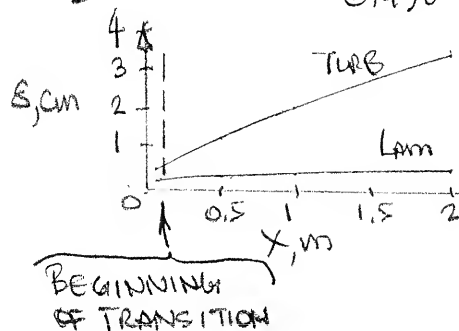
$$12.23 \text{ LAMINAR FLOW: } \frac{\delta}{x} = 5 Re_x^{-1/2}$$

$$\text{TURBULENT " } \frac{\delta}{x} = 0.376 Re_x^{-0.2}$$

$$\text{FOR AIR @ } 20^\circ C \quad \nu = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Re_x = \frac{30 \times x}{1.505 \times 10^{-5}} = 2 \times 10^6 x$$

x, m	Re <sub>x</sub>	δ <sub>L</sub> , cm	δ <sub>t</sub> , cm
0	0	0	0
0.1	2 × 10 <sup>5</sup>	0.111	0.327
0.5		0.249	1.126
1		0.352	2.063
2		0.498	3.591



$$12.24 \quad \dot{V} = 0.006 \text{ m}^3/\text{s}$$

$$V = \frac{0.006}{(\pi/4)(0.15)^2} = 0.34 \text{ m/s}$$

TO CALCULATE  $y^+ \frac{1}{2} U^+$ :

$$\frac{\nu}{\bar{U}} = 0.0225 \bar{U}_{\text{MAX}} \left[ \frac{\nu}{U_{x\text{MAX}} y_{\text{MAX}}} \right]$$

FROM RESULTS OF 1/7 POWER LAW:

$$\bar{U} = 0.817 U_{x\text{MAX}} \sim U_{x\text{MAX}} = 0.416 \text{ m/s}$$

$$y_{\text{MAX}} = 0.075 \text{ m}$$

$$\text{AT } 20^\circ C \quad \nu = 0.995 \times 10^{-6} \text{ m}^2/\text{s}$$

SUBSTITUTING INTO  $\nu/\bar{U}$  EXPRESSION

$$\sqrt{\nu/\bar{U}} = 0.0171 \text{ m/s}$$

12.24 CONTINUED

- LAMINAR SUBLAYER:

$$y^+ = \frac{\sqrt{f/8} y}{D} = 5$$

$$y = \frac{5(0.995 \times 10^{-6})}{0.0171} = \underline{\underline{0.291 \text{ mm}}}$$

BUFFER LAYER -

EXTENDS FOR  $5 < y^+ < 30$

$$@ y^+ = 30 \quad y = 1.746 \text{ mm}$$

$$\text{THICKNESS}_{BL} = \underline{\underline{1.455 \text{ mm}}}$$

TURBULENT CORE EXTENDS

FROM  $y = 1.455 \text{ mm}$

TO  $y = 75 \text{ mm}$

$$\text{THICKNESS}_{T.C.} = \underline{\underline{73.55 \text{ mm}}}$$

12.25 FOR (12-68)

$$\delta_0/\delta = 0.0225 U_{\max}^2 \left[ \frac{D}{U_{x\max} y_{\max}} \right]^{1/4}$$

$$C_{fx} = \frac{\delta_0/\delta}{U_{\text{AV}}^2/2}$$

$$= 0.045 \left( \frac{U_m}{U_{\text{AV}}} \right)^2 \left[ \right]$$

$$\left[ \right] = \left[ \frac{U_m}{U_{\text{AV}}} \frac{U_{\text{AV}} R}{D} \right]^{1/4}$$

$$= \left( \frac{U_m}{U_{\text{AV}}} \right)^{-1/4} \left( 2^{1/4} \right) Re_D^{-1/4}$$

$$\text{GIVEN } C_{fx} = 0.0535 \left( \frac{U_m}{U_{\text{AV}}} \right)^{7/4} Re_D^{-1/4}$$

NOW - TO FIND  $\frac{U_m}{U_{\text{AV}}}$  FOR PIPE FLOW

12.25 CONTINUED

$$U_{\text{AV}} (\pi R^2) = \int_A U dA$$

$$= 2\pi \int_0^R U r dr$$

$$U_{\text{AV}} = \frac{2}{R^2} \int_0^R U_{\max} \left( 1 - \frac{r}{R} \right)^{1/4} r dr$$

$$\text{DOING THE MATH: } \frac{U_m}{U_{\text{AV}}} = 1.225$$

$$C_{fx} = 0.0763 Re_D^{-1/4}$$

$$12.26 \quad Re_L = \frac{LU}{\nu} = \frac{0.5(40)}{0.159 \times 10^{-3}} = 125,800$$

$$C_{fL} = \frac{1}{L} \int_0^L C_{fx} dx = \frac{1}{L} \int_0^L \frac{0.0576 x}{Re_x^{1/5}} dx$$

$$= 0.072 Re_L^{-1/5}$$

$$C_{fL} = 0.072 (125,800)^{-1/5} = 6.877 \times 10^{-3}$$

FOR 2 SIDES  $\frac{1}{2}$  60°F AIR

$$C_A = 2 C_{fL} A P \frac{U^2}{2}$$

$$= \frac{2 (6.877 \times 10^{-3}) (1.5) (0.0764) (40)^2}{2 (32.2)}$$

$$= \underline{\underline{0.0392 \text{ lbf}}} \quad (a)$$

FOR LAMINAR FLOW

$$C_{fL} = 1.328 Re_L^{-1/2} = 0.00375$$

$$F_D = 2 C_{fL} A P \frac{U^2}{2}$$

$$= \underline{\underline{0.0213 \text{ lbf}}} \quad (b)$$

12.27

$$Re = 10^5$$

LAMINAR FLOW -  $\delta_L = 5 Re_x^{-1/2}$

TURBULENT "  $\delta_t = 0.375 Re_x^{-0.2}$

for  $Re = 10^5$   $\frac{\delta_t}{\delta_L} = 2.38$

FROM CHAPTER 5: MOMENTUM  $\sim \rho U^2$

" " 6: ENERGY  $\sim \rho U^3$

FOR  $U = U_p f(\frac{y}{\delta})$

MOMENTUM =  $\rho U_p^2 f^2(\frac{y}{\delta})$

ENERGY =  $\rho U_p^3 \frac{f^3(\frac{y}{\delta})}{2}$

$$\frac{M}{\rho U_p^2} = f^2\left(\frac{y}{\delta}\right)$$

$$\frac{E}{\rho U_p^3/2} = f^3\left(\frac{y}{\delta}\right)$$

FOR LAMINAR CASE:

$$\frac{M}{\rho U_p^2} = \sin^2\left(\frac{y}{\delta_L} \frac{\pi}{2}\right)$$

$$\frac{E}{\rho U_p^3/2} = \sin^3\left(\frac{y}{\delta_L} \frac{\pi}{2}\right)$$

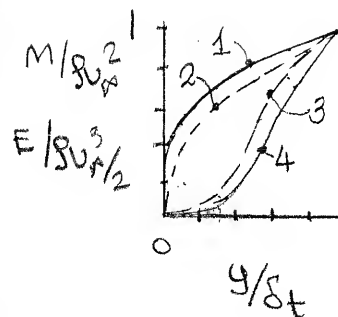
$y/\delta_L$	$\sin \frac{y}{\delta_L} \frac{\pi}{2}$	$\frac{M}{\rho U_p^2}$	$\frac{E}{\rho U_p^3/2}$
0	0	0	0
0.1	0.156	0.0244	0.0038
0.3	0.455	0.207	0.094
0.5	0.707	0.50	0.354
0.7	0.89	0.795	0.708
0.9	0.99	0.98	0.97
1	1.0	1.0	1.0

12.27 CONTINUED

FOR TURBULENT CASE:

$$\frac{M}{\rho U_p^2} = \left(\frac{y}{\delta_t}\right)^{2/7} \quad \frac{E}{\rho U_p^3/2} = \left(\frac{y}{\delta_t}\right)^{3/7}$$

$y/\delta_t$	$\frac{M}{\rho U_p^2}$	$\frac{E}{\rho U_p^3/2}$
0	0	0
0.1	0.518	0.373
0.3	0.709	0.600
0.5	0.820	0.743
0.7	0.903	0.858
0.9	0.970	0.956
1	1	1



- 1 - MOM TURB
- 2 - ENERGY "
- 3 - MOM LAM
- 4 - ENERGY "

12.28

$$F_D = C_F L A \rho U^2/2$$

$$A = (7)(40)(2) = 560 \text{ FT}^2 \quad \{2 \text{ SIDES}\}$$

$$U = 140 \text{ MPH} = 205.3 \text{ FT/S}$$

@ 500 FT -  $\rho = 0.0660 \text{ LB}_m/\text{FT}^3$

$$\mu = 1.165 \times 10^{-5} \text{ LB}_m/\text{FT}\cdot\text{S}$$

$$Re = \frac{LU}{\mu} = \frac{7(205.3)}{1.165 \times 10^{-5} / 0.0660} = 8.14 \times 10^6$$

12.28 CONTINUED -

a) LAMINAR

$$C_{fL} = 1.328 Re_L^{-1/2} = 0.000465$$

$$F_D = \frac{(0.000465)(560)(0.0660)(205.3)^2}{2(32.2)}$$

$$= \underline{11.26 \text{ lbf}}$$

b) TURBULENT

$$C_{fL} = 0.072 Re_L^{-0.2} = 0.00299$$

$$F_D = \underline{72.3 \text{ lbf}}$$

12.29  $Re_x = 10^6$

B.L. THICKNESS -

LAM:  $\delta = 5 \times Re_x^{-1/2}$

TURB:  $\delta = 0.376 \times Re_x^{-1/5}$

$$\delta_{1/5} = \frac{0.376}{5} Re_x^{0.3} = \underline{4.74}$$

COEF. OF SKIN FRICTION:

LAM  $C_{fx} = 0.664 Re_x^{-1/2}$

TURB  $C_{fx} = 0.0576 Re_x^{-0.2}$

$$\frac{C_{fxT}}{C_{fxL}} = \frac{0.0576}{0.664} Re_x^{0.3} = \underline{5.47}$$

12.30 FOR TURBULENT B.L. - WATER

@ 60°F  $\rho = 62.3 \text{ lbm/ft}^3$

$\nu = 1.22 \times 10^{-5} \text{ ft}^2/\text{s}$

$U = 20 \text{ ft/s}$

$$Re_L = \frac{20(20)}{1.22 \times 10^{-5}} = 3.28 \times 10^7$$

12.30 CONTINUED

$$\delta = 0.376 \times Re_x^{-0.2}$$

$$= 0.376(20)(3.28 \times 10^7)^{-0.2} = \underline{0.240 \text{ ft}}$$

$$C_{fL} = 0.072 Re_L^{-0.2} = 0.00226$$

$$F_D = C_{fL} A S U^2/2$$

$$= \frac{(0.00226)(200)(2)(62.3)(20)^2}{2(32.2)}$$

$$= \underline{350 \text{ lbf}}$$

IF FLOW IS LAMINAR -

$$C_{fL} = 1.328 Re_L^{-1/2} = 2.319 \times 10^{-4}$$

$$F_D = \underline{35.91 \text{ lbf}}$$

12.31 EXPANDING  $U'_x(x,y)$  IN TAYLOR SERIES:

$$U'_x(x,y) = U'_x(0,0) + x \frac{\partial U'_x}{\partial x}(0,0) + y \frac{\partial U'_x}{\partial y}(0,0) + \frac{x^2}{2} \frac{\partial^2 U'_x}{\partial x^2}(0,0) + \frac{y^2}{2} \frac{\partial^2 U'_x}{\partial y^2}(0,0) + \frac{x^2}{2} \frac{\partial^2 U'_x}{\partial x \partial y}(0,0) + \dots$$

As  $y \rightarrow 0$   $x' \rightarrow 0$

THE EXPRESSION FOR  $U'_x$  REDUCES TO

$$U'_x(x,y) = a_1 y + a_2 y^2 + a_3 x y + \dots$$

WHERE  $a_1 = \frac{\partial U'_x}{\partial y} \Big|_0$  - ETC.

SIMILARLY

$$U'_y(x,y) = b_1 y + b_2 y^2 + b_3 x y + \dots$$

WHERE  $b_1 = \frac{\partial U'_y}{\partial y} \Big|_0$  - ETC.

12.31 CONTINUED

CONTINUITY EQN REQUIRES THAT

$$\frac{\partial u_x'}{\partial x} + \frac{\partial u_y'}{\partial y} = 0$$

GIVEN:  $a_3 y + b_1 + 2b_2 y + b_3 x = 0$

COEFFICIENTS OF LIKE PWS OF  $x \frac{1}{2} y$

REQUIRE  $a_3 + 2b_2 = 0$

$$b_1 = b_3 = 0$$

SO  $u_x'(x, y) = a_1 y + a_2 y^2 + a_3 x y$

$$u_y'(x, y) = -a_3 y^2$$

$$u_x' u_y' = -a_3 a_1 y^3 - a_3 a_3 y^4$$

TAKING TIME AVERAGE

$$\overline{u_x' u_y'} = -a_3 a_1 y^3 + \dots$$

i.e.  $\overline{u_x' u_y'} \approx y^3$

WHILE MIXING LENGTH THEORY

SAYS  $\overline{u_x' u_y'} \approx y^2$

12.32 POWER LAW PROFILE -

$$\frac{u_x}{u_{max}} = \left(\frac{y}{R}\right)^{1/n}$$

$$\frac{\partial u_x}{\partial y} = \frac{u_{max}}{n} \frac{y^{1/n-1}}{R^{1/n}}$$

AS  $y \rightarrow 0$   $\frac{\partial u_x}{\partial y} \rightarrow \infty$

AS  $y \rightarrow R$   $\frac{\partial u_x}{\partial y} \rightarrow \frac{u_{max}}{nR}$

12.33  $\delta_0 = 0.0225 \delta u_{max}^2 \left(\frac{\nu}{u_{max} \delta}\right)^{1/4}$  (1)

FOR  $\frac{u_x}{u_{max}} = \left(\frac{y}{\delta}\right)^{1/n}$

$$\frac{\delta_0}{\delta u_{max}^2} = \frac{2}{\delta x} \int_0^\delta \frac{u_x}{u_{max}} \left(1 - \frac{u_x}{u_{max}}\right) dy$$

$$= \frac{2\delta}{\delta x} \int_0^1 \left[\left(\frac{y}{\delta}\right)^{1/n} - \left(\frac{y}{\delta}\right)^{2/n}\right] d\left(\frac{y}{\delta}\right)$$

$$= \left[\frac{1}{1+1/n} - \frac{1}{1+2/n}\right] \frac{2\delta}{\delta x}$$

EQUATING WITH (1) & DOING ALGEBRA

$$0.0225 \left(\frac{\nu}{u_{max} \delta}\right)^{1/4} = \frac{n}{(n+1)(n+2)} \frac{2\delta}{\delta x}$$

$$0.0225 \left(\frac{\nu}{u_{max}}\right)^{1/4} \int_0^x \frac{1}{\delta^{1/4}} dx = \frac{n}{(n+1)(n+2)} \int_0^\delta \delta^{1/4} d\delta$$

BECOMES

$$\left(\frac{\delta}{x}\right)^{5/4} = \frac{(n+1)(n+2)}{n} (0.0225) Re_x^{-1/4}$$

& FINALLY

$$\frac{\delta}{x} = \left[0.0281 \frac{(n+1)(n+2)}{n}\right]^{0.8} Re_x^{-0.2}$$

# CHAPTER 13

- 13.1 OIL -  $\nu = 0.08 \times 10^{-3} \text{ ft}^2/\text{s}$   
 $\rho = 57 \text{ lbm/ft}^3$   
 $\dot{V} = 10 \text{ gal/hr}$   
 TUBE - DIAM = 0.24 IN.,  $L = 50 \text{ FT}$

$$V = \frac{\dot{V}}{\pi D^2/4} = 1.18 \text{ ft/s}$$

$$Re = \frac{DV}{\nu} = \frac{(0.24/12)(1.18)}{0.08 \times 10^{-3}} = 295 \quad \left\{ \text{LAMINAR} \right\}$$

$$\frac{\Delta P}{8} = h_L = 2 f_F \frac{L}{D} V^2 \quad \left\{ f_F = 16/Re \right\}$$

$$= 2 \frac{(16)}{295} \frac{50}{0.24/12} (1.18)^2$$

$$= 377.6 \text{ ft}^2/\text{s}^2$$

$$\Delta P = \frac{57(377.6)}{32.2} = \underline{\underline{668 \text{ lbf/ft}^2}}$$

- 13.2 OIL - SAME PROPERTIES AS IN PROB 13.1

TUBE -  $D = 0.1 \text{ IN.}$ ,  $L = 30 \text{ IN.}$   
 $\Delta P = 15 \text{ lbf/in}^2$

FOR LAMINAR FLOW - USE H.P. EQN

$$\Delta P = 32 \mu V \Delta x / D^2$$

$$\text{OR } \frac{\Delta P}{8 \mu} = \frac{32 \nu V \Delta x}{g D^2}$$

$$V = \frac{(15)(144)(0.1/12)^2 (32.2)}{32(57)(0.08 \times 10^{-3})(30/12)}$$

$$= 13.24 \text{ ft/s}$$

$$\dot{V} = VA = \underline{\underline{7.22 \times 10^{-4} \text{ ft}^3/\text{s}}}$$

$$Re = \frac{(0.1/12)(13.24)}{0.08 \times 10^{-3}} = 1379 \quad \left\{ \begin{array}{l} \text{LAMINAR} \\ \text{FLOW} \\ \text{OK.} \end{array} \right\}$$

13.3  $\Delta P = 2 f_F \frac{L}{D} V^2$

FOR A SPECIFIED PIPE:  $\Delta P \propto f_F V^2$

IF FULLY TURBULENT -  $f_F \sim e/D$  ONLY

$$\therefore \Delta P \sim V^2$$

FOR  $\text{H}_2\text{O}$   $\Delta P = 13 \text{ PSI}$  FOR  $\dot{m} = 28.3 \text{ lbm/s}$

FOR LOX  $\rho = 70 \text{ lbm/ft}^3$   $\dot{m} = 35 \text{ lbm/s}$

$$\frac{\Delta P_{\text{LOX}}}{\Delta P_{\text{H}_2\text{O}}} = \frac{(\dot{m}/\rho A)^2_{\text{LOX}}}{(\dot{m}/\rho A)^2_{\text{H}_2\text{O}}}$$

$$= \left( \frac{35}{70} \right)^2 \left( \frac{62.4}{28.3} \right)^2 = 1.21$$

FOR LOX -  $\Delta P = 13(1.21) = \underline{\underline{15.8 \text{ PSI}}}$

- 13.4 ENERGY EQN:

$$-\frac{dW_s}{dt} = \dot{m} \left[ \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g \Delta y + h_L \right]$$

OIL:  $\rho = 810 \text{ kg/m}^3$   $\nu = 4.5 \times 10^{-6} \text{ m}^2/\text{s}$   
 $\dot{V} = 0.56 \text{ m}^3/\text{s}$

LINE:  $D = 0.162 \text{ m}$   $y_2 - y_1 = -250 \text{ m}$

$$\Delta P = 101.3 - 300 = -198.7 \text{ kPa}$$

COMMERCIAL STEEL

$$V = \dot{V}/A = \frac{0.56}{\pi/4 (0.162)^2} = 1.855 \text{ m/s}$$

$$\frac{P_2 - P_1}{\rho} = \frac{+198.7(1000)}{810} = +245.3 \text{ m}^2/\text{s}^2$$

$$\frac{V_2^2 - V_1^2}{2} = 0$$

$$g \Delta y = 9.81(-250) = -2452 \text{ m}^2/\text{s}^2$$

13.4 CONTINUED -

$$Re = \frac{Dv}{\omega} = \frac{0.62(1.855)}{4.5 \times 10^{-6}} = 256,000$$

FOR THIS  $Re$  VALUE  $\frac{1}{2}$  COMMERCIAL  
STEEL -  $\frac{e}{D} \approx 0.00075$

FIG 13.1 -  $f_f \approx 0.0045$

$$h_L = 2 \left( 0.0045 \right) \left( \frac{280,000}{0.62} \right) (1.855)^2$$

$$= 13990 \text{ m}^2/\text{s}^2$$

$$-\frac{\delta W_s}{dt} = (810)(0.56) [245.3 - 2452 + 13990]$$

$$= \underline{5.34 \text{ MW}}$$

13.5 SAME CONDITIONS AS IN  
PROB 13.4 EXCEPT

2 PIPES IN SERIES -

270 KM OF ORIGINAL  
PIPE

10 KM OF NEW PIPE  
WITH  $D = 0.42 \text{ m}$

FOR THE NEW SYSTEM:

$$-\frac{\delta W_s}{dt} = \dot{m} \left[ \frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g\Delta y + h_L \right]$$

$$\frac{P_2 - P_1}{\rho} = \{ \text{SAME} \} = 245.3 \text{ m}^2/\text{s}^2$$

$$\frac{v_2^2 - v_1^2}{2} = \{ \text{SAME} \} = 0$$

$$g\Delta y = \{ \text{SAME} \} = -2452 \text{ m}^2/\text{s}^2$$

$$h_L = h_{L1} + h_{L2}$$

1 --- ORIGINAL

2 --- NEW

13.5 CONTINUED -

$$h_{L1} = \frac{270,000}{280,000} (13990) = 13490 \text{ m}^2/\text{s}^2$$

FOR NEW SECTION:

$$v = \frac{0.56}{\pi/4 (0.42)^2} = 4.04 \text{ m/s}$$

$$Re = \frac{0.42(4.04)}{4.5 \times 10^{-6}} = 3.773 \times 10^5$$

$$\frac{e}{D} = 0.00012 \quad f_f \approx 0.0038$$

$$h_{L2} = 2(0.0038) \frac{10000}{0.42} (4.04)^2$$

$$= 2953 \text{ m}^2/\text{s}^2$$

$$\text{TOTAL } h_L = 13490 + 2953 = 16440 \text{ m}^2/\text{s}^2$$

NEW CASE -

$$-\frac{\delta W_s}{dt} = (810)(0.56) [245.3 - 2452 + 16440]$$

$$= \underline{6.46 \text{ MW}}$$

13.6 STEADY FLOW BETWEEN PUMPING  
STATIONS

$$0 = \frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g\Delta y + h_L$$

$$\frac{v_2^2 - v_1^2}{2} = 0$$

$$\Delta y = 0$$

$$\text{SO } \frac{P_2 - P_1}{\rho} = h_L = 2 f_f \frac{L}{D} v^2$$

$$Re = \frac{Dv}{\omega} = \frac{(0.71)(1.1)}{6.7 \times 10^{-6}} = 1.166 \times 10^5$$

$$\frac{e}{D} = 0.000068 \quad f_f \approx 0.0046$$

$$h_L = 2(0.0046) \left( \frac{320 \times 10^3}{0.71} \right) (1.1)^2$$

$$= 5017 \text{ m}^2/\text{s}^2$$

$$\Delta P = h_L / g = \underline{511 \text{ m OF OIL}}$$

13.6 CONTINUED -

$$-\frac{\delta W_s}{dt} = \dot{m} g h_L$$

$$= 801 \left( \frac{\pi}{4} \right) (0.71)^2 (1.1) (9.81) (51)$$

$$= \underline{\underline{1,749 \text{ MW}}}$$

13.7 ENERGY EQN IN STEADY FLOW.

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g \Delta y + h_L = 0$$

$$-\frac{\Delta P}{\rho} = -\frac{60(144)(32.2)}{62.4} = -4460 \text{ FT}^2/\text{s}^2$$

$$\frac{V_2^2 - V_1^2}{2} = \frac{V_2^2}{2}$$

$$g \Delta y = 0$$

$$h_L = 2f_F \frac{L}{D} V^2 + \sum K \frac{V^2}{2}$$

$$= V^2 \left[ 2f_F \frac{L}{D} + \frac{1}{2} \sum K \right]$$

$$\sum K = (6)(0.7) + 3.8 + 7.5 = 19.5$$

$$2f_F \frac{L}{D} = 2(0.007) \frac{160}{0.75/12} = 35.84$$

$$h_L = V^2 [35.84 + 7.75] = 43.6 V^2$$

$$V_2 = \frac{VA}{A_2} = V \left( \frac{D}{D_2} \right)^2 = V \left( \frac{0.75}{0.11} \right)^2 = 56.25 V$$

ENERGY EQN. BECOMES

$$-4460 + \frac{1}{2} (56.25 V)^2 + 43.6 V^2 = 0$$

$$V^2 = \frac{4460}{1625} = 2.74 \text{ FT}^2/\text{s}^2$$

$$V = 1.656 \text{ FT/s}$$

13.7 CONTINUED

NOW TO CHECK  $f_F$ :

$$Re = \left( \frac{0.75}{12} \right) (1.656) / (1.22 \times 10^{-5})$$

$$= 8480$$

$$\epsilon/D = \frac{5 \times 10^{-6} (12)}{0.75} = 0.00008$$

$$\text{FIG 13.1} - f_F \approx 0.75$$

THIS MAKES A NEGLIGIBLE CHANGE IN THE  $h_L$  CALCULATION -  $\therefore$

$$V = 1.656 \text{ FT/s}$$

$$\dot{V} = \frac{\pi}{4} \left( \frac{0.75}{12} \right)^2 (1.656) = 0.0051 \text{ FT}^3/\text{s}$$

13.8 FOR THIS CASE -

$$\frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g \Delta y + h_L = 0$$

$$-\frac{\Delta P}{\rho} = \frac{4.55(144)(32.2)}{62.4} = -338.1 \text{ FT}^2/\text{s}^2$$

$$\Delta V^2 = 0$$

$$g \Delta y = 0$$

$$h_L = 2f_F \frac{L}{D} V^2$$

$$V = \frac{118 \text{ FT}^3/\text{min}}{(60) \pi / 4 (D^2)} = \frac{2.50}{D^2}$$

$$h_L = 2f_F \frac{250}{D} \left( \frac{2.5}{D^2} \right)^2 = \frac{3125}{D^5} f_F$$

GOVERNING EQN. IS

$$-338.1 + \frac{3125}{D^5} f_F = 0$$

$$f_F = 0.1082 D^5$$

OTHER CONSTRAINT IS  $f_F(Re) \sim \text{FIG 13.1}$

13.8 CONTINUED -

$$Re = \frac{DV}{\nu} = \frac{DV}{\pi D^2/4 \nu}$$

$$= \frac{118}{60 (\pi) D/4 (1.22 \times 10^{-5})}$$

$$\frac{2.052 \times 10^5}{D}$$

TRIAL & ERROR -

ASSUME  $f_f = 0.004$

$$D = \left[ \frac{0.004}{0.1082} \right]^{1/5} = 0.517 \text{ FT}$$

$$Re = \frac{2.052 \times 10^5}{0.517} = 3.97 \times 10^5$$

FIG 13.1 --  $f_f = 0.00325$

USING THIS VALUE -

$$D = 0.496 \text{ FT} \quad Re = 4.137 \times 10^5$$

$$f_f = 0.0031$$

$$\Rightarrow D = \underline{0.491 \text{ FT}} \quad (5.9 \text{ IN})$$

13.9 CONTINUED -

$$gAy = 9.81(3) = 29.4 \text{ FT}^2/\text{s}^2$$

$$h_L = 2 f_f \frac{L}{D} V^2$$

$$Re = \frac{(6/12)(5.67)}{1.22 \times 10^{-5}} = 2.32 \times 10^5$$

$$e/D = 0.003$$

$$f_f (\text{FIG 13.1}) \approx 0.0066$$

$$h_L = 2(0.0066) \left( \frac{6}{0.5} \right) (5.67)^2 = 5.09 \text{ FT}^2/\text{s}^2$$

SUBSTITUTING INTO ENERGY EQN:

$$-\frac{P_2}{\rho} = 16.1 + 29.4 + 5.09 = 50.6 \text{ FT}^2/\text{s}^2$$

$$-P_2 = \frac{62.4(50.6)}{32.2} = \underline{98.0 \text{ Lbf/ft}^2}$$

$$P_2 = \underline{-0.681 \text{ PSIG}}$$

$$P_2 \text{ ABSOLUTE} = 14.7 - 0.681$$

$$= \underline{14.02 \text{ PSIA}}$$

13.9 ENERGY EQN - STEADY FLOW

$$\frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + gAy + h_L = 0$$

LOCATION 1 - SUMP ( $V_1 = 0$ )

2 - PUMP INLET

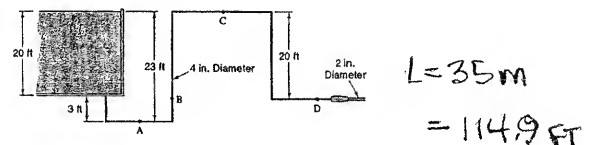
$$\frac{\Delta P}{\rho} = \frac{P_2 - P_{\text{ATM}}}{\rho} = \frac{P_{2g}}{\rho}$$

$$V_2 = \frac{V}{\pi D^2/4} = \frac{500}{(7.48)(60) \left( \frac{\pi}{4} \right) (6/12)^2}$$

$$= 5.67 \text{ FT/s}$$

$$\Delta V^2/2 = 16.1 \text{ FT}^2/\text{s}^2$$

13.10



BETWEEN RESERVOIR SURFACE (1)  
& NOZZLE EXIT (2).

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + gAy + h_L = 0$$

$$\Delta P = 0$$

$$V_1^2 = 0$$

$$gAy = -(32.2)(20) = -644 \text{ FT}^2/\text{s}^2$$

$$\text{IN PIPE: } V_p = \frac{V_2 A_2}{A_p} = V_2 \left( \frac{D_2}{D_1} \right)^2 = \frac{V_2}{4}$$

$$98 \quad V_p^2 = V_2^2/16$$

13.10 CONTINUED

$$h_L = 2 f_F \frac{L}{D} \frac{V_p^2}{2} + \sum K \frac{V_p^2}{2}$$

$$= V_p^2 \left[ 2 f_F \frac{114.9}{4/12} + \frac{\sum K}{2} \right]$$

$$\sum K = (5)(0.7) + 1$$

ELBOWS ENTRANCE

$$h_L = V_p^2 [689.4 f_F + 2.25]$$

ENERGY EQN. IS

$$\frac{V_p^2}{8} - 644 + V_p^2 [ ] = 0$$

$$\text{OR } V_p^2 [689.4 f_F + 2.63] = 644$$

TRIAL & ERROR -

$$\text{ASSUME } f_F = 0.005$$

$$V_p = 10.29 \text{ FT/S}$$

$$Re = \frac{(4/12)(10.29)}{1.22 \times 10^{-5}} = 2.811 \times 10^5$$

$$e/D = 0.0005$$

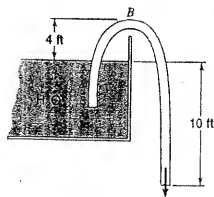
$$\text{FIG 13.1} - f_F \approx 0.0045$$

$$\text{WITH } f_F = 0.0045 \quad V_p = 10.6 \text{ FT/S}$$

Re CHECKS

$$\dot{V} = \frac{\pi}{4} \left( \frac{4}{12} \right)^2 (10.6) = \underline{\underline{0.925 \text{ FT}^3/\text{S}}}$$

13.11



$$L = 23 \text{ FT}$$

$$D = 1 \text{ INCH}$$

BETWEEN RESERVOIR SURFACE (1)  
& EXIT (2)

13.11 CONTINUED -

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g \Delta y + h_L = 0$$

$$\Delta P = 0$$

$$V_1 = 0$$

$$g \Delta y = 32.2(-10) = -322 \text{ FT}^2/\text{S}^2$$

$$h_L = 2 f_F \frac{L}{D} \frac{V^2}{2} + \sum K \frac{V^2}{2}$$

$$= 2 f_F \frac{23}{1/12} \frac{V^2}{2} + V^2 \sum K \frac{1}{2}$$

$$K = 1 - \text{ENTRANCE LOSS}$$

$$h_L = V^2 [552 f_F + 0.5]$$

ENERGY EQN. IS

$$\frac{V^2}{2} - 322 + V^2 [552 f_F + 0.5] = 0$$

$$V^2 [552 f_F + 1] = 322$$

TRIAL & ERROR:

$$\text{ASSUME } f_F = 0.005$$

$$V = 9.25 \text{ FT/S}$$

$$Re = \frac{(1/12)(9.25)}{1.22 \times 10^{-5}} = 6.32 \times 10^4$$

$$\text{FIG 13.1 - SMOOTH TUBE - } f_F = 0.0047$$

$$\text{FOR } f_F = 0.0047 \quad V = 9.46 \text{ FT/S}$$

$$Re = 6.46 \times 10^4 \quad f_F = 0.0047$$

$$\dot{V} = \frac{\pi}{4} \left( \frac{1}{12} \right)^2 (9.46) = \underline{\underline{0.0516 \text{ FT}^3/\text{S}}}$$

BETWEEN (2) & B

$$\frac{P_B - P_2}{\rho} + \frac{V_B^2 - V_2^2}{2} + g \Delta y + h_L = 0$$

13.11 CONTINUED -

$$\frac{P_B - P_2}{\rho} = \frac{P_{B9}}{\rho}$$

$$\frac{V_B^2 - V_2^2}{2} = 0$$

$$g \Delta y = 32.2(14) = 450.8 \text{ FT}^2/\text{s}^2$$

$$h_L = 2 f_F \frac{L}{D} V^2 + \sum \frac{K V^2}{2}$$

$$= \frac{2(0.0047)(14)(9.46)^2}{1/12}$$

$$= 141.3 \text{ FT}^2/\text{s}^2$$

INTO ENERGY EQN:

$$\frac{P_{B9}}{\rho} = -450.8 - 141.3 = -592.1 \text{ FT}^2/\text{s}^2$$

$$P_{B9} = \frac{(-592.1)(62.4)}{32.2} = -1147 \text{ PSF}$$

$$= \underline{\underline{-7.97 \text{ PSI}}}$$

$$P_{\text{ABSOLUTE}} = 14.7 - 7.97 = \underline{\underline{6.73 \text{ PSI}}}$$

13.12 RECTANGULAR DUCT - 8' x 8" x 25 FT

$$\dot{V} = 600 \text{ FT}^3/\text{min STD AIR}$$

$$D_{\text{EQ}} = \frac{4(8 \times 8)}{4/8} = 8 \text{ IN}$$

$$V = \frac{600/60}{8(8)/144} = 22.5 \text{ FT/S}$$

ENERGY EQN. REDUCES TO

$$\frac{\Delta P}{\rho} = 2 f_F \frac{L}{D} V^2$$

$$Re = \frac{(8/12)(22.5)}{1.56 \times 10^{-5}} = 9.59 \times 10^4$$

13.12 CONTINUED -

$$e/D = \frac{0.0005}{8/12} = 0.00075$$

$$\text{FIG 13.1} - f_F \approx 0.0054$$

$$\frac{\Delta P}{\rho} = 2(0.0054) \left( \frac{25}{8/12} \right) (22.5)^2 = 205 \text{ FT}^2/\text{s}^2$$

$$\Delta P = \frac{205(0.0766)}{32.2} = 0.4876 \text{ PSF}$$

$$= 6.366 \text{ FT AIR} = 76.4 \text{ IN AIR}$$

$$= (76.4) \frac{0.0766}{62.4} = \underline{\underline{0.0938 \text{ IN H}_2\text{O}}}$$

13.13 ENERGY EQN.

$$\frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g \Delta y + h_L = 0$$

$$g \Delta y = (32.2)(175) = 5635 \text{ FT}^2/\text{s}^2$$

$$\dot{V} = 3 \times 10^6 \frac{\text{GAL}}{\text{DAY}} = 4.642 \text{ FT}^3/\text{s}$$

$$V = \frac{4.642}{\frac{\pi}{4} D^2} = \frac{5.91}{D^2} \text{ FT/S}$$

$$h_L = 2 f_F \frac{L}{D} V^2 = 2 f_F \frac{10560 V^2}{D^5}$$

$$= 2.112 \times 10^4 \frac{f_F V^2}{D^5}$$

$$\text{FOR 10-IN PIPE: } V = \frac{5.91}{(10/12)^2} = 8.51 \text{ FT/S}$$

$$Re = \frac{(10/12)(8.51)}{1.22 \times 10^{-5}} = 5.81 \times 10^5$$

$$e/D = 0.00011 - f_F = 0.0051$$

$$h_L = \frac{2.112 \times 10^4 (0.0051)(8.51)^2}{(10/12)^5}$$

$$= \underline{\underline{19410 \text{ FT}^2/\text{s}^2}}$$

13.13 CONTINUED -

FOR 12" PIPE:  $U = 5.91 \text{ FT/s}$

$$Re = 4.84 \times 10^5 \quad e/d = 0.00085$$

$$f_f \approx 0.0048 \quad h_L = 3540 \text{ FT}^2/\text{s}^2$$

FOR 14" PIPE:  $U = 4.34 \text{ FT/s}$

$$Re = 4.15 \times 10^5 \quad e/d = 0.00073$$

$$f_f \approx 0.0047 \quad h_L = 865 \text{ FT}^2/\text{s}^2$$

$$\begin{aligned} \text{COST/HR} &= \left( \text{POWER}_{\text{LOST}} \right) + \left( \frac{1}{20} \text{ INITIAL}_{\text{LOST}} \right) \\ &\quad + 0.06 \left( \text{INITIAL}_{\text{LOST}} \right) \\ &= \left( \text{POWER}_{\text{LOST}} \right) + 0.11 \left( \text{INITIAL}_{\text{LOST}} \right) \end{aligned}$$

$$\text{POWER}_{\text{LOST}} = \frac{\$0.07}{\text{KWH}} (P)$$

$$P = \dot{m} (h_L + g \Delta y) \frac{(1.356)(365)(24)}{(32.2)(1000)}$$

$$= 106.86 (h_L + g \Delta y) \text{ KWH}$$

FOR 10-IN PIPE:

$$\begin{aligned} \text{COST} &= 0.11 (\$11.40)(2)(5280) \\ &\quad + \$0.07 (106.86)(19410 + 5635) \\ &= \$2,60,580 \end{aligned}$$

FOR 12" PIPE -

$$\begin{aligned} \text{COST} &= 0.11 (\$14.70)(2)(5280) \\ &\quad + \$0.07 (106.86)(3540 + 5635) \\ &= \$85,670 \end{aligned}$$

FOR 14" PIPE -

$$\begin{aligned} \text{COST} &= 0.11 (\$16.80)(2)(5280) \\ &\quad + 0.07 (106.86)(865 + 5635) \\ &= \$68,108 \leftarrow \text{CHEAPEST} \end{aligned}$$

13.14 ENERGY EQN. REDUCES TO

$$\frac{\Delta P}{\rho} + 2 f_f \frac{L}{D} U^2 = 0$$

$$\begin{aligned} -\frac{\Delta P}{\rho} &= \frac{P_1 - P_{\text{atm}}}{\rho_w} = \frac{P_{\text{in}}}{\rho_w} = \frac{40 \text{ PSI}}{\rho_w} \\ &= \frac{40(144)(32.2)}{62.4} = 2970 \text{ FT}^2/\text{s}^2 \end{aligned}$$

$$2 f_f \frac{L}{D} U^2 = 2 f_f \frac{50}{0.5/12} U^2 = 2400 f_f U^2$$

~ FOR 1/2-IN. DIAM HOSE -

$$\text{INTO ENERGY EQN: } f_f U^2 = 1.2375$$

TRIAL & ERROR:

$$\text{ASSUME } f_f = 0.005 \quad U = 15.73 \text{ FT/s}$$

$$Re = \frac{(0.5)(15.73)}{1.22 \times 10^{-5}} = 5.373 \times 10^4$$

FIG 13.1 - ASSUME SMOOTH -  $f_f = 0.0049$

$$\text{WITH } f_f = 0.0049 \quad U = 15.89 \text{ FT/s}$$

$$Re = 5.427 \times 10^4 \rightarrow f_f = 0.0049$$

$\therefore$  FOR 1/2-IN. HOSE -  $U = 15.89 \text{ FT/s}$

$$\dot{V} = 15.89 \left( \frac{\pi}{4} \right) \left( \frac{0.5}{12} \right)^2 = 0.0217 \text{ FT}^3/\text{s}$$

FOR 3/4-IN DIAM HOSE:

$$h_L = 1600 f_f U^2 \sim f_f U^2 = 1.856$$

- ASSUME  $f_f = 0.004$  -  $U = 21.54 \text{ FT/s}$

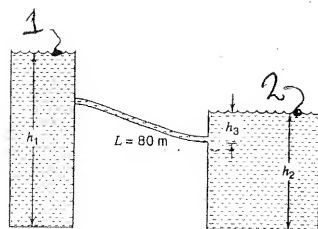
$$Re = \frac{(0.75/12)(21.54)}{1.22 \times 10^{-5}} = 1.1035 \times 10^5 \quad f_f = 0.0042$$

WITH  $f_f = 0.0042 \quad U = 21.02 \text{ FT/s}$

$$Re = 1.071 \times 10^5 \quad f_f = 0.00425$$

$$U = 20.9 \text{ FT/s} \quad \dot{V} = 0.0641 \text{ FT}^3/\text{s}$$

13.15



$$h_1 = 60\text{ m}, h_2 = 30\text{ m}, h_3 = 8\text{ m}$$

$$L = 80\text{ m} \quad D = 0.35\text{ m}$$

ENERGY EQN - BETWEEN 1 &amp; 2

$$\frac{\Delta P}{\rho} + \frac{\Delta U^2}{2} + g\Delta y + h_L = 0$$

$$\frac{\Delta P}{\rho} = 0 \quad \frac{\Delta U^2}{2} = 0$$

$$g\Delta y = -(9.81)(30) = -294.3 \text{ m}^2/\text{s}^2$$

$$h_L = 2f_f \frac{L}{D} U^2 + \sum K \frac{U^2}{2}$$

$$= 2(0.004) \left( \frac{80}{0.35} \right) U^2$$

$$+ 0.5 U^2$$

$$= 2.329 U^2$$

INTO ENERGY EQN:

$$2.329 U^2 = 294.3$$

$$U = 11.24 \text{ m/s}$$

$$a) \quad \dot{V} = 11.24 \left( \frac{\pi}{4} \right) (0.35)^2 = \underline{1.082 \text{ m}^3/\text{s}}$$

for  $e/D = 0.0004$ 

$$h_L = 2f_f \frac{80}{0.35} U^2 = 457 f_f U^2$$

INTO ENERGY EQN:

$$457 f_f U^2 = 294.3 \quad f_f U^2 = 0.6438$$

TRIAL &amp; ERROR:

$$\text{ASSUME } f_f = 0.0072$$

$$U = 9.46 \text{ m/s}$$

13.15 CONTINUED-

$$Re = \frac{(0.35)(9.46)}{0.995 \times 10^{-6}} = 3.328 \times 10^6$$

$$\text{FULLY TURBULENT} - f_f = 0.0072$$

$$\therefore U = 9.46 \text{ m/s} \quad \underline{\underline{\dot{V} = 0.910 \text{ m}^3/\text{s}}}$$

13.16 ENERGY EQN IS -

$$\frac{\Delta P}{\rho} + \frac{\Delta U^2}{2} + g\Delta y + h_L = 0$$

$$\frac{\Delta U^2}{2} = 0$$

$$g\Delta y = (9.81)(-6.68) = -6553 \text{ m}^2/\text{s}^2$$

$$h_L = 2f_f \frac{L}{D} U^2$$

$$\dot{V} = 90 \text{ m}^3/\text{s} \quad U = \frac{90}{\pi/4 (5)^2} = 4.584 \text{ m/s}$$

$$Re = \frac{5(4.584)}{0.995 \times 10^{-6}} = 2.3 \times 10^7$$

$$e/D \approx \frac{(0.003 \text{ Ft})(0.3048)}{5} = 0.00018$$

$$f_f \approx 0.0034$$

$$h_L = 2(0.0034) \left( \frac{8000}{5} \right) (4.584)^2 = 228.6 \text{ m}^2/\text{s}^2$$

INTO ENERGY EQN:

$$\frac{\Delta P}{\rho} = 6553 - 228.6 = 6324 \text{ m}^2/\text{s}^2$$

$$\Delta P = 6324 (1000) = 6324 \times 10^3 \text{ N/m}^2$$

$$= \underline{\underline{6.324 \text{ MPa}}}$$

13.17 GATE VALVE -

$$\frac{\Delta P}{\rho} = K \frac{U^2}{2}$$

$$P_1 = 236 \text{ kPa} \quad P_2 = P_{\text{atm}} = 101.4 \text{ kPa}$$

$$\Delta P = 134.6 \text{ kPa} \quad \frac{\Delta P}{\rho} = 134.6 \frac{\text{m}^2}{\text{s}^2}$$

a) VALVE FULLY OPEN:  $K = 0.15$

$$U = \left[ \frac{(134.6)^2}{0.15} \right]^{1/2} = 42.36 \text{ m/s}$$

$$\dot{V} = (42.36) \left( \frac{\pi}{4} \right) (0.2)^2 = 1.331 \text{ m}^3/\text{s}$$

b) VALVE 1/4 CLOSED -  $K = 0.85$

$$U = 17.8 \text{ m/s} \quad \dot{V} = 0.559 \text{ m}^3/\text{s}$$

c) VALVE 1/2 CLOSED -  $K = 4.4$

$$U = 7.82 \text{ m/s} \quad \dot{V} = 0.246 \text{ m}^3/\text{s}$$

d) VALVE 3/4 CLOSED -  $K = 20$

$$U = 3.67 \text{ m/s} \quad \dot{V} = 0.115 \text{ m}^3/\text{s}$$

13.18  $h_L = 2 f_f \frac{L}{D} U^2$

$$Re = \frac{DU}{\nu} = \frac{(0.18)(34)}{0.995 \times 10^{-6}} = 6.15 \times 10^6$$

$$\epsilon/D = 0.0014 \quad f_f = 0.0053$$

$$h_L = 2(0.0053) \frac{400}{0.18} (34)^2$$

$$= 127230 \text{ m}^2/\text{s}^2$$

$$= 2776 \text{ m of H}_2\text{O}$$

13.19  $\text{H}_2\text{O @ } 15^\circ\text{C} \quad \frac{\Delta P}{\rho} = 0.50 \text{ m}$

$$L = 300 \text{ m} \quad D = 2.20 \text{ m}$$

$$\nu = 1.195 \times 10^{-6} \text{ m}^2/\text{s}$$

$$h_L = 2 f_f \frac{L}{D} U^2$$

13.19 CONTINUED

$$Re = \frac{DU}{\nu} = \frac{(2.2)(U)}{1.195 \times 10^{-6}} = 1.841 \times 10^6 U$$

$$h_L = 9.81(0.5) = 2 f_f \frac{300}{2.2} U^2$$

$$f_f U^2 = 0.01799$$

TRIAL & ERROR -

ASSUME TURBULENT FLOW - SMOOTH PIPE

$$\text{ASSUME } f_f = 0.003$$

$$U = 2.448 \text{ m/s} \quad Re = 4.508 \times 10^6$$

$$\text{FIG 13.1} - f_f = 0.0022$$

$$U = 2.86 \text{ m/s} \quad Re = 5.26 \times 10^6$$

$$\text{FIG 13.1} - f_f \approx 0.0021$$

$$U = 2.93 \text{ m/s} \rightarrow \text{CLOSE ENOUGH}$$

$$\dot{V} = 2.93 \left( \frac{\pi}{4} \right) (2.2)^2 = 11.13 \text{ m}^3/\text{s}$$

13.20 ENERGY EQUATION:

$$\frac{\Delta P}{\rho} + \frac{\Delta U^2}{2} + g\Delta y + h_L = 0$$

$$\frac{\Delta P}{\rho} = 0$$

(1) = SURFACE OF TANK

$$\frac{\Delta U^2}{2} = \frac{U_2^2}{2}$$

(2) = PIPE EXIT

$$g\Delta y = 9.81(-16.9) = -165.8 \text{ m}^2/\text{s}^2$$

$$h_L = 2 f_f \frac{L}{D} U^2 = 2 f_f \frac{30}{0.6} U^2 = 600 f_f U^2$$

INTO ENERGY EQN:

$$U^2 [600 f_f + 0.5] = 165.8$$

TRIAL & ERROR -

$$0.6\text{-m CAST IRON PIPE} - \epsilon/D = 0.00045$$

13,20 CONTINUED -

$$Re = \frac{0.65}{0.995 \times 10^{-6}} = 6.03 \times 10^5$$

ASSUME  $f_f = 0.003$   $U = 6.36 \text{ m/s}$

$$Re = 3.83 \times 10^6 \quad f_f = 0.0041$$

THIS IS IN FULLY TURBULENT REGION

$\therefore f_f$  IS 0.0041  $\&$   $U = 7.48 \text{ m/s}$

$$\dot{V} = (7.48) \left( \frac{\pi}{4} \right) (0.6)^2 = 2.116 \text{ m}^3/\text{s}$$

13,21  $D = 0.15 \text{ m}$   $L = 100 \text{ m}$

$20^\circ \text{C H}_2\text{O} - \nu = 0.995 \times 10^{-6} \text{ m}^2/\text{s}$

$$\Delta P = 30 \text{ kPa} \sim \frac{\Delta P}{\rho} = 30 \text{ m}^2/\text{s}^2$$

WEIGHT IRON PIPE  $\frac{\rho}{D} = 0.00035$

$$Re = \frac{(0.15)U}{0.995 \times 10^{-6}} = 1.507 \times 10^5$$

ENERGY EQN:  $\frac{\Delta P}{\rho} + h_L = 0$

$$2f_f \frac{100}{0.15} U^2 = 30 \quad f_f U^2 = 0.0225$$

TRIAL & ERROR -

ASSUME  $f_f = 0.004$   $U = 2.37 \text{ m/s}$

$$Re = 3.574 \times 10^5 \quad f_f \approx 0.0042$$

@  $f_f = 0.0042$   $U = 2.31 \text{ m/s}$

$$Re = 3.488 \times 10^5 \quad f_f = 0.0042$$

$$\dot{V} = (2.31) \left( \frac{\pi}{4} \right) (0.15)^2 = 0.0408 \text{ m}^3/\text{s}$$

13,22  $\Delta P = 1.3 \text{ m H}_2\text{O}$   $L = 10 \text{ m}$

$D = 0.2 \text{ m}$   $e = 0.0004 \text{ m}$

ASSUME  $20^\circ - \nu = 0.995 \times 10^{-6} \text{ m}^2/\text{s}$

$$\frac{\Delta P}{\rho} = 1.3(9.81) = 12.75 \text{ m}^2/\text{s}^2$$

13,22 CONTINUED -

ENERGY EQN -  $\frac{\Delta P}{\rho} = 2f_f \frac{L}{D} U^2$

$$12.75 = 2f_f \frac{10}{0.2} U^2 = 100f_f U^2$$

$$f_f U^2 = 0.1275$$

$$Re = \frac{0.2U}{0.995 \times 10^{-6}} = 2.01 \times 10^5$$

ASSUME SMOOTH PIPE -

IF  $f_f = 0.004$  -  $U = 5.646 \text{ m/s}$

$$Re = 1.135 \times 10^6 \quad f_f \approx 0.002565$$

@  $f_f = 0.003$   $U = 6.52 \text{ m/s}$

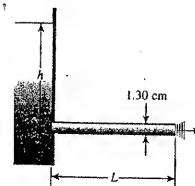
$$Re = 1.31 \times 10^6 \quad f_f \approx 0.0027$$

@  $f_f = 0.0027$   $U = 6.87 \text{ m/s}$

$$Re = 1.381 \times 10^6 \quad f_f \approx 0.0027$$

$$\dot{m} = (6.87) \left( \frac{\pi}{4} \right) (0.2)^2 (1000) = 0.216 \text{ kg/s}$$

13,23



$$\dot{V} = 5.675 \times 10^{-4} \text{ m}^3/\text{s}$$

$$L = 20 \text{ m}$$

PIPE IS COM. STEEL  
 $e/D = 2.446 \times 10^{-5}$

$$\frac{\Delta P}{\rho} = h_L + \sum K \frac{U^2}{2}$$

$$= 2f_f \frac{L}{D} U^2 + 0.5 U^2 \quad \left\{ \text{FOR ENTRANCE } K=1 \right\}$$

$$U = \frac{5.675 \times 10^{-4}}{\pi/4 (0.013)^2} = 4.275 \text{ m/s}$$

$$Re = \frac{(0.013)(4.275)}{0.995 \times 10^{-6}} = 55900$$

FIG 13.1 -  $f_f \approx 0.0049$

13,23 CONTINUED

$$\frac{\Delta P}{\rho} = V^2 \left[ 2(0.0049) \left( \frac{20}{0.013} \right) + 0.5 \right]$$

$$= 284.7 \text{ m}^2/\text{s}^2$$

$$h = \frac{284.7}{g} = \underline{\underline{29.02 \text{ m}}}$$

13,24  $\dot{V} = 0.25 \text{ m}^3/\text{s}$ 

PIPE 1:  $U = \frac{0.25}{\frac{\pi}{4}(0.16)^2} = 12.43 \text{ m/s}$

PIPE 2  $U = \frac{0.25}{\frac{\pi}{4}(0.18)^2} = 9.82 \text{ m/s}$

PIPE 3  $U = \frac{0.25}{\frac{\pi}{4}(0.2)^2} = 7.96 \text{ m/s}$

PIPE 1 -  $\frac{\Delta P}{\rho} = 2f_f \frac{L}{D} U^2$

$$Re = \frac{0.16(12.43)}{0.995 \times 10^{-6}} = 1.998 \times 10^6$$

$$e/D = 0.0055 - f_f = 0.0079$$

$$\frac{\Delta P}{\rho g} = \frac{2(0.0079)(900)(12.43)^2}{0.16(9.81)} = \underline{\underline{1400 \text{ m}}}$$

PIPE 2 -

$$Re = \frac{0.18(9.82)}{0.995 \times 10^{-6}} = 1.776 \times 10^6$$

$$e/D = 0.005 - f_f = 0.0075$$

$$\frac{\Delta P}{\rho g} = \frac{2(0.0075)(1500)(9.82)^2}{0.18(9.81)} = \underline{\underline{1241 \text{ m}}}$$

PIPE 3 -

$$Re = \frac{0.2(7.96)}{0.995 \times 10^{-6}} = 1.6 \times 10^6$$

$$e/D \approx 0.0045 - f_f \approx 0.0073$$

$$\frac{\Delta P}{\rho g} = \frac{2(0.0073)(200)(7.96)^2}{0.2(9.81)} = \underline{\underline{377 \text{ m}}}$$

13,25

Pipe	Length, m	Diameter, cm	Roughness, mm
1	125	8	0.240
2	150	6	0.120
3	100	4	0.200

PIPES IN SERIES -  $H_2O @ 20^\circ C - \nu = 0.995 \times 10^{-6} \text{ m}^2/\text{s}$ 

$$U_1 = \dot{V} / \frac{\pi D_1^2}{4} = 199 \dot{V}$$

$$U_2 = \dots = 354 \dot{V}$$

$$U_3 = \dots = 796 \dot{V}$$

$$\frac{\Delta P}{\rho} = h_{L1} + h_{L2} + h_{L3} + 5(9.81)$$

$$h_{L1} = 2f_{f1} \frac{125}{0.08} (199 \dot{V})^2 = 1.238 \times 10^8 f_{f1} \dot{V}^2$$

$$h_{L2} = 2f_{f2} \frac{150}{0.06} (354 \dot{V})^2 = 6.266 \times 10^8 f_{f2} \dot{V}^2$$

$$h_{L3} = 2f_{f3} \frac{100}{0.04} (796 \dot{V})^2 = 3.176 \times 10^9 f_{f3} \dot{V}^2$$

PIPE 1 -  $e/D = \frac{0.24}{80} = 0.003$

ASSUME FULLY TURBULENT -  $f_{f1} = 0.0065$ 

PIPE 2 -  $e/D = \frac{0.12}{60} = 0.002$

~ SAME ASSUMPTION -  $f_{f2} = 0.00585$ 

PIPE 3  $e/D = \frac{0.20}{40} = 0.005$

~ SAME ASSUMPTION  $f_{f3} = 0.0077$ 

$$\Sigma h_L = \dot{V}^2 \left[ 8.047 \times 10^5 + 36.66 \times 10^5 + 244.55 \times 10^5 \right]$$

$$= 289.3 \times 10^5 \dot{V}^2$$

$$\Sigma h_L = \frac{\Delta P}{\rho} + g\Delta y = 180 + 9.81 \times 21 = 276.3 \text{ m}^2/\text{s}^2$$

SOLVING -  $\dot{V} = 0.00309 \text{ m}^3/\text{s}$

$$U_1 = 0.615 \text{ m/s} \quad Re_1 = 4.94 \times 10^4 \quad f_{f1} = 0.0071$$

$$U_2 = 1.094 \text{ m/s} \quad Re_2 = 6.60 \times 10^4 \quad f_{f2} = 0.0065$$

$$U_3 = 2.460 \text{ m/s} \quad Re_3 = 9.87 \times 10^4 \quad f_{f3} = 0.0077$$

13.25 CONTINUED —

USING NEW VALUES FOR  $f_F$  —

$$\sum h_L = \left[ (8.79 + 40.73 + 2.45) \times 10^5 \right] \dot{V}^2$$

$$= 294.5 \times 10^5 \dot{V}^2 = 276.3$$

$$\dot{V} = 0.00306 \text{ m}^3/\text{s}$$

13.26 CONCRETE PIPES IN SERIES

$$H_2O @ 20^\circ C - \dot{V} = 0.18 \text{ m}^3/\text{s}$$

$$h_{L1} + h_{L2} = 18 \text{ m} = 176.6 \text{ m}^2/\text{s}^2$$

$$\text{FOR PIPE 1} - h_{L1} = 2 f_F \frac{L}{D} V_1^2$$

$$V_1 = \frac{0.18}{\left(\frac{\pi}{4}\right)(0.3)^2} = 2.55 \text{ m/s}$$

$$Re = \frac{(0.3)(2.55)}{0.995 \times 10^{-6}} = 7.678 \times 10^5$$

$$e/D = \frac{0.0035}{0.3} = 0.00117$$

$$\text{FIG 13.1} - f_F \approx 0.0051$$

$$\frac{\Delta P}{8} = h_L = 2(0.0051) \left( \frac{312.5}{0.3} \right) (2.55)^2$$

$$= 69.09 \text{ m}^2/\text{s}^2$$

THIS REQUIRES  $h_L$  FOR PIPE 2

$$\text{TO BE } 176.6 - 69.09 = 107.5 \text{ m}^2/\text{s}^2$$

$$107.5 = 2 f_F \frac{312.5}{D} V^2$$

$$\frac{f_F V^2}{D} = 0.172$$

$$\therefore V = \frac{\dot{V}}{\frac{\pi D^2}{4}} = \frac{0.18}{\frac{\pi (0.3)^2}{4}} = 0.2292$$

$$\therefore \frac{f_F}{D^5} = 3.275$$

13.26 CONTINUED —

$$Re = \frac{DV}{\nu} = \frac{D(0.2292)}{0.2(0.995 \times 10^{-6})}$$

$$= \frac{2.304 \times 10^5}{D}$$

TRIAL  $\approx$  ERROR —

$$\text{ASSUME } f_F = 0.006 - D = 0.2835 \text{ m}$$

$$e/D = 0.0123 \quad Re = 8.127 \times 10^5$$

$$\text{FIG 13.1} - f_F = 0.01$$

$$D = 0.314 \text{ m} \quad e/D = 0.0111$$

$$Re = 7.338 \times 10^5 \quad f_F = 0.01$$

$$\sim \underline{D = 0.314 \text{ m}}$$

13.27 2 PIPES IN PARALLEL:

$$\text{PIPE 1} - D = 0.2 \text{ m} \quad L = 150 \text{ m}$$

$$\text{CAST IRON: } e/D = 0.0013$$

$$\text{PIPE 2} - D = 0.067 \text{ m} \quad L = 150 \text{ m}$$

$$\text{STEEL} - e/D = 0.0007$$

$$\Delta P = 210 \text{ kPa} \quad \frac{\Delta P}{8} = 210 \text{ m}^2/\text{s}^2$$

$$\text{PIPE 1: } \frac{\Delta P}{8} = 2 f_F \frac{L}{D} V^2$$

$$\text{ASSUME FULLY TURBULENT} - f_F \approx 0.0055$$

$$210 = 2(0.0055) \frac{150}{0.2} V^2 - V = 5.045 \text{ m/s}$$

$$Re = \frac{0.2(5.045)}{0.995 \times 10^{-6}} = 1.014 \times 10^6$$

$$\text{THIS CONFIRMS FULLY TURBULENT}$$

$$\underline{V_1 = 5.045 \text{ m/s}}$$

PIPE 2: AGAIN ASSUME FULLY TURBULENT

$$f_F = 0.0045 \sim V_2 = 3.228 \text{ m/s}$$

13.27 CONTINUED -

$$Re_2 = \frac{0.067(3.228)}{0.995 \times 10^{-6}} = 2.173 \times 10^5$$

FIG 13.1: REVISED  $f_F \approx 0.0049$

WITH THIS VALUE  $U_2 = 3.094 \text{ m/s}$

$$Re = 2.083 \times 10^5 \quad f_F \approx 0.0049$$

$$\therefore U_2 = 3.049 \text{ m/s}$$

$$\dot{V} = 5.045 \left( \frac{\pi}{4} \right) (0.2)^2 + 3.049 \left( \frac{\pi}{4} \right) (0.067)^2$$

$$= 0.1585 \text{ FT}^3/\text{s} + 0.0107 \text{ FT}^3/\text{s}$$

$$\dot{V}_1 = 0.1585 \text{ FT}^3/\text{s} \quad \dot{V}_2 = 0.0107 \text{ FT}^3/\text{s}$$

13.28 3 PIPES IN PARALLEL

Pipe	Length, m	Diameter, cm	Roughness, mm
1	100	8	0.240
2	150	6	0.120
3	80	4	0.200

$$\text{TOTAL } h_L = 24 \text{ m} = 235.4 \text{ m}^2/\text{s}^2$$

$$\text{PIPE 1} - 2f_F \frac{L}{D} U_1^2 = 2f_F \frac{100}{0.08} U^2$$

$$\sim f_F U_1^2 = 0.0824$$

$$Re_1 = \frac{0.08 U_1}{0.995 \times 10^{-6}} = 8.04 \times 10^5 U_1$$

TRIAL  $\frac{1}{2}$  ERROR -

$$e/p = 0.24/100 = 0.0024$$

ASSUME FULLY TURBULENT -

$$f_F = 0.0063 - U_1 = 3.617 \text{ m/s}$$

$$Re = \frac{(0.08)(3.617)}{0.995 \times 10^{-6}} = 2.91 \times 10^5$$

$$f_F = 0.0062 -$$

$$\text{REVISED VALUE} - U_1 = 3.65 \text{ m/s}$$

13.28 CONTINUED

$$\text{PIPE 2} \quad 235.4 = 2f_F \frac{150}{0.06} U_2^2$$

$$f_F U_2^2 = 0.0412$$

$$Re_2 = \frac{(0.06) U_2}{0.995 \times 10^{-6}} = 6.03 \times 10^4 U_2$$

$$e/p_2 = 0.002 - \text{ASSUME } f_F = 0.006$$

$$U_2 = 2.62 \text{ m/s} \quad Re = 1.58 \times 10^5$$

$$\sim f_F = 0.0061$$

$$\text{REVISED VALUE FOR } U_2: U_2 = 2.60 \text{ m/s}$$

$$\text{PIPE 3:} \quad 235.4 = 2f_F \frac{80}{0.04} U_3^2$$

$$f_F U_3^2 = 0.059$$

$$Re = \frac{(0.04) U_3}{0.995 \times 10^{-6}} = 4.02 \times 10^4 U_3$$

$$e/p = 0.005 - \text{ASSUME } f_F = 0.008$$

$$U_3 = 2.716 \text{ m/s} \quad Re = 1.092 \times 10^5$$

$$f_F = 0.0077$$

$$\sim \text{REVISED VALUE: } U_3 = 2.77 \text{ m/s}$$

TOTAL SYSTEM FLOW RATE:

$$\dot{V} = 3.65 \left( \frac{\pi}{4} \right) (0.08)^2 + (2.60) \left( \frac{\pi}{4} \right) (0.06)^2$$

$$+ 2.77 \left( \frac{\pi}{4} \right) (0.04)^2$$

$$= 0.0292 \text{ FT}^3/\text{s}$$

## CHAPTER 14

### 14.1 CENTRIFUGAL PUMP:

$$\dot{V} = 0,2 \text{ m}^3/\text{s} \quad \omega = 850 \text{ rpm}$$

$$r_2 = 0,225 \text{ m} \quad \rho = 1000 \text{ kg/m}^3$$

$$L = 0,05 \text{ m}$$

$$\beta_2 = 24^\circ$$

TORQUE - Eqn. 14.9

$$M_2 = \rho \dot{V} r_2 \left[ r_2 \omega - \frac{\dot{V}}{2\pi r_2 L} \cot \beta_2 \right]$$

$$\omega = 850 \left( \frac{2\pi}{60} \right) = 89,0 \text{ rad/s}$$

$$M_2 = (1000)(0,2)(0,225) \times$$

$$\left[ (0,225)(89) - \frac{0,2 \cot 24}{2\pi(0,225)(0,05)} \right]$$

$$= \underline{615 \text{ N}\cdot\text{m}} \quad \text{a)}$$

$$\dot{W} = M_2 \omega = 615(89)$$

$$= \underline{54,75 \text{ kW}} \quad \text{a)}$$

$$\left. \frac{\Delta P}{\rho g} \right|_{\text{max}} = - \frac{\dot{W}}{\dot{V}} = - \frac{\dot{W}}{\rho \dot{V}}$$

$$\Delta P_{\text{max}} = - \frac{54,75 \times 10^3 \text{ N}\cdot\text{m/s}}{0,2 \text{ m}^3/\text{s}}$$

$$= \underline{-274 \text{ kPa}} \quad \text{b)}$$

### 14.2 CENTRIFUGAL PUMP:

$$\rho = 680 \text{ kg/m}^3 \quad r_1 = 0,075 \text{ m}$$

$$L = 0,09 \text{ m} \quad r_2 = 0,14 \text{ m}$$

$$\beta_1 = 25^\circ \quad \beta_2 = 40^\circ$$

$$\omega = (1200) \left( \frac{2\pi}{60} \right) = 125,7 \text{ rad/s}$$

### 14.2 CONTINUED

$$\dot{V} = 2\pi r_1^2 L \omega \tan \beta_1$$

$$= 2\pi (0,075)^2 (0,09) (125,7) \tan 25^\circ$$

$$= \underline{0,186 \text{ m}^3/\text{s}} \quad \text{a)}$$

$$\dot{W} = M\omega = \rho \dot{V} r_2 \omega \left[ r_2 \omega - \frac{\dot{V} \cot \beta_2}{2\pi r_2 L} \right]$$

$$= (680)(0,186)(0,14)(125,7) \times$$

$$\left[ (0,14)(125,7) - \frac{0,186 \cot 40^\circ}{2\pi(0,14)(0,09)} \right]$$

$$= \underline{32,94 \text{ kW}} \quad \text{b)}$$

$$\left. \frac{\Delta P}{\rho g} \right|_{\text{max}} = \frac{\dot{W}}{\rho g \dot{V}}$$

$$= \frac{32,94 \times 10^3}{680(9,81)(0,186)}$$

$$= \underline{26,5 \text{ m}} \quad \text{c)}$$

### 14.3 CENTRIFUGAL PUMP -

$$r_2 = 0,21 \text{ m} \quad L = 0,05 \text{ m} \quad \beta_2 = 33^\circ$$

$$\omega = 1200 \left( \frac{2\pi}{60} \right) = 125,7 \text{ rad/s}$$

$$\frac{\Delta P}{\rho g} = 52 \text{ m H}_2\text{O}$$

$$\dot{W} = \rho \dot{V} r_2 \omega \left[ r_2 \omega - \frac{\dot{V} \cot \beta_2}{2\pi r_2 L} \right]$$

$$= \frac{\dot{m} \Delta P}{\rho} = \dot{V} \Delta P$$

EQUATION:

$$\Delta P = \rho r_2 \omega \left[ r_2 \omega - \frac{\dot{V} \cot \beta_2}{2\pi r_2 L} \right]$$

#### 14.3 CONTINUED

$$\begin{aligned}\Delta P &= 52(1000)(9.81) = 490 \text{ kPa} \\ &= (1000)(0.21)(125.7) \times \\ &\quad \left[ (0.21)(125.7) - \frac{\dot{V} \cot 33^\circ}{2\pi(0.21)(0.05)} \right] \\ &= 26400 [26.4 - 23.34 \dot{V}]\end{aligned}$$

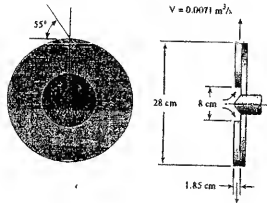
EQUATING:

$$\begin{aligned}18.56 &= 26.4 - 23.34 \dot{V} \\ \dot{V} &= 0.336 \text{ m}^3/\text{s} \quad \text{a)}\end{aligned}$$

$$\begin{aligned}\dot{W} &= \dot{V} \Delta P \\ &= 0.336 (490 \times 10^3) \\ &= 164.6 \text{ kW} \quad \text{b)}\end{aligned}$$

#### 14.4

Pump Depicted  
 $\omega = 1020 \text{ rpm}$   
 $= 106.8 \text{ rad/s}$



$$\begin{aligned}r_1 &= 0.04 \text{ m} & \dot{V} &= 0.0071 \text{ m}^3/\text{s} \\ r_2 &= 0.14 \text{ m} & \beta_2 &= 55^\circ \\ L &= 0.0185 \text{ m} & \rho &= 1000 \text{ kg/m}^3\end{aligned}$$

$$\begin{aligned}\dot{W} &= \rho \dot{V} r_2 \omega \left[ r_2 \omega - \frac{\dot{V} \cot \beta_2}{2\pi r_2 L} \right] \\ &= (1000)(0.0071)(0.14)(106.8) \times \\ &\quad \left[ (0.14)(106.8) - \frac{0.0071 \cot 55^\circ}{2\pi(0.14)(0.0185)} \right] \\ &= 1555 \text{ W} = 1.555 \text{ kW}\end{aligned}$$

#### 14.5 CENTRIFUGAL Pump -

$$\begin{aligned}\rho &= 1000 \text{ kg/m}^3 & \dot{V} &= 0.018 \text{ m}^3/\text{s} \\ \dot{W} &= 4.5 \text{ kW} & \eta &= 63\%\end{aligned}$$

$$\eta = \frac{\dot{m} \Delta P}{\rho \dot{W}}$$

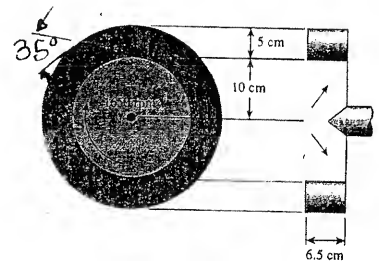
$$\begin{aligned}\Delta P &= \frac{\eta \rho \dot{W}}{\dot{m}} = \eta \frac{\dot{W}}{\dot{V}} \\ &= \frac{0.63 (4500)}{0.018} = 157.5 \text{ kPa}\end{aligned}$$

$$\frac{\Delta P}{\rho g} = \frac{157500}{(1000)(9.81)} = 16.05 \text{ m H}_2\text{O}$$

#### 14.6

Pump Depicted

$$\begin{aligned}\dot{V} &= 0.032 \text{ m}^3/\text{s} \\ \rho &= 680 \text{ kg/m}^3 \\ \omega &= 1650 \text{ rpm} \\ &= 172.8 \text{ rad/s}\end{aligned}$$



$$\begin{aligned}\dot{W} &= \rho \dot{V} r_2 \omega \left[ r_2 \omega - \frac{\dot{V} \cot \beta_2}{2\pi r_2 L} \right] \\ &= (680)(0.032)(0.15)(172.8) \times \\ &\quad \left[ (0.15)(172.8) - \frac{(0.032) \cot 35^\circ}{2\pi(0.15)(0.065)} \right] \\ &= 14.2 \text{ kW} = 19.0 \text{ Hp} \quad \text{a)}\end{aligned}$$

$$\begin{aligned}\Delta P &= \frac{\dot{W}}{\dot{V}} = \frac{14200}{0.032} = 444 \text{ kPa} \\ &= \frac{444000}{(680)(9.81)} = 66.5 \text{ m} \quad \text{b)}\end{aligned}$$

14.6 CONTINUED-

$$\dot{V} = 2\pi r_1^2 L \omega \tan \beta_1$$

$$\tan \beta_1 = \frac{0.032}{2\pi(0.10)^2(0.065)(1128)}$$

$$= 0.0453$$

$$\underline{\underline{\beta_1 = 2.6^\circ}} \quad (c)$$

14.7 CENTRIFUGAL PUMP

$$\rho = 1000 \text{ kg/m}^3 \quad r_1 = 0.12 \text{ m}$$

$$\beta_1 = 32^\circ \quad r_2 = 0.20 \text{ m}$$

$$\beta_2 = 20^\circ \quad L = 0.042 \text{ m}$$

$$\omega = 1500 \text{ rpm} = 157.1 \text{ rad/s}$$

$$\dot{V} = 2\pi r_1^2 L \omega \tan \beta_1$$

$$= 2\pi (0.12)^2 (0.042) (157.1) \tan 32^\circ$$

$$= \underline{\underline{0.373 \text{ m}^3/\text{s}}} \quad (a)$$

$$\dot{W} = \rho \dot{V} r_2 \omega \left[ r_2 \omega - \frac{\dot{V} \cot \beta_2}{2\pi r_2 L} \right]$$

$$= (1000)(0.373)(0.2)(157.1) \times$$

$$\left[ (0.2)(157.1) - \frac{0.373 \cot 20^\circ}{2\pi(0.2)(0.042)} \right]$$

$$= 140.7 \text{ kW} = \underline{\underline{189 \text{ HP}}} \quad (b)$$

$$\Delta p = \frac{\dot{W}}{\dot{V}} = \frac{140.7}{0.373} = 377 \text{ kPa}$$

$$\frac{\Delta p}{\rho g} = \frac{377 \times 10^3}{(1000)(9.81)} = \underline{\underline{38.5 \text{ m H}_2\text{O}}}$$

14.8  $\text{H}_2\text{O} @ 15^\circ\text{C}$

$$\rho = 999 \text{ kg/m}^3$$

$$D = 0.45 \text{ m}$$

$$\omega = 1600 \left( \frac{2\pi}{60} \right) = 167.6 \text{ rad/s}$$

@  $\eta$  MAY -

$$C_a \approx 0.012$$

$$C_H \approx 0.0515$$

$$C_p \approx 0.0068$$

$$\eta \approx 0.89$$

$$C_H = \frac{gh}{n^2 D^2}$$

$$h = \frac{(0.0515)(167.6)^2 (0.45)^2}{9.81} = \underline{\underline{29.9 \text{ m}}} \quad (a)$$

$$C_a = \frac{\dot{V}}{n D^3}$$

$$\dot{V} = 0.012 (167.6)(0.45)^3 = \underline{\underline{0.183 \text{ m}^3/\text{s}}} \quad (b)$$

$$\Delta p = \rho gh = (999)(9.81)(29.9) = \underline{\underline{293 \text{ kPa}}} \quad (c)$$

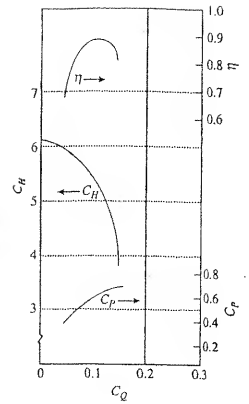
$$C_p = \frac{\dot{W}}{\rho n^3 D^5}$$

$$\dot{W} = (0.0068)(999)(167.6)^3 (0.45)^5$$

$$= 587 \text{ kW}$$

$$BHP = \frac{587 \times 10^3}{0.89} = 659.6 \text{ kW}$$

$$= \underline{\underline{884 \text{ HP}}} \quad (d)$$



14.9 CENTRIFUGAL PUMP WITH SAME CHARACTERISTICS AS IN PROB 14.8

$$\dot{V} = 0.2 \text{ m}^3/\text{s}$$

$$\omega = 1400 \left( \frac{2\pi}{60} \right) = 146.6 \text{ rad/s}$$

$$\rho = 1000 \text{ kg/m}^3$$

14.9 CONTINUED -

$$\begin{aligned} \text{At } \eta_{\max}: C_Q &\approx 0.012 \\ C_H &\approx 0.0515 \\ C_P &\approx 0.0068 \end{aligned}$$

$$C_Q = \frac{\dot{V}}{nD^3} \quad D^3 = \frac{0.2}{(146.6)(0.012)} \\ D = \underline{0.485 \text{ m}} \quad (a)$$

$$C_H = \frac{gh}{n^2 D^2} \quad h = \frac{(0.0515)(146.6)^2 (0.485)^2}{9.81} \\ = 26.5 \text{ m}$$

$$P_{\max} = \rho gh = (1000)(9.81)(26.5) \\ = \underline{260 \text{ kPa}} \quad (b)$$

14.10 SAME PUMP FAMILY AS IN PROB 14.8 BUT:

$$\begin{aligned} D &= 0.4 \text{ m} \\ \omega &= 2200 \left( \frac{2\pi}{60} \right) = 230.4 \text{ Rad/s} \\ \rho &= 999 \text{ kg/m}^3 \end{aligned}$$

$$@ \eta_{\max} \approx 0.89$$

$$\begin{aligned} C_Q &\approx 0.012 \\ C_H &\approx 0.0515 \\ C_P &\approx 0.0068 \end{aligned}$$

$$C_H = \frac{gh}{n^2 D^2} \quad h = \frac{(0.0515)(230.4)^2 (0.4)^2}{9.81} \\ = \underline{44.6 \text{ m H}_2\text{O}} \quad (a)$$

14.10 CONTINUED

$$C_Q = \frac{\dot{V}}{nD^3} \quad \dot{V} = (0.012)(230.4)(0.4)^3 \\ = \underline{0.177 \text{ m}^3/\text{s}} \quad (b)$$

$$\Delta P = \rho gh = (999)(9.81)(44.6) \\ = \underline{437 \text{ kPa}} \quad (c)$$

$$P = \frac{\dot{W}}{\rho n^3 D^5} \quad \dot{W} = (0.0068)(999)(230.4)^3 (0.4)^5 \\ = 850.8 \text{ kW} \\ \text{BHP} = \frac{850.8}{(0.89)(0.746)} = \underline{1280 \text{ hp}} \quad (d)$$

14.11 SAME PUMP FAMILY AS IN PROB 14.8

$$\begin{aligned} D &= 0.35 \text{ m} \quad \omega = 2400 \left( \frac{2\pi}{60} \right) = 251.3 \text{ rad/s} \\ \rho &= 999 \text{ kg/m}^3 \end{aligned}$$

$$\begin{aligned} \eta_{\max} &= 0.89 \\ C_Q &\approx 0.012 \\ C_H &\approx 0.0515 \\ C_P &\approx 0.0068 \end{aligned}$$

$$C_H = \frac{gh}{n^2 D^2} \quad h = \frac{(0.0515)(251.3)^2 (0.35)^2}{9.81} \\ = \underline{40.61 \text{ m H}_2\text{O}} \quad (a)$$

$$C_Q = \frac{\dot{V}}{nD^3} \quad \dot{V} = (0.012)(251.3)(0.35)^3 \\ = \underline{0.129 \text{ m}^3/\text{s}} \quad (b)$$

$$\Delta P = (999)(9.81)(44.6) \\ = \underline{437 \text{ kPa}} \quad (c)$$

14.11 CONTINUED -

$$C_p = \frac{\dot{W}}{8W^3 D^5}$$

$$\dot{W} = (0.0068)(999)(251.3)^3 (0.35)^5$$

$$= 566 \text{ kW}$$

$$BHP = \frac{566}{(0.89)(0.746)} = 853 \text{ Hp} \quad (a)$$

14.12 - SAME PUMP FAMILY AS IN PROB 14.8

$$\dot{V} = 0.30 \text{ m}^3/\text{s} \quad \eta = 1800 \left( \frac{2\pi}{60} \right) = 188.5 \text{ r/s}$$

$$@ \eta_{\max} = 0.89 \quad C_Q \approx 0.12$$

$$C_H \approx 0.0515$$

$$C_p \approx 0.0068$$

$$C_Q = \frac{\dot{V}}{nD^3} \quad D = \left[ \frac{0.30}{(188.5)(0.12)} \right]^{1/3}$$

$$D = 0.430 \text{ m} \quad (a)$$

$$C_H = \frac{gh}{n^2 D^2}$$

$$h = \frac{(0.0515)(188.5)^2 (0.43)^2}{9.81}$$

$$= 34.49 \text{ m H}_2\text{O}$$

$$\Delta p = \rho gh = (1000)(9.81)(34.49) = 338 \text{ kPa} \quad (b)$$

14.13 - SAME PUMP FAMILY AS IN PROB 14.8

$$\dot{V} = 0.201 \text{ m}^3/\text{s} \quad \omega = (1800) \left( \frac{2\pi}{60} \right) = 188.5 \text{ r/s}$$

14.13 - CONTINUED

$$@ \eta_{\max} = 0.89$$

$$C_Q \approx 0.12$$

$$C_H \approx 0.0515$$

$$C_p \approx 0.0068$$

$$C_Q = \frac{\dot{V}}{nD^3} \quad D = \left[ \frac{0.201}{(188.5)(0.12)} \right]^{1/3} = 0.207 \text{ m} \quad (a)$$

$$C_H = \frac{gh}{n^2 D^2} \quad h = \frac{(0.0515)(188.5)^2 (0.207)^2}{9.81} = 7.99 \text{ m H}_2\text{O}$$

$$\Delta p = \rho gh = (1000)(9.81)(7.99) = 78.4 \text{ kPa} \quad (b)$$

14.14

$$@ \eta_{\max} \approx 0.89$$

$$C_Q \approx 0.12$$

$$C_H \approx 5.3$$

$$h = 90 \text{ m H}_2\text{O}$$

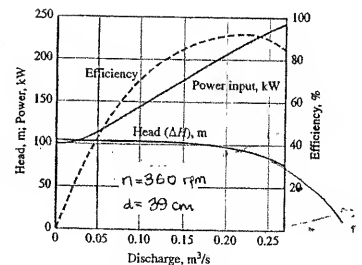
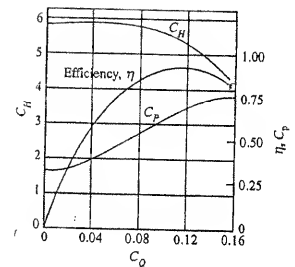
$$C_H = \frac{gh}{n^2 D^2} = 5.3$$

$$\omega^2 = \frac{gh}{C_H D^2} = \frac{9.81(90)}{5.3(0.39)^2}$$

$$\omega = 33.1 \text{ rad/s} = 316 \text{ rpm} \quad (a)$$

$$C_Q = 0.12 = \frac{\dot{V}}{nD^3}$$

$$\dot{V} = (0.12)(33.1)(0.39)^3 = 0.236 \text{ m}^3/\text{s} \quad (b)$$



14.15 SAME PUMP FAMILY AS IN 14.14

NEW PUMP:  $n = 400 \text{ rpm}$   
 $= 41.89 \text{ rad/s}$

$D_{\text{NEW}} = 6 \text{ DOLD}$

AT  $\eta_{\text{max}} - C_Q \approx 0.12 = \dot{V} / n D^3$

$C_H \approx 5.3 = g h / n^2 D^2$

$C_P \approx 0.7 = P / \rho n^3 D^5$

$P_1 = 0.70 (1000) (37.70)^3 (0.371)^5$

$= 263.6 \text{ kW}$

$P_{\text{NEW}} = P_1 \left( \frac{\omega_2}{\omega_1} \right)^3 \left( \frac{D_2}{D_1} \right)^5$

$= 263.6 \left( \frac{400}{360} \right)^3 (6)^5$

$= \underline{2.81 \text{ MW}} \quad (a)$

$h_1 = \frac{5.3 (37.70)^2 (0.371)^2}{9.81} = 105.7 \text{ m}$

$h_2 = h_1 \left( \frac{\omega_2}{\omega_1} \right)^2 \left( \frac{D_2}{D_1} \right)^2$

$= 105.7 \left( \frac{400}{360} \right)^2 (6)^2$

$= \underline{4.7 \text{ km}} \quad (b)$

$\dot{V}_1 = 0.12 n_1 D_1^3$

$= 0.12 (37.70) (0.371)^3$

$= 0.231 \text{ m}^3/\text{s}$

$\dot{V}_2 = \dot{V}_1 \left( \frac{n_2}{n_1} \right) \left( \frac{D_2}{D_1} \right)^3$

$= 0.231 \left( \frac{400}{360} \right) (6)^3$

$= \underline{55.4 \text{ m}^3/\text{s}} \quad (c)$

14.16 SAME PUMP FAMILY AS IN PROB 14.14

NEW  $n = 1000 \text{ rpm}$

$C_Q \approx 0.12 = \dot{V} / n D^3 \quad C_P \approx 0.7 = P / \rho n^3 D^5$

$\dot{V} = 0.12 \left( 1000 \times \frac{2\pi}{60} \right) (0.371)^3$

$= \underline{0.642 \text{ m}^3/\text{s}} \quad (a)$

$P = 0.7 (1000) \left( 1000 \times \frac{2\pi}{60} \right)^3 (0.371)^5$

$= \underline{5.65 \text{ MW}} \quad (b)$

14.17 SAME PUMP FAMILY AS IN PROB 14.14

NEW  $\omega = 800 \text{ rpm} = 83.8 \text{ rad/s}$

$h = 410 \text{ m}$

$C_H = \frac{g h}{n^2 D^2} = \frac{9.81 (410)}{(83.8)^2 (0.371)^2} = 4.161$

AT THIS VALUE OF  $C_H$ ,  $C_Q \approx 0.16$

$C_Q = 0.16 = \dot{V} / n D^3$

$\dot{V} = 0.16 (83.8) (0.371)^3$

$= \underline{0.685 \text{ m}^3/\text{s}}$

14.18 SAME PUMP FAMILY AS PROB 14.14

$D_2 = 3 D_1, \quad n_2 = 0.5 n_1$

@  $\eta_{\text{max}} \quad C_Q \approx 0.12 = \dot{V} / n D^3$

$C_H \approx 5.3 = g h / n^2 D^2$

$\frac{\dot{V}_2}{\dot{V}_1} = \left( \frac{n_2}{n_1} \right) \left( \frac{D_2}{D_1} \right)^3 = \frac{1}{2} (3)^3 = 13.5$

$\frac{h_2}{h_1} = \left( \frac{n_2}{n_1} \right)^2 \left( \frac{D_2}{D_1} \right)^2 = \left( \frac{1}{2} \right)^2 (3)^2 = 2.25$

14.18 CONTINUED

$$\dot{V}_1 = 0.12 (37.7) (0.371)^3 = 0.231 \text{ m}^3/\text{s}$$

$$\dot{V}_2 = 0.231 (135) = \underline{\underline{3.12 \text{ m}^3/\text{s}}}$$

$$h_1 = \frac{5.3 (37.7)^2 (0.371)^2}{9.81} = 105.7 \text{ m}$$

$$h_2 = (105.7) (2.25) = \underline{\underline{238 \text{ m}}}$$

14.19 Pump Performance AS IN  
PROB 14.14 -  $\Delta y = 95 \text{ m}$

H<sub>2</sub>O Pump -  $D = 0.28 \text{ m}$

$L = 550 \text{ m}$

$e = 0.457 \times 10^{-4} \text{ m}$

$$e/D = 0.000163$$

ENERGY EQN:

$$-\dot{W}_s = \dot{m} \left[ \frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g \Delta y + h_L \right]$$

$$h_L = 2 f_f \frac{L}{D} U^2$$

~ ASSUME FULLY TURBULENT

$$f_f \approx 0.0033$$

$$h_L = 2 (0.0033) \left( \frac{550}{0.28} \right) U^2$$

$$= 12.96 U^2$$

1<sup>ST</sup> LAW EXPRESSION BECOMES -

$$-\dot{W} = \dot{m} [90g + 12.96 U^2]$$

SYSTEM HEAD -

$$-\frac{\dot{W}}{\dot{m}g} = h = 90 + 1.32 U^2 \quad (1)$$

THIS MUST MATCH PUMP PERFORMANCE -

14.19 CONTINUED -

SYSTEM PERFORMANCE - EQN (1)

$\dot{V}$	$h$
0.10	93.48
0.15	95.93
0.20	100.54
0.25	106.5

$$\dot{V} = \frac{\pi}{4} D^2 U = 0.0616 U$$

SYSTEM & PUMP PERFORMANCE INTERSECT

AT  $\dot{V} \approx 0.21 \text{ m}^3/\text{s}$  -  $U = 3.41$

$$Re = \frac{(0.28)(3.41)}{0.995 \times 10^{-6}} = 9.59 \times 10^5$$

$$f_f = 0.0035 \sim \text{CLOSE ENOUGH}$$

SO: WITHIN ACCURACY OF READING PLOTS

$$\dot{V} = \underline{\underline{0.21 \text{ m}^3/\text{s}}}$$

14.20

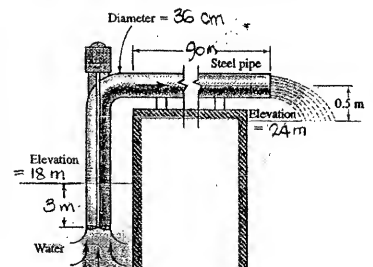
FOR STEEL

$$e = 0.457 \times 10^{-4} \text{ m}$$

$$e/D = 0.000127$$

FOR FULLY-TURBULENT

$$\text{Flow } f_f \approx 0.0031$$



$$\text{ENERGY EQN: } -\dot{W} = \dot{m} \left[ \frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g \Delta y + h_L \right]$$

BETWEEN RESERVOIR SURFACE (1)  
& DISCHARGE (2) -

$$\frac{\Delta P}{\rho} = V_1^2 = 0$$

$$g \Delta y = 6.5g$$

$$\frac{\Delta V^2}{2} = \frac{U^2}{2}$$

14.20 CONTINUED -

$$h_L = 2f_f \frac{L}{D} V^2 + \sum K \frac{V^2}{2}$$

$$= 2(0.0031) \frac{99.5}{0.36} V^2 + \underbrace{\frac{V^2}{2}}_{\text{ENTRANCE}}$$

$$= 2.21 V^2$$

ENERGY EQN NOW BECOMES:

$$-\dot{W} = \dot{m} \left[ 0.5 V^2 + 16.5g + 2.21 V^2 \right]$$

$$-\frac{\dot{W}}{\dot{m}g} = h_{\text{sys}} = 6.5 + 0.276 V^2 \quad (1)$$

SYSTEM PERFORMANCE - EQN (1)

$\dot{V}$	$h_{\text{sys}}$
0.20	7.27
0.25	7.71
0.30	8.24
0.35	8.87

PUMP & SYSTEM AREN'T WELL MATCHED - PUMP PERFORMANCE HEAD CURVE MUST BE EXTRAPOLATED

$$\dot{V} \approx 0.33 \text{ m}^3/\text{s}$$

$$-\dot{W} \approx 8.8 (1000)(0.33)(9.81)$$

$$= \underline{\underline{28.5 \text{ kW}}}$$

14.21 SAME PUMP FAMILY AS IN PROB 14.14

$$\frac{h_2}{h_1} = \left( \frac{v_2}{v_1} \right)^2 \left( \frac{D_2}{D_1} \right)^2$$

$$h_2 = h_1 \left( \frac{900}{360} \right)^2 = 6.25 h_1$$

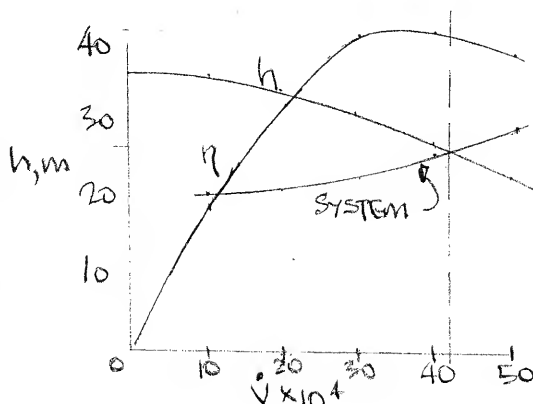
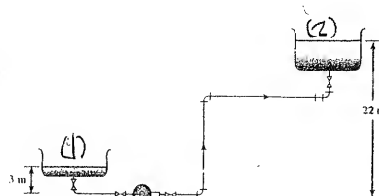
TOTAL MISMATCH -

14.22.

PUMP PERFORMANCE →

Capacity, $\text{m}^3/\text{s} \times 10^4$	Developed head, m	Efficiency, %
0	36.6	0
10	35.9	19.1
20	34.1	32.9
30	31.2	41.6
40	27.5	42.2
50	23.3	39.7

SYSTEM CONFIGURATION →



SYSTEM - INLET -  $D = 0.06 \text{ m}$   
 $L = 8.5 \text{ m}$

DISCHARGE -  $D = 0.06 \text{ m}$   
 $L = 60 \text{ m}$

STEEL -  $e = 0.457 \times 10^{-4} \text{ m}$   
 $e/D = 0.000762$

MINOR LOSSES - 4 VALVES  $K = 10$   
4 ELBOWS  $K = 0.3$   
1 CONTRACTION  $K = 1.0$

BETWEEN RESERVOIRS - (1) & (2)

$$-\dot{W} = \dot{m} \left[ \frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g \Delta y + h_L \right]$$

$$\frac{\Delta P}{\rho} = \frac{\Delta V^2}{2} = 0$$

$$g \Delta y = 19g \text{ m}^2/\text{s}^2$$

$$h_L = 2f_f \frac{L}{D} V^2 + \sum K \frac{V^2}{2}$$

14.22 CONTINUED -

ASSUME FLOW IS FULLY TURBULENT

$$f_f \approx 0,0046$$

$$\sum K = 4(10) + 4(0,3) + 1 = 42,2$$

$$h_L = \left[ 2(0,0046) \frac{68,5}{0,06} + \frac{42,2}{2} \right] U^2$$

$$= 31,6 U^2$$

ENERGY LOSS BECOMES:

$$-\frac{\dot{W}}{\dot{m}g} = \Delta y + \frac{h_L}{g} = h$$

$$= 19 + 3,22 U^2$$

$$\dot{V} \times 10^4 \quad h, m$$

20	20,61
30	22,63
40	25,45
50	29,07

INTERSECTION OCCURS AT

$$\dot{V} \approx 42 \times 10^{-4} \text{ m}^3/\text{s}$$

$$\text{AT } \dot{V} = 42 \times 10^{-4} \text{ m}^3/\text{s}$$

$$U = 1,485 \text{ m/s}$$

$$Re = \frac{(0,06)(1,485)}{0,995 \times 10^{-6}} = 8,957 \times 10^5$$

USING FIG 13.1 -

CONDITIONS ARE VERY CLOSE TO FULLY TURBULENT FLOW -

INITIAL ASSUMPTION FOR  $f_f$  WAS O.K.

$$\dot{V} = 0,0042 \text{ m}^3/\text{s}$$

14.23 Pump -  $D = 0,25 \text{ m}$

$$n = 2000 \text{ rpm}$$

$$\dot{V} = 0,065 \text{ m}^3/\text{s}$$

$$U_{\text{INLET}} = 6,1 \text{ m/s}$$

$$H_2O @ 20^\circ C \sim P_v = 2,34 \text{ kPa}$$

CAVITATION OCCURS AT  $P_c = 82,7 \text{ kPa}$

$$NPSH + \frac{P_v}{\rho g} = \frac{U_c^2}{2g} + \frac{P_c}{\rho g}$$

$$NPSH = \frac{U_c^2}{2g} + \frac{P_c - P_v}{\rho g}$$

$$= \frac{(6,1)^2}{2(9,81)} + \frac{(82,7 - 2,34)(10^3)}{1000(9,81)}$$

$$= 10,09 \text{ m H}_2\text{O}$$

14.24 - SAME PUMP AS DESCRIBED IN PROB 14.23 -

NEW TEMP IS  $80^\circ C$  ( $P_v = 47,35 \text{ kPa}$ )

$$NPSH = \frac{U_c^2}{2g} + \frac{P_c - P_v}{\rho g}$$

$$= \frac{(6,1)^2}{2(9,81)} + \frac{(82,7 - 47,35)(1000)}{1000(9,81)}$$

$$= 5,50 \text{ m H}_2\text{O}$$

CHANGE FROM  $20^\circ C$  CASE IS

$$\Delta = 10,09 - 5,50 = 4,59 \text{ m}$$

14,25

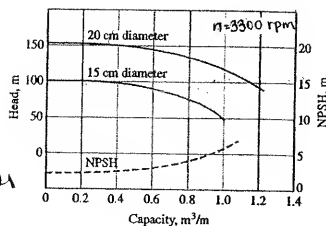
Pump -

$$D = 0.18 \text{ m}$$

INLET AT  $y = 3.8 \text{ m}$   
ABOVE SUPPLY  
RESERVOIR

$$\dot{V} = 0.760 \text{ m}^3/\text{s}$$

BETWEEN RESERVOIR SURFACE &  
PUMP INLET -  $h_L = 1.8 \text{ m H}_2\text{O}$



ENERGY EQUATION

$$NPSH = \frac{P_{\text{atm}} - P_v}{\rho g} - y_2 - h_L$$

AT  $20^\circ\text{C}$   $P_v = 2.34 \text{ kPa}$

$$\frac{P_{\text{atm}} - P_v}{\rho g} = \frac{(101.3 - 2.34)(10^3)}{(1000)(9.81)} = 10.09 \text{ m}$$

$$NPSH = 10.09 - 3.8 - 1.8 = 4.49 \text{ m H}_2\text{O}$$

FROM PERFORMANCE CURVE -

@  $\dot{V} = 0.760 \text{ m}^3/\text{s}$

$$NPSH \approx 3.9 \text{ m}$$

CAVITATION SHOULD NOT OCCUR

14,26  $\dot{V} = 220 \text{ m}^3/\text{s} = 3.487 \times 10^6 \text{ gpm}$

$$h = 420 \text{ m} = 1378 \text{ ft}$$

$$N_s = \frac{(400)(3.487 \times 10^6)^{1/2}}{(1378)^{3/4}} = 3302$$

ACCORDING TO FIG 14.11

THIS IS PROBABLY A HIGH  
CAPACITY CENTRIFUGAL PUMP

14,27 PUMP TO DELIVER  $60,000 \text{ gpm}$   
WITH  $h = 300 \text{ m}$  @  $2000 \text{ rpm}$ .

$$N_s = \frac{(2000)(6 \times 10^4)^{1/2}}{(300/0.3048)^{3/4}} \approx 2790$$

USING FIG 14.11 - PUMP IS PROBABLY  
A HIGH-CAPACITY CENTRIFUGAL PUMP.

14,28 AXIAL FLOW PUMP -  $N_s = 6.0$

$$N_s = \frac{C_a^{1/2}}{C_h^{3/4}} = \frac{\dot{V}^{1/2} \omega}{h^{3/4} g^{3/4}} \quad (1)$$

THIS RATIO IS (OBVIOUSLY) DIMENSIONLESS -  
BY CONVERTING TO UNITS ON ABSCISSA  
OF FIG 14.11 -

THE RATIO OF  $N_s$  GIVEN BY (1)  
TO THE VALUE ON FIG 14.11 IS  
2733

- SO A VALUE OF 6 FOR EQN (1)  
IS EQUIVALENT TO  $6(2733) = 1.64 \times 10^4$   
ON ABSCISSA OF FIG 14.11.

$$1.64 \times 10^4 = \frac{n(2400)^{1/2}}{(18)^{3/4}}$$

$$n = 2925 \text{ rpm}$$

14,29 Pump @ 520 rpm

$$\dot{V} = 3.3 \text{ m}^3/\text{s}$$

$$h = 16 \text{ m}$$

$$\dot{V} = (3.3) \left( \frac{1}{0.3048} \right)^3 (7.48) (60)$$

$$= 52302 \text{ gpm}$$

$$h = (16) / 0.3048 = 42.65 \text{ ft}$$

$$N_s = \frac{(520)(5.23 \times 10^5)^{1/2}}{(42.65)^{3/4}}$$

$$= 22532$$

FIG 14.11: AXIAL FLOW

14,30  $n = 2400 \text{ rpm}$

$$\dot{V} = 3.2 \text{ m}^3/\text{s}$$

$$h = 21 \text{ m}$$

$$\dot{V} = 3.2 \left( \frac{1}{0.3048} \right)^3 (7.48) (60)$$

$$= 5.072 \times 10^4 \text{ gpm}$$

$$h = \frac{21}{0.3048} = 68.9 \text{ ft}$$

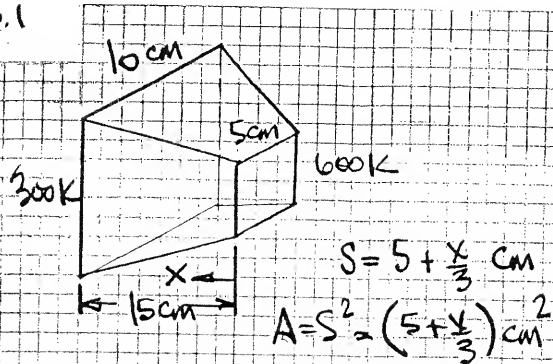
$$N_s = \frac{(5.072 \times 10^4)^{1/2} (2400)}{(68.9)^{3/4}}$$

$$= 22601$$

FIGURE 14.11 AXIAL FLOW

# CHAPTER 15

15.1



$$q = -kA \frac{dT}{dx}$$

$$q dx = -kA dT$$

$$q \int_0^{15} \frac{dx}{(5+x/3)^2} = k \int_{600}^{300} dT$$

$$q \left[ -\frac{3}{5+x/3} \right]_0^{15} = -300 k$$

$$q [0.6 - 0.3] = 300 (0.173 \text{ W/m}\cdot\text{K})$$

$$q = 1.73 \text{ W}$$

15.2 SAME VALUE AS IN PREVIOUS PROBLEM EXCEPT HEAT FLOWS IN OPPOSITE DIRECTION

$$q = 1.73 \text{ W}$$

15.3

$$q \int_0^{15} \frac{dx}{(5+x/3)^2} = -k_0 \int_{300}^{600} (1+\beta T) dT$$

$$q \left[ -\frac{3}{5+x/3} \right]_0^{15} = k_0 \Delta T \left[ 1 + \frac{\beta}{2} (T_1 + T_2) \right]$$

$$q [0.3 \text{ cm}^{-1}] = [0.135 \text{ W/m}\cdot\text{K}] (300 \text{ K}) \times [1 + 1.95 \times 10^{-4}] (450)$$

$$q = 1.50 \text{ W}$$

15.4

$$R_{\text{BOLT}} = \frac{L}{kA} = \frac{0.15}{(40)(\pi/4)(0.01905)^2} = 13.16 \text{ K/W}$$

NEGLECTING CHANGE IN CROSS-SECTIONAL AREA OF ASBESTOS:



$$R_{AS} = \frac{\Delta T}{q} = \frac{300}{1.73} = 173.4 \text{ K/W}$$

$$\frac{1}{R_{\text{equiv}}} = \frac{1}{13.16} + \frac{1}{173.4} = \frac{1}{12.23}$$

$$q = \frac{\Delta T}{R_{\text{equiv}}} = 24.5 \text{ W}$$

15.5

$$q = \frac{kA}{L} \Delta T$$

$$\Delta T = \frac{4000 \text{ W} (0.02 \text{ m})}{(0.22 \text{ W/m}\cdot\text{K})(2.97 \text{ m}^2)} = 122.4 \text{ K}$$

$$T_c = 55 + 122.4 = 177.4 \text{ C}$$

15.6

$$q = \frac{\Delta T}{\sum R}$$

$$\sum R = \frac{L}{kA} + \frac{1}{hA}$$

$$= \frac{0.02}{(0.22)(2.97)} + \frac{1}{(284)(2.97)}$$

$$= 4.246 \times 10^{-2} \text{ K/W}$$

$$\Delta T_{\text{TOTAL}} = (4000)(4.246 \times 10^{-2}) = 169.9 \text{ K}$$

$$T_{\text{HOT}} = 30 + 169.9 = 199.9 \text{ C}$$

$$T_{\text{SURF}} = 30 + \frac{4000}{(284)(2.97)} = 7.4 \text{ C}$$

15.7



$$q_{\max} = -k \left. \frac{dT}{dx} \right|_{\max}$$

$$= (1.35 \text{ W/m}\cdot\text{K})(15 \text{ K/cm})(100 \text{ cm/m})$$

$$= 2025 \text{ W/m}^2 = \Delta T/R$$

$$\Delta T = 2025 (1/5) = 405 \text{ K}$$

$$T_{\min} = 850 - 405 = \underline{445 \text{ K}}$$

15.8  $q_{\max} (\text{FROM PREVIOUS PROBS}) = 2025 \text{ W/m}^2$

$$= \frac{\Delta T}{R} + \sigma (T_{\text{SURF}}^4 - T_A^4)$$

$$= \frac{850 - T}{1/5} + 5.676 \left[ 8.5^4 - \left( \frac{T}{100} \right)^4 \right]$$

By TRIAL & ERROR:  $T = \underline{836 \text{ K}}$

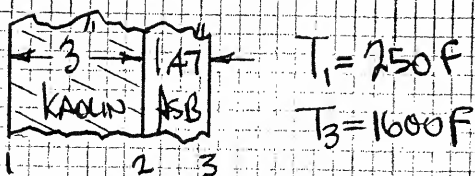
15.9

$$\frac{q}{A} = \frac{k \Delta T}{L} \text{ or } L = \frac{k \Delta T}{q/A}$$

$$L = \frac{(0.10 \text{ Btu/hr}\cdot\text{ft}\cdot\text{F})(1100 \text{ F})}{900 \text{ Btu/hr}\cdot\text{ft}^2}$$

$$= \underline{0.122 \text{ ft}} = \underline{1.47 \text{ in.}}$$

15.10 ADDING 3 IN. OF KAOLIN:



$$\frac{q}{A} = \frac{\Delta T}{\sum R} = \frac{1350}{\frac{1.47/12}{0.10} + \frac{3/12}{0.007}}$$

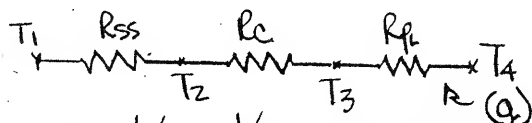
$$= \underline{281 \text{ Btu/hr}\cdot\text{ft}^2} \text{ (a)}$$

15.10 CONTINUED -

$$\frac{q}{A} = \frac{1600 - T_2}{\frac{1.47/12}{0.10}} = \frac{T_2 - 250}{\frac{3/12}{0.07}}$$

$$T_2 = \underline{1254 \text{ F}} \text{ (b)}$$

15.11



$$R_{ss} = L/k_{ss} = \frac{1/48}{10} = 0.0028 \text{ (a)}$$

$$R_c = L/k_c = \frac{3/12}{0.025} = 10 \text{ (b)}$$

$$R_p = L/k_p = \frac{1/24}{1.5} = 0.0278$$

$$\frac{q}{A} = \frac{\Delta T}{\sum R} = \frac{170}{10.03} = \underline{16.95 \frac{\text{Btu}}{\text{hr}\cdot\text{ft}^2}} \text{ (c)}$$

$$\frac{250 - T_2}{0.0028} = \frac{T_2 - T_3}{10} = \frac{T_3 - 80}{0.0278} = 16.95$$

$$T_2 = \underline{249.95 \text{ F}} \quad T_3 = \underline{80.47 \text{ F}} \text{ (d)}$$

15.12

$$R_{ins} = \frac{1}{40} = 0.025 \text{ hr}\cdot\text{ft}^2\cdot\text{F/Btu}$$

$$R_{con} = 1/5 = 0.20 \text{ " (a)}$$

$$\frac{q}{A} = \frac{\Delta T}{\sum R} = \frac{180}{10.225} = 17.6 \text{ Btu/hr}\cdot\text{ft}^2$$

$$= \frac{T_3 - 70}{0.0278 + 0.2} \quad T_3 = \underline{74.0 \text{ F}} \text{ (b)}$$

CONTROLLING RESISTANCE IS  
THE CORK BOARD.

15.13

$$q_{\text{TOTAL}} = q_{\text{CONV}} + q_{\text{RAD}}$$

FOR BLACK BODY RADIATION TO SPACE (NONE INCOMING)

$$q = \frac{\pi (10)^2}{4} \left[ 5(80) + 0.1714(6.2)^4 \right]$$

$$= 356 \text{ Btu/hr} \quad (a)$$

By CONVECTION: PERCENT =  $\frac{61.2}{38.8}$  (b)

RADIATION

IF SURROUNDINGS RADIATE @ 540 R

$$q_{\text{CONV}} = \frac{\pi (10)^2}{4} (5)(80) = 218.2 \text{ Btu/hr}$$

$$q_{\text{CONV}} = \frac{\pi (10)^2}{4} (0.1714) [6.2^4 - 5.4^4]$$

$$= 58.6 \text{ Btu/hr}$$

$$q = 276.8 \text{ Btu/hr} \quad \% \text{ CONV} = \frac{78.8}{21.2}$$

" RAD = 21.2

15.14

$$800 \text{ W} = 2730 \text{ Btu/hr}$$

IF ALL HEAT LEAVES TOP SURFACE

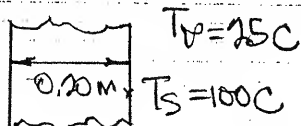
$$q = hA\Delta T + \sigma A\epsilon [T^4 - T_s^4]$$

$$\frac{2700}{A} = 5(T-40) + 0.1714(1) \left[ \left( \frac{T}{100} \right)^4 - 5.4^4 \right]$$

By TRIAL & ERROR:  $T = 1086 \text{ R}$

$= 626 \text{ F}$

15.15



$$\frac{q}{A} = \frac{k}{L} \Delta T_w = h(T_S - T_D)$$

15.15 CONTINUED -

$$\Delta T_w = \frac{(18 \text{ W/m}\cdot\text{K})(75 \text{ K})(20 \text{ m})}{1.3 \text{ W/m}\cdot\text{K}}$$

$$= 207 \text{ K}$$

$$T_{\text{INSIDE}} = 100 + 207 = \underline{307 \text{ C}}$$

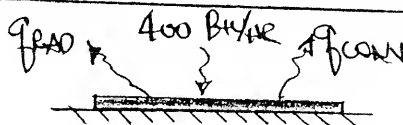
15.16

$$\frac{q}{A} = \frac{k}{L\Delta T_w} = h(T_S - T_D) + \sigma [T_S^4 - T_D^4]$$

$$\Delta T_w = \frac{0.2}{1.3} \left[ 18(75) + 5.676(3.73^4 - 2.98^4) \right]$$

$$= 308 \text{ K} \quad T_{\text{INSIDE}} = 408 \text{ C}$$

15.17



$$400 \frac{\text{Btu}}{\text{hr}} = A \left[ h(T_S - T_D) + \sigma (T_S^4 - T_D^4) \right]$$

$$100 \frac{\text{Btu}}{\text{hr}} \cdot \text{ft}^2 = 4(T-550) + 0.1714 \left[ \left( \frac{T}{100} \right)^4 - 5.5^4 \right]$$

TRIAL & ERROR:  $T = 570 \text{ R} = 110 \text{ F}$

15.18

AT TOP:

$$60 \frac{\text{Btu}}{\text{hr}} \cdot \text{ft}^2 = q_{\text{RAD}} + q_{\text{CONV}} + q_{\text{COND}}$$

$$100 = 0.1714 \left[ \left( \frac{T}{100} \right)^4 - 5.5^4 \right] + 4(T-550) + \frac{24.8}{1.4/12} (T-T_b) \quad (1)$$

AT BOTTOM:

$$\frac{24.8}{1.4/12} (T-T_b) = 3(T_b - T_D) \quad (2)$$

FROM (1)

$$8.05 \times 10^{-4} \left( \frac{T}{100} \right)^4 + 1.09T - T_b = 11.53$$

FROM (2)

$$T_b = 0.986T + 7.65$$

TRIAL &amp; ERROR:

$$T = 559 \text{ R} = 99 \text{ F}$$

15.18 CONTINUED -

WITH RADIATION FROM TOP

WITHOUT " " "

$$\text{Eqn ①: } T_B = 1.019T - 11.53$$

$$\text{② } T_b = 0.986T + 7.65$$

$$T = 582 \text{ R} = 122 \text{ F}$$

$$15.19 \quad A = 2[(0.3)(0.25) + (0.3)(0.5) + (0.25)(0.5)] \\ = 0.7 \text{ m}^2$$

$$q = \frac{kA}{L} \Delta T \quad L = \frac{kA \Delta T}{q}$$

$$L = \frac{(0.30 \text{ W/m}\cdot\text{K})(0.7 \text{ m}^2)(43 \text{ K})}{400 \text{ W}}$$

$$= 0.0226 \text{ m} = 2.26 \text{ cm}$$

$$15.20 \quad A = 0.7 \text{ m}^2$$

$$q = \Delta T / \sum R$$

$$R_i = 1/h_i A = \frac{1}{16(0.7)} = 8.93 \times 10^{-2} \text{ K/W}$$

$$R_{\text{cond}} = L/kA = \frac{L}{(0.30)(0.7)} = \frac{L}{0.21} "$$

$$R_o = 1/h_o A = \frac{1}{32(0.7)} = 4.46 \times 10^{-2} "$$

$$\sum R = 0.1340 + L/0.21 = \frac{43 \text{ K}}{400 \text{ W}}$$

FOR THESE CONDITIONS - NO INSULATION IS NECESSARY & THE SYSTEM CANNOT TRANSFER 400 W.

15.20 CONTINUED

$$q = \frac{\Delta T}{\sum R} = \frac{43 \text{ K}}{0.1340} = 321 \text{ K}$$

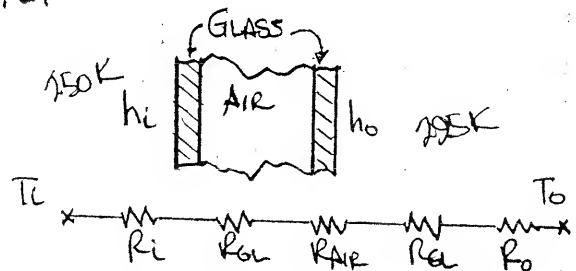
EFFECTIVE WALL Temp IS

$$q = h_i(0.7)\Delta T_i = h_o(0.7)\Delta T_o$$

$$\Delta T_i = \frac{321}{16(0.7)} = 28.6 \text{ K}$$

$$T_{\text{wall}} = 86 \text{ C}$$

15.21



$$q = \frac{\Delta T}{\sum R} \quad R_i = \frac{1}{(20)(1.83)(3.66)} = 7.46 \times 10^{-3}$$

$$R_{GL} = \frac{0.0032}{(0.78)(1.83)(3.66)} = 6.125 \times 10^{-4}$$

$$R_{AIR} = \frac{0.008}{(0.0245)(1.83)(3.66)} = 9.0488$$

$$R_o = \frac{1}{(15)(1.83)(3.66)} = 9.95 \times 10^{-3}$$

$$\sum R = 0.06744$$

$$q = \frac{\Delta T}{\sum R} = \frac{45}{0.06744} = 667 \text{ W}$$

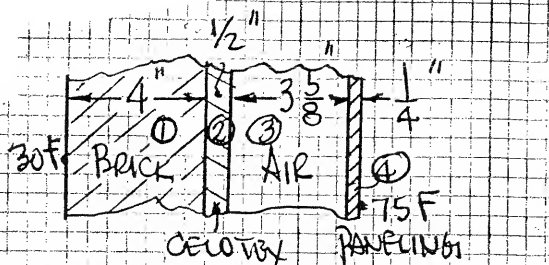
15.22 FOR 1 PAIR OF GLASS ONLY

$$\sum R = R_i + R_{GL} + R_o$$

$$= 0.0180$$

$$q = \frac{\Delta T}{\sum R} = \frac{45}{0.0180} = 2500 \text{ W}$$

15.23



$$\frac{q}{A} = -k \frac{dT}{dx} = k \frac{\Delta T}{\Delta x} = \frac{\Delta T}{R}; R = \frac{\Delta x}{k}$$

$$\frac{q}{A} = \frac{75-30 \text{ F}}{\frac{1/3}{0.38} + \frac{1/24}{0.028} + \frac{29/96}{0.015} + \frac{1/48}{0.012}}$$

$$= \frac{45}{0.876 + 1.49 + 2.02 + 0.174}$$

$$= \underline{1.98 \text{ BTU/hr-ft}^2} \quad (a)$$

$$R_{\text{AIR}} = \frac{\Delta x}{k} = \frac{1}{1.8} = 0.555$$

$$\Sigma R = 0.876 + 1.49 + 0.555 + 0.174$$

$$= 3.095$$

$$\frac{q}{A} = \frac{45}{3.095} = \underline{14.54 \text{ BTU/hr-ft}^2} \quad (b)$$

$$R_{\text{GLASS}} = \frac{29/96}{0.025} = 12.1$$

$$\Sigma R = 14.64$$

$$\frac{q}{A} = \frac{45}{14.64} = \underline{3.07 \text{ BTU/hr-ft}^2} \quad (c)$$

$$15.24 \quad \frac{q}{A} = \Delta T / \Sigma R$$

$$R_{\text{INSIDE}} = 1/h_i = 1/7 \frac{\text{hr-ft}^2}{\text{Btu}}$$

$$R_{\text{OUTSIDE}} = 1/h_o = 1/2$$

15.24 CONTINUED

$$\text{Part a) } \Sigma R = 23.4 \quad \frac{q}{A} = 1.92 \text{ BTU/hr-ft}^2$$

$$b) \Sigma R = 3.74 \quad \frac{q}{A} = 12.04$$

$$c) \Sigma R = 15.28 \quad \frac{q}{A} = 2.94$$

15.25



$$R_i = 1/A_i h_i = \frac{1}{(5110)(1)} = 1.97 \times 10^{-4}$$

$$R_o = 1/A_o h_o = \frac{1}{(45)(1)} = 0.22$$

$$R_1 = \frac{\Delta x}{kA} = \frac{0.106}{(1.13)(1)} = 0.0938$$

$$R_2 = \frac{0.00635}{42.9} = 1.48 \times 10^{-4}$$

$$\Sigma R = 0.1161$$

$$q = \frac{1340 - 295}{0.1161} = 9000 \text{ W}$$

$$9000 = \frac{1340 - T_1}{R_i} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2}$$

$$= \frac{T_3 - 295}{R_o}$$

$$\underline{T_1 = 1328 \text{ K} \quad T_2 = 494 \text{ K} \quad T_3 = 493 \text{ K}}$$

15.26

From previous problem

$$R_i + R_o + R_1 + R_2 + R_{\text{celotex}}$$

$$= 0.1161 + 1/0.069$$

$$= 0.1161 + 14.49 \text{ L}$$

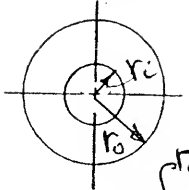
15.26 CONTINUED

$$q = \frac{340 - 295}{0.022} = 2027 \text{ W}$$

$$2027 = \frac{340 - 295}{0.1161 + 14.49 L}$$

$$L = 0.0276 \text{ m}$$

15.27



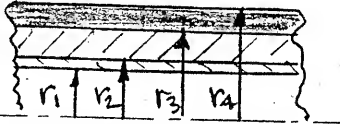
$$\int \frac{q}{A} dr = \int k dt$$

$$q \int_{r_i}^{r_o} \frac{dr}{2\pi r L} = -0.08 \int_{300}^{400} (1 - 0.0003T) dt$$

$$q \frac{\ln r_o/r_i}{2\pi L} = 65.52$$

$$q = \frac{2\pi (65.52)}{\ln 4} = 297 \frac{\text{Btu}}{\text{hr}}$$

15.28



FOR UNIT LENGTH:

$$R_{\text{INS}} = \frac{1}{2\pi r_i h_i} = 0.0238$$

$$R_1 = \frac{\ln r_2/r_1}{2\pi k_1} = 0.00105$$

$$R_2 = \frac{\ln r_3/r_2}{2\pi k_2} = 0.115/k_2$$

$$R_3 = \frac{\ln r_4/r_3}{2\pi k_3} = 0.0659/k_3$$

$$R_4 = \frac{1}{2\pi r_4 h_o} = 0.1296$$

15.28 CONTINUED -

$$\Sigma R = 0.1545 + 0.115/k_2 + 0.0659/k_3$$

CASE 1:  $k_2$  FOR MAGNESIA

$$\Sigma R = 6.134$$

CASE 2:  $k_2$  FOR GLASS WOOL

$$\Sigma R = 7.096$$

GLASS WOOL CASE IS BEST

$$q = \frac{\Delta T}{\Sigma R} = \frac{60}{7.096} = 8.47 \frac{\text{Btu}}{\text{hr-ft}}$$

$$\frac{q}{A} = \frac{8.47}{2\pi (2.95/12)} = 5.55 \frac{\text{Btu}}{\text{hr-ft}^2}$$

15.29 FOR BASE PIPE:

$$q = \pi D_i h \Delta T = \pi \left( \frac{1.315}{12} \right) (1.5) (310) = 160 \frac{\text{Btu}}{\text{hr per ft}}$$

FOR INSULATED PIPE:

$$q_o = \frac{T_s - T_\infty}{\frac{\ln D_2/D_1}{2\pi k} + \frac{1}{\pi D_2 h}}$$

$$\frac{\pi h}{2\pi k} \ln \frac{D_2}{D_1} + \frac{1}{D_2} = \frac{\pi h}{80} \Delta T$$

$$12.5 \ln D_2 / 0.1905 + \frac{1}{D_2} = 18.25$$

BY TRIAL & ERROR:  $D_2 = 0.382 \text{ FT}$ 

$$2t = D_2 - D_1 = 3.265 \text{ IN}$$

$$t = 1.63 \text{ IN.}$$

15.30

For Bare Pipe, Per Foot:

$$q = hA\Delta T = \frac{0.575}{(1.315/12)^{1/4}} \pi (1.315/12) (310) = 106.7 \text{ Btu/hr}$$

With Insulation -  $q = 53.3$ 

$$q = \frac{\Delta T}{\frac{\ln(D_o/D_i)}{2\pi k} + \frac{1}{\pi D_o h_o}}$$

$$53.3 = \frac{310}{\frac{\ln(D_o/1.315)}{2\pi(0.06)} + \frac{(D_o/12)^{1/4}}{0.575\pi D_o/12}}$$

By Trial & Error!  $D_o = 9.22 \text{ in}$ 

$$\text{Insulation Thickness} = \frac{9.22 - 1.315}{2} = \underline{\underline{3.95 \text{ in}}}$$

15.31 CONTINUED -

By Trial &amp; Error!

$$r_o = 0.177 \text{ m}$$

Insulation Thickness =  $r_o - r_i$ 

$$= 0.177 - 0.137 = \underline{\underline{0.04 \text{ m}}} = \underline{\underline{4 \text{ cm}}}$$

$$15.31 \quad q_o = \frac{\Delta T}{\sum R}$$

Without Insulation:

$$\sum R_{wo} = \frac{1}{2\pi L} \left[ \frac{\ln(13.7/12.5)}{17.3} + \frac{1}{(12)(0.137)} \right] = \frac{0.6136}{2\pi L} \text{ K/W}$$

With Insulation:

$$\sum R_w = \frac{1}{2\pi L} \left[ \frac{\ln(13.7/12.5)}{17.3} + \frac{1}{12r_o} + \frac{\ln(r_o/0.137)}{0.13} \right] = \frac{1}{2\pi L} \left[ 5.299 \times 10^{-3} + \frac{1}{12r_o} + \frac{\ln(r_o/13.7)}{0.13} \right]$$

$$k q_w = q_{wo} / 4$$

$$\sum R_w = 4 \sum R_{wo} = 2.454 / 2\pi L$$

$$\therefore 5.299 \times 10^{-3} + \frac{1}{12r_o} + \frac{\ln(r_o/13.7)}{0.13} = 2.454$$

# CHAPTER 16

16.1 IN CYLINDRICAL COORDINATES: (a)

$$\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = 0 \quad \text{or} \quad \frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) = 0$$

$$r \frac{dT}{dr} = C_1$$

$$T = C_1 \ln r + C_2$$

$$\text{B.C. } T_i = C_1 \ln r_i + C_2$$

$$T_o = C_1 \ln r_o + C_2$$

$$C_1 = -\frac{T_i - T_o}{\ln r_o / r_i} \quad C_2 = T_i - C_1 \ln r_i$$

$$T = T_i - (T_i - T_o) \frac{\ln r / r_i}{\ln r_o / r_i} \quad (b)$$

$$\begin{aligned} \dot{q} &= -kA \frac{dT}{dr} = -k(2\pi r L) \frac{dT}{dr} \\ &= -2\pi k L C_1 = \frac{2\pi k L}{\ln r_o / r_i} (T_i - T_o) \quad (c) \end{aligned}$$

$$16.2 \quad \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = 0 \quad (a)$$

$$r^2 \frac{dT}{dr} = C_1 \quad T = -\frac{C_1}{r} + C_2$$

$$\text{B.C. } T_i = -\frac{C_1}{r_i} + C_2$$

$$T_o = -\frac{C_1}{r_o} + C_2$$

$$C_1 = \frac{T_i - T_o}{\frac{1}{r_o} - \frac{1}{r_i}} \quad C_2 = T_i + \frac{C_1}{r_i}$$

$$T = T_i - \frac{\frac{1}{r} - \frac{1}{r_i}}{\frac{1}{r_o} - \frac{1}{r_i}} (T_i - T_o) \quad (b)$$

$$\begin{aligned} \dot{q} &= -k(4\pi r^2) \frac{dT}{dr} = -4\pi k C_1 \\ &= \frac{4\pi k}{\frac{1}{r_o} - \frac{1}{r_i}} (T_i - T_o) \quad (c) \end{aligned}$$

$$16.3 \quad \frac{d^2 T}{d\theta^2} = 0 \quad (a)$$

$$T = C_1 \theta + C_2$$

$$\text{B.C. } T_o = C_2$$

$$T_\pi = C_1 \pi + C_2$$

$$C_1 = \frac{T_\pi - T_o}{\pi} \quad C_2 = T_o$$

$$T = T_o - \frac{\theta}{\pi} (T_o - T_\pi) \quad (b)$$

$$\begin{aligned} dq_\theta &= -k dA \frac{dT}{dn} \\ &= -k(L dr) \frac{dT}{r d\theta} = -kL \frac{dT}{d\theta} dr \\ \int_0^{2\pi} dq_\theta &= -kL C_1 \int_{r_i}^{r_o} \frac{dr}{r} \\ \dot{q}_\theta &= -kL C_1 \ln r_o / r_i \\ &= \frac{kL}{\pi} \ln r_o / r_i (T_o - T_\pi) \quad (c) \end{aligned}$$

16.4 PROBLEM STATEMENT REQUIRES THAT WE DEMONSTRATE

$$\frac{S Du}{Dt} + \frac{S D}{Dt} (gy) + \vec{v} \cdot \vec{sg} = S \frac{DT}{Dt}$$

$$\text{FOR } C_v \text{ CONSTANT: } \frac{S Du}{Dt} = S C_v \frac{DT}{Dt} \quad (1)$$

$$\begin{aligned} \frac{S D}{Dt} (gy) &= S \left[ \frac{\partial}{\partial t} (gy) + u_x \frac{\partial}{\partial x} (gy) + u_y \frac{\partial}{\partial y} (gy) + u_z \frac{\partial}{\partial z} (gy) \right] \\ &= S g u_y \quad (2) \end{aligned}$$

$$\vec{v} \cdot \vec{sg} = \vec{v} \cdot \vec{sg} \vec{e}_y = -S v_y g \quad (3)$$

SUBSTITUTING (1), (2), & (3)

THE DESIRED RESULT IS OBTAINED

16.5 EON (16-7) WITH CONSTANT  $k$ :

$$k \nabla^2 T + \dot{q} + \mu \nabla^2 \vec{v} = \nabla \cdot S \vec{v} + S \frac{D}{Dt} \left( \frac{v^2}{2} \right) + S \frac{Du}{Dt} + S \frac{D}{Dt} (gy)$$

For  $\dot{q} = 0$  & NO VISCOS DISSIPATION

$$k \nabla^2 T + \vec{v} \cdot \mu \nabla^2 \vec{v} = \nabla \cdot S \vec{v} + S \frac{D}{Dt} \left( \frac{v^2}{2} \right) + S \frac{Du}{Dt} + S \frac{D}{Dt} (gy) \quad (1)$$

FROM NAVIER-STOKES:

$$S \frac{D \vec{v}}{Dt} = S \vec{g} - \nabla P + \mu \nabla^2 \vec{v} \quad (9-19)$$

DOT PRODUCT WITH  $\vec{v}$  YIELDS

$$S \frac{D}{Dt} \left( \frac{v^2}{2} \right) = \vec{v} \cdot S \vec{g} - \vec{v} \cdot \nabla P + \vec{v} \cdot \mu \nabla^2 \vec{v}$$

SUBSTITUTING INTO (1) & CANCELLING:

$$k \nabla^2 T = P \nabla \cdot \vec{v} + \vec{v} \cdot S \vec{g} + S \frac{Du}{Dt} + S \frac{D}{Dt} (gy)$$

FOR A POTENTIAL FUNCTION  $\phi = gy$

$$\nabla \phi = -\vec{g}$$

$$\text{THEN } S \frac{D}{Dt} (gy) = S \frac{D}{Dt} \phi = S \left( \frac{\partial \phi}{\partial t} + \vec{v} \cdot \nabla \phi \right)$$

COMBINING WITH THE ENERGY EON:

$$k \nabla^2 T = P \nabla \cdot \vec{v} + S \frac{Du}{Dt}$$

NOW - FROM THERMODYNAMICS:

$$u = u(v, T) \quad du = \left( \frac{\partial u}{\partial v} \right)_T dv + \left( \frac{\partial u}{\partial T} \right)_v dT$$

$$\Rightarrow S \frac{Du}{Dt} = S \left( \frac{\partial u}{\partial v} \right)_T \frac{Dv}{Dt} + S \left( \frac{\partial u}{\partial T} \right)_v \frac{DT}{Dt}$$

GIVING:

$$k \nabla^2 T = P \nabla \cdot \vec{v} + S c_v \frac{DT}{Dt} + S \frac{Dv}{Dt} \left[ -P + T \left( \frac{\partial P}{\partial v} \right)_T \right]$$

$$S \frac{Dv}{Dt} = S \frac{D}{Dt} \left( \frac{1}{\beta} \right) = -\frac{1}{\beta} \frac{D\beta}{Dt}$$

16.5 CONT. - BY CONTINUITY  $-\frac{1}{\beta} \frac{D\beta}{Dt} = \nabla \cdot \vec{v}$

$$\text{SO } k \nabla^2 T = P \nabla \cdot \vec{v} + S c_v \frac{DT}{Dt} - P \nabla \cdot \vec{v} + T \left( \frac{\partial P}{\partial T} \right)_v \nabla \cdot \vec{v}$$

FOR INCOMPRESSIBLE FLOW  $\nabla \cdot \vec{v} = 0$

$$k \nabla^2 T = S c_v \frac{DT}{Dt}$$

Q.E.D.

16.6 @ STEADY STATE  $\nabla T + \frac{\dot{q}}{k} = 0$

$$\frac{dT}{dx^2} + \frac{\dot{q}_0}{k} \frac{1}{\beta} e^{-\beta x/L} = 0$$

$$\frac{dT}{dx} = \frac{\dot{q}_0}{k} \frac{L}{\beta} e^{-\beta x/L} + C_1$$

$$T = -\frac{\dot{q}_0}{k} \frac{L^2}{\beta^2} e^{-\beta x/L} + C_1 x + C_2$$

$$\text{B.C. } T(0) = T_0 \quad T_0 = -\frac{\dot{q}_0}{k} \frac{L^2}{\beta^2} + C_2$$

$$T(L) = T_L \quad T_L = -\frac{\dot{q}_0}{k} \frac{L^2}{\beta^2} e^{-\beta} + C_1 L + C_2$$

$$T = T_0 + (T_L - T_0) \frac{x}{L} + \frac{\dot{q}_0 L^2}{k \beta^2} \left[ \frac{-\beta x/L}{1 - e^{-\beta}} - \frac{x}{L} (1 - e^{-\beta}) \right]$$

16.7 SAME PROBLEM EXCEPT 2ND B.C. IS

$$\frac{\partial T}{\partial x}(L) = 0 \Rightarrow 0 = \frac{\dot{q}_0}{k} \frac{L}{\beta} e^{-\beta} + C_1$$

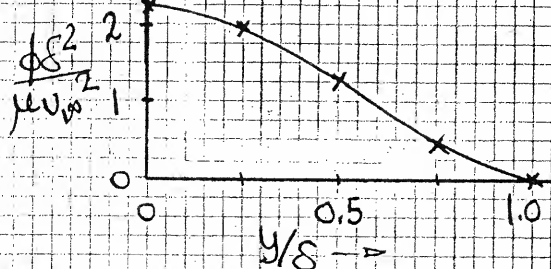
$$T = T_0 + \frac{\dot{q}_0}{k} \frac{L^2}{\beta^2} \left( (1 - \beta \frac{x}{L}) - e^{-\beta x/L} \right)$$

16.8. SAME PROBLEM AS 16.6 BUT 2ND 16.10 CONT.

B.C.  $\frac{dT}{dx}(L) = \frac{q}{k}$  (A CONSTANT)

$$\frac{q}{k} = \frac{q_0}{k} \frac{L}{\beta} e^{-\beta x} + C_2$$

$$T = T_0 + \frac{q_0}{k} x - \frac{q_0}{k} \frac{L^2}{\beta^2} \left( \frac{\beta x}{L} e^{-\beta x} + e^{-\beta x/L} \right)$$



16.9  $T ds = dh - \frac{dp}{\rho} = du + p dv$

FORMING SUBSTANTIAL DERIVATIVES

$$T \frac{Ds}{Dt} = \frac{Du}{Dt} + p \frac{Dv}{Dt} = \frac{Du}{Dt} + \frac{p}{s^2} \frac{Ds}{Dt}$$

BY CONTINUITY  $\frac{Ds}{Dt} = -s \nabla \cdot \vec{v}$

So  $\frac{Du}{Dt} = T \frac{Ds}{Dt} + \frac{p}{s} \nabla \cdot \vec{v}$

FROM THE ENERGY EQN

$$\frac{Du}{Dt} = \nabla \cdot k \nabla T + \dot{q} + \phi$$

$$\therefore T \frac{Ds}{Dt} = \nabla \cdot k \nabla T + \dot{q} + \phi + \frac{p}{s} \nabla \cdot \vec{v}$$

SINCE  $\phi$  IS ALWAYS  $> 0$  ITS EFFECT IS ALWAYS TO INCREASE  $s$

SINCE  $\nabla \cdot k \nabla T$  CAN BE EITHER  $+$  OR  $-$  AT TX CAN EITHER INCREASE OR DECREASE  $s$

16.10 For  $\frac{v_x}{v_w} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3$

WITH  $v_y = v_z = 0$  THE DISSIPATION FUNCTION REDUCES TO

$$\phi = \mu \left[ \partial v_x / \partial y \right]^2$$

FOR THIS CASE

$$\frac{\phi \delta^2}{\mu v_w^2} = \frac{9}{4} \left[ 1 - \left( \frac{y}{\delta} \right)^2 \right]$$

16.11 SPHERICAL COORDINATES

$$\frac{1}{r} \frac{d}{dr} \left( r^2 \frac{dT}{dr} \right) = 0$$

$$r^2 \frac{dT}{dr} = \text{CONST} \quad \frac{dT}{dr} = \frac{\text{CONST}}{r}$$



AS  $r$  INCREASES  $\frac{dT}{dr}$  DECREASES

16.12 FOR THE TRUNCATED CONE:

$$A_1 = \pi r_1^2 \quad A_2 = \pi r_2^2 \quad r = r_1 + \frac{r_2 - r_1}{L} x$$

$$\Rightarrow A = A_0 \left[ 1 + \left( \frac{r_2}{r_1} - 1 \right) \frac{x}{L} \right]^2 = A_0 \left( 1 + \beta \frac{x}{L} \right)^2$$

$$\beta = \left( \frac{r_2}{r_1} - 1 \right) \frac{1}{L}$$

SINCE  $q = -kA \frac{dT}{dx}$  WE HAVE

$$q = -kA_0 \left( 1 + \beta \frac{x}{L} \right) \frac{dT}{dx}$$

$$q \int_0^L \frac{dx}{1 + \beta x^2} = -kA_0 \int_{T_1}^{T_2} dT$$

$$q = kA_0 \left[ \tan^{-1}(\sqrt{\beta} L) \right] (T_1 - T_2)$$

### 16.12 CONTINUED -

IF, IN ADDITION,  $k = k_0 - \alpha T$

WE HAVE

$$q = -(k_0 - \alpha T)(1 + \beta x^2) \frac{dT}{dx}$$

$$q \int_0^L \frac{dx}{(1 + \beta x^2)} = - \int_{T_1}^{T_2} (k_0 - \alpha T) dT$$

$$q = A_0 \left[ \tan^{-1}(\sqrt{\beta} L) \right] [k_0 - \alpha(T_1 + T_2)] \times (T_1 - T_2)$$

### 16.13. HT GENERATION IN PLANE WALL

$$\dot{q} = \dot{q}_{\max} \left(1 - \frac{x}{L}\right)$$

FOURIER FIELD EQN, FOR STEADY STATE 1-D CONDUCTION, REDUCES TO

$$\frac{d^2 T}{dx^2} + \frac{\dot{q}_{\max}}{k} \left(1 - \frac{x}{L}\right) = 0$$

1ST INTEGRATION:

$$\frac{dT}{dx} + \frac{\dot{q}_{\max}}{k} \left(x - \frac{x^2}{2L}\right) = C_1$$

SYMMETRY,  $\frac{dT}{dx} = 0$  @  $x=0 \therefore C_1 = 0$

SECOND INTEGRATION:

$$\int_{T_c}^{T_o} dT + \frac{\dot{q}_{\max}}{k} \int_0^L \left(x - \frac{x^2}{2L}\right) dx = 0$$

$$T_o - T_c + \frac{\dot{q}_{\max}}{k} \left(\frac{x^2}{2} - \frac{x^3}{6L}\right)_0^L = 0$$

$$T_c - T_o = \frac{\dot{q}_{\max} L^2}{3k}$$

### 16.14 HT GENERATION IN A CYLINDER

$$\dot{q} = \dot{q}_{\max} \left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

FOURIER FIELD EQN REDUCES TO

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr}\right) + \frac{\dot{q}_{\max}}{k} \left[1 - \left(\frac{r}{r_o}\right)^2\right] = 0$$

SEPARATING VARIABLES - 1<sup>st</sup> INTEGRATION

$$\int d\left(r \frac{dT}{dr}\right) + \frac{\dot{q}_{\max}}{k} \int \left[r - \frac{r^3}{r_o^2}\right] dr = 0$$

$$r \frac{dT}{dr} + \frac{\dot{q}_{\max}}{k} \left(\frac{r^2}{2} - \frac{r^4}{4r_o^2}\right) = C_1$$

SYMMETRY:  $\frac{dT}{dr} = 0$  @  $r=0 \therefore C_1 = 0$

SECOND SEPARATION & INTEGRATION

$$\int_{T_c}^{T_o} dT + \frac{\dot{q}_{\max}}{k} \int_0^{r_o} \left(\frac{r}{2} - \frac{r^3}{4r_o^2}\right) dr = 0$$

$$T_c - T_o = \frac{\dot{q}_{\max}}{k} \frac{3}{16} r_o^2$$

### 16.15 HT GENERATION IN A SPHERE -

FOURIER FIELD EQN REDUCES TO:

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr}\right) + \frac{\dot{q}_{\max}}{k} \left[1 - \left(\frac{r}{r_o}\right)^3\right] = 0$$

1ST INTEGRATION YIELDS 0 - SYMMETRY

$$r^2 \frac{dT}{dr} + \frac{\dot{q}_{\max}}{k} \left(\frac{r^3}{3} - \frac{r^4}{4r_o^3}\right) = C_1$$

SECOND INTEGRATION

$$\int_{T_c}^{T_o} dT + \frac{\dot{q}_{\max}}{k} \int_0^{r_o} \left(\frac{r}{3} - \frac{r^4}{4r_o^3}\right) dr = 0$$

$$T_c - T_o = \frac{\dot{q}_{\max}}{k} \frac{2}{15} r_o^2$$

# CHAPTER 17

## 17.1 STEADY-STATE X-DIRECTIONAL CONDUCTION THROUGH A PLANE WALL

$$q_x = -kA \frac{dT}{dx} = \frac{kA}{L} (T_1 - T_2)$$

$$\text{For } T_1 - T_2 = 75 \text{ K}$$

$$q = \frac{(30 \text{ W/m}\cdot\text{K})(1 \text{ m}^2)(75 \text{ K})}{0.30 \text{ m}}$$

$$= 7500 \text{ W/m}^2$$

$$\Delta T/\Delta x = \frac{\Delta T}{L} = \frac{75 \text{ K}}{0.30 \text{ m}} = 250 \text{ K/m}$$

$$\text{For } T_1 = 300 \text{ K } q = -2000 \text{ W/m}^2$$

$$\Delta T/\Delta x = -\frac{q}{kA} = \frac{2000 \text{ W/m}^2}{(30 \text{ W/m}\cdot\text{K})} = 66.7 \text{ K/m}$$

$$\Delta T = \frac{qL}{kA} = \frac{(-2000)(0.3)}{(30)} = -20 \text{ K}$$

$$T_2 = 320 \text{ K}$$

$$\text{For } T_2 = 350 \text{ K } \Delta T/\Delta x = -300 \text{ K/m}$$

$$q = -(30)(-300) = 9000 \text{ W/m}^2$$

$$\Delta T = -300 \text{ K/m}(0.3 \text{ m}) = 90 \text{ K}$$

$$T_1 = 440 \text{ K}$$

$$\text{For } T_1 = 250 \text{ K } \Delta T/\Delta x = 200 \text{ K/m}$$

$$q = -(30)(200 \text{ K/m}) = -6000 \text{ W/m}^2$$

$$\Delta T = -(200)(0.3) = -60 \text{ K } T_2 = 310 \text{ K}$$

$$17.2 \quad q = \frac{k\bar{A}}{r_o r_i} \Delta T = \frac{k}{r_o r_i} 2\pi(r_o - r_i) \Delta T$$

$$= \frac{2\pi k}{\ln r_o/r_i} \Delta T \quad (a)$$

$$\% \text{ ERROR} = \frac{A_{lm} - A_{pm}}{A_{lm}} \times 100$$

## 17.2 CONTINUED

$$= \frac{\frac{2\pi(r_o - r_i)}{\ln r_o/r_i} - \pi(r_o r_i)}{2\pi(r_o - r_i)/\ln r_o/r_i} \times 100$$

$$= \left| 1 - \frac{(r_o r_i) \ln r_o/r_i}{2(r_o - r_i)} \right| \times 100$$

$$= \left| 1 - \frac{(r_o/r_i + 1) \ln r_o/r_i}{2(r_o/r_i - 1)} \right| \times 100$$

$$\text{For } r_o/r_i = 1.5 \quad \% \text{ ERROR} = 1.3 \%$$

$$3 \quad " = 10.0 \%$$

$$5 \quad " = 20.7 \%$$

## 17.3

$$q = \frac{4\pi k r_o r_i}{r_o - r_i} \Delta T \quad \bar{A} = 4\pi r_o r_i \quad (a)$$

$$A_m = \frac{4\pi(r_o^2 + r_i^2)}{2} = 2\pi(r_o^2 + r_i^2)$$

$$\% \text{ ERROR} = \frac{4\pi r_o r_i - 2\pi(r_o^2 + r_i^2)}{4\pi r_o r_i}$$

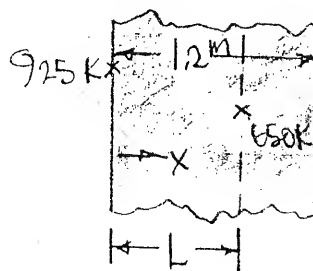
$$= 1 - \frac{1}{2} \left( \frac{r_o}{r_i} + \frac{r_i}{r_o} \right)$$

$$r_o/r_i = 1.5 \quad \% \text{ ERROR} = 8.3 \%$$

$$3 \quad " = 66.6 \%$$

$$5 \quad " = 160 \%$$

## 17.4



$$T_{\infty} = 300 \text{ K}$$

17.4 CONTINUED

$$q'' = -k \frac{dT}{dx} = -k_0(1+bT) \frac{dT}{dx}$$

From 0 to 1.2: T

$$q'' \int_0^{1.2} dx = -k_0 \int_{925}^{925} (1+bT) dT$$

$$q'' = \frac{k_0}{1.2} \left[ T + \frac{b}{2} T^2 \right]_{925}^{925} = 23(T-300)$$

Solving:  $T_{RH WALL} = 307.1 \text{ K}$   
 $q'' = 163.3 \text{ W/m}^2$

From 0 to L:

$$q'' \int_0^L dx = -k_0 \int_{925}^{650} (1+bT) dT$$

$$L = \frac{k_0}{q''} \left[ T + \frac{b}{2} T^2 \right]_{650}^{925}$$

Solving:

$$L = 0.646 \text{ m}$$

17.5

GOVERNING EQN-  $\nabla \cdot k \nabla T = 0$ IN ONE DIMENSION:  $\frac{d}{dx} \left( k \frac{dT}{dx} \right) = 0$ FOR CONSTANT k:  $\frac{d^2 T}{dx^2} = 0$ 

$$\frac{dT}{dx} = C_1$$

$$T = C_1 x + C_2$$

$$T(0) = T_0 = C_1(0) + C_2 \quad C_2 = T_0$$

$$T(L) = T_L = C_1 L + C_2 \quad C_1 = \frac{T_L - T_0}{L}$$

FOR VARIABLE k:  $\frac{d}{dx} \left( k_0(1+bT) \frac{dT}{dx} \right) = 0$ 

$$(1+bT) \frac{dT}{dx} = C_3$$

$$T + \frac{b}{2} T^2 = C_3 x + C_4$$

$$T(0) = T_0 \quad T_0 + \frac{b}{2} T_0^2 = C_4$$

$$T(L) = T_L \quad T_L + \frac{b}{2} T_L^2 = C_3 L + C_4$$

17.5 CONTINUED -

$$C_3 = \frac{T_L}{L} \left( 1 + \frac{bL}{2} \right) - \frac{C_4}{L}$$

$$T^2 + \frac{2}{\beta} T - \frac{2}{\beta} C_3 x - \frac{2}{\beta} C_4$$

$$T^2 + BT - C = 0 \quad B = 2/\beta$$

$$T = -\frac{B}{2} \pm \sqrt{\frac{B^2}{4} - C} \quad C = \frac{2}{\beta} (C_3 x + C_4)$$

NOW - THE TEMPERATURE DIFFERENCE WE'RE SEEKING IS:

$$\Delta = C_1 x + C_2 - \left[ -\frac{B}{2} \pm \sqrt{\frac{B^2}{4} - C} \right]$$

MINIMUM IS WHERE  $\frac{d}{dx} \Delta = 0$ 

$$\frac{d\Delta}{dx} = C_1 + \left( \frac{B^2}{4} - C \right)^{-1/2} \left( -\frac{2C_3}{\beta} \right) = 0$$

$$\frac{C_1 \beta}{2C_3} + \left( \frac{B^2}{4} - C \right)^{-1/2} = 0$$

$$\frac{B^2}{4} - C = \frac{4C_3^2}{\beta^2 C_1^2}$$

$$C = \frac{B^2}{4} - \frac{4C_3^2}{\beta^2 C_1^2}$$

$$\frac{2}{\beta} C_3 x + \frac{2}{\beta} C_4 = \frac{1}{\beta^2} - \frac{4C_3^2}{\beta^2 C_1^2}$$

$$x = \frac{1}{2\beta C_3} - \frac{2}{\beta C_1^2} - \frac{C_4}{C_3}$$

 $C_1, C_3, \frac{1}{\beta} C_4$  ARE AS DETERMINED ABOVE

17.6 SAME GENERAL PROCEDURE AS PREVIOUS PROBLEM:

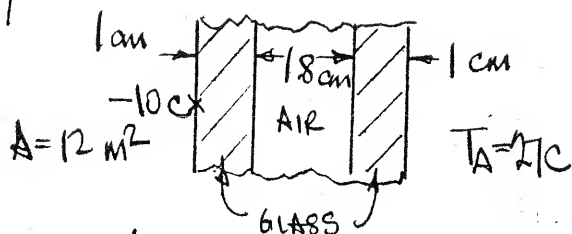
$$\text{D.E. IS } \frac{1}{r} \frac{d}{dr} \left( k r \frac{dT}{dr} \right) = 0$$

$$\text{FOR CONSTANT } k: \frac{d}{dr} \left( r \frac{dT}{dr} \right) = 0$$

$$\text{FOR VARIABLE } k: \frac{d}{dr} \left( r(1+bT) \frac{dT}{dr} \right) = 0$$

- MESSY BUT STRAIGHTFORWARD -

17.7



$$q = \frac{\Delta T}{\sum R} \quad R_{GL} = \frac{0.01}{(0.78)(12)} = 1.068 \times 10^{-3}$$

$$R_{AIR} = \frac{0.018}{0.10262(12)} = 5.725 \times 10^{-2}$$

$$R_{GLW} = \frac{1}{12(12)} = 6.944 \times 10^{-3}$$

$$\sum R = 2(1.068 \times 10^{-3}) + R_{AIR} + R_{GLW}$$

$$= 0.06633 \text{ K/W}$$

$$q = 37 / 0.06633 = 585 \text{ W}$$

$$T_i = 27 - \frac{585}{(12)(12)} = 22.94 \text{ C}$$

17.8 - Brick Sree = 9" x 4.5' x 3"

Brick #1  $k = 0.44 \frac{\text{Btu}}{\text{hr ft F}}$   $T_{max} = 1500 \text{ F}$ #2  $k = 0.94$  "  $T_{max} = 2100$  "

MOST ECONOMICAL ARRANGEMENT IS TO USE AS MUCH OF #1 AS POSSIBLE (LOW  $k$ ). USE #2 NEXT TO HIGH TEMP SUCH THAT ITS COOLER SURFACE HAS  $T \leq 1500 \text{ F}$ .

$$q'' = \frac{2000 - T}{L/k} \quad L_2 = \frac{k(2000 - T_m)}{200}$$

$$= 2.35 \text{ FT}$$

$$= 28.2 \text{ IN.}$$

$$L_{ACTUAL} = 28.5 \text{ IN.} \quad (9 \times 2 + 4.5 + 2 \times 3)$$

$$T_{INTERFACE} = T_H - \frac{q''}{k_2/L_2} = 1495 \text{ F}$$

$$L_{MIN} = \frac{0.44(1495 - 300)}{200} = 2.63 \text{ FT}$$

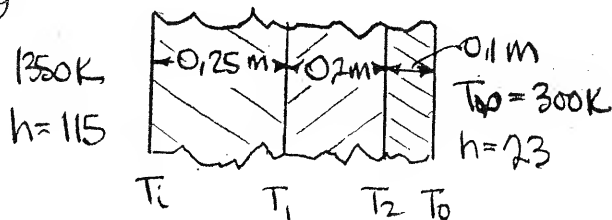
$$= 31.6 \text{ IN.}$$

$$L_{ACT} = 33 \text{ IN}$$

17.8 CONTINUED -

MOST ECONOMICAL:  $L_1 = 33 \text{ IN}$   
 $L_2 = 28.5$  "

17.9



$$R_i = 1/115 = 8.696 \times 10^{-5} \text{ K/W}$$

$$R_1 = 0.25/1.13 = 0.221 \text{ "}$$

$$R_2 = 0.20/1.45 = 0.138 \text{ "}$$

$$R_3 = 0.10/0.66 = 0.152 \text{ "}$$

$$R_o = 1/23 = 0.0435 \text{ K/W}$$

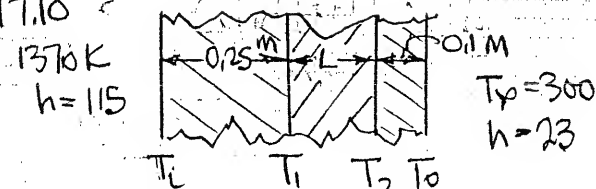
$$\sum R = 0.563$$

$$q = \Delta T / \sum R = \frac{1070}{0.563} = 1900 \text{ W/m}^2$$

$$= 176.5 \text{ W/ft}^2$$

$$q = 23(T_0 - 300) \quad T_0 = 383 \text{ K}$$

17.10



$$T_0 = 325 \text{ K} \quad q = 23(325 - 300)$$

$$= 575 \text{ W/m}^2$$

$$R_i = 1/115 = 8.696 \times 10^{-5} \text{ K/W}$$

$$R_1 = 0.25/1.13 = 0.221 \text{ "}$$

$$R_2 = L/1.45$$

$$R_3 = 0.10/0.66 = 0.152 \text{ "}$$

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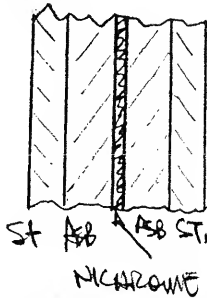
17.10 CONTINUED

$$\Sigma R = 0.416 + \frac{1}{1.45} = \frac{\Delta T}{q}$$

$$0.416 + \frac{1}{1.45} = \frac{1045}{575}$$

$$L = 2.03 \text{ m}$$

17.11


 $T_{\infty} \approx 70^\circ \text{F}$  (ASSUME)

$$L_{ST} = \frac{1}{8} \text{ ft} \quad R = 10 \frac{\text{hr} \cdot \text{ft}^2}{\text{Btu}}$$

$$L_{ASB} = \frac{1}{8} \text{ in} \quad R = 0.15 \text{ hr} \cdot \text{ft}^2 / \text{Btu}$$

$$q = \frac{\Delta T}{\Sigma R} = \frac{1000 - T_{\infty}}{\frac{0.125/12}{0.15} + \frac{0.125/12}{10} + \frac{1}{3}}$$

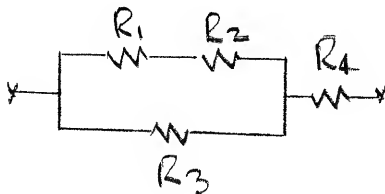
$$= \frac{930}{0.403} = 2305 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2} \text{ PER SIDE}$$

$$\frac{q}{A} = \frac{2305(2)}{3.413} = 1351 \frac{\text{W}}{\text{ft}^2} \text{ (a)}$$

$$2305 = h \Delta T = 3 \Delta T$$

$$\Delta T = 768^\circ \text{F} \quad T_{\text{surf}} = 838^\circ \text{F} \text{ (b)}$$

17.12



$$R_1 = \frac{0.125/12}{0.15} = 0.0694$$

$$R_2 = \frac{0.125/12}{10} = 0.00104$$

$$R_3 = \frac{0.25/12}{(12)(2)(\pi/4)(0.75/12)^2}$$

$$= 0.154$$

17.12 CONTINUED -

$$R_{\text{CONDUCTION, EQUIV}} = \frac{1}{\frac{1}{R_1 + R_2} + \frac{1}{R_3}}$$

$$= \frac{1}{\frac{1}{0.07044} + \frac{1}{0.154}} = 0.0483$$

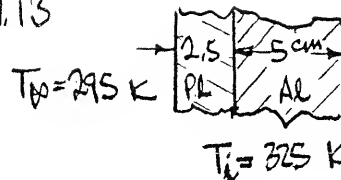
$$\Sigma R_{\text{PER SIDE}} = 0.0483 + \frac{1}{3} = 0.3817$$

$$\text{NEW HT FLUX} = \frac{930}{0.3817} = 2437 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2}$$

$$\text{INCREASE} = \frac{2437 - 2305}{2305}$$

$$= 0.057 = 5.7\%$$

17.13



$$T_{\infty} = 295 \text{ K}$$

$$h_1 = 12 \text{ W/m}^2 \cdot \text{K}$$

$$h_2 = \text{ "}$$

$$k_1 = 2.42 \text{ W/m} \cdot \text{K}$$

$$k_2 = 229 \text{ W/m} \cdot \text{K}$$

a) APPLIED TO PLASTIC:

$$q = 12(T_1 - 295) + \frac{2.42}{0.025}(T_1 - 325)$$

$$\frac{229}{0.05}(325 - T_2) = 12(T_2 - 295)$$

$$T_2 = 324.9 \quad T_1 = 328.7$$

$$q = 359 + 404 = 763 \text{ W}$$

b) APPLIED TO AL:

$$q = 12(T_2 - 295) + \frac{229}{0.05}(T_2 - 325)$$

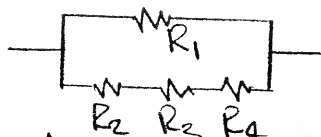
$$\frac{229}{0.05}(T_2 - 325) = \frac{2.42}{0.025}(325 - T_1)$$

$$T_1 = 322 \text{ K}$$

$$T_2 = 325$$

$$q = 320 + 361 = 681 \text{ W}$$

17.14



BOLTS IN A SQUARE ARRAY  
WITH 4 EQUIV. BOLTS/FT<sup>2</sup>

$$R_1 (\text{BOLTS}) = \frac{L}{kA} = \frac{375/12}{(10 \times 4) \times (1/4) \times \left(\frac{1}{144}\right)}$$

$$= 5.7 \text{ HR-F/FT}^2 \text{ - STEEL}$$

$$= 0.475 \text{ " - ALUM.}$$

$$R_{2ss} = \frac{1/48}{(10 \times 1)} = 0.0021$$

$$R_{3cb} = \frac{3/12}{(0.25 \times 1)} = 10$$

$$R_{4pl} = \frac{1/24}{(1.5 \times 1)} = 0.0278$$

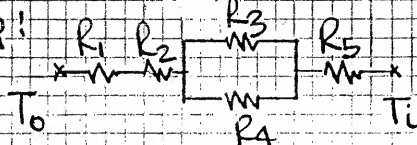
$$\sum R = 10.03$$

PER FT<sup>2</sup> OF X-SECTION

$$R_{\text{equiv}} = \frac{1}{1/5.7 + 1/10.03} = 3.63 \frac{\text{HR-F}}{\text{FT}^2} \text{ (a)}$$

$$= \frac{1}{1/0.475 + 1/10.03} = 0.454 \text{ " (b)}$$

17.15 FOR A SECTION 36 CM WIDE &  
1 m DEEP:



$$R_1 = \frac{1}{A h_0} = \frac{1}{(0.36 \times 20)} = 0.139 \text{ K/W}$$

$$R_2 = \frac{L}{kA|_p} = \frac{0.02}{(0.814)(0.36)} = 0.0683$$

$$R_3 = \frac{L}{kA|_w} = \frac{0.15}{0.115(0.06)} = 16.67 \text{ "}$$

17.15 CONTINUED-

$$R_4 = \frac{L}{kA|_{fg}} = \frac{0.15}{0.035(0.30)} = 14.28 \text{ "}$$

$$R_5 = \frac{1}{A h_i} = \frac{1}{(0.36 \times 10)} = 0.278 \text{ "}$$

$$R_{\text{stud equiv}} = \frac{1}{1/16.67 + 1/14.28} = 7.691 \text{ "}$$

$$q = \frac{\Delta T}{\sum R_{\text{equiv}}} = \frac{35 \text{ K}}{8.116 \text{ K/W}} = 4.28 \text{ W}$$

$$q_3 = \frac{\Delta T_{sw}}{R_3} \quad q_4 = \frac{\Delta T_{sw}}{R_4}$$

$$\Delta T_{sw} = \frac{q}{1/R_3 + 1/R_4} = 32.92 \text{ K}$$

$$q_3 = \frac{32.92}{16.67} = 1.975 \text{ W}$$

$$q_4 = \frac{32.92}{14.28} = 2.305 \text{ W}$$

17.16

$$q_{\text{loss}} = q_1 + q_2$$

$$q_1 = \text{HT TO AIR} = h A \Delta T$$

$$= (23 \text{ W/m}^2 \cdot \text{K}) (2.5 \times 0.1 \times 2 \text{ m}^2 + 2.5 \times 0.05 \times 2 \text{ m}^2 + 0.1 \times 0.05 \times 2 \text{ m}^2 - (0.08)^2 \times 2 \text{ m}^2) (T - 300)$$

$$= 17.19 (T - 300)$$

$$q_2 = \text{HT THROUGH PEDESTALS}$$

$$= 2 k A m \theta_0 \left[ 1 - 2 \frac{e^{mL} - e^{-mL}}{e^{mL} + e^{-mL}} \right]$$

$$m = \left[ \frac{h p}{k A} \right]^{1/2} = \left[ \frac{(23 \times 0.08)(4)}{(2.6)(0.08)(0.08)} \right]^{1/2} = 21$$

$$e^{mL} = 23.45 \quad e^{-mL} = 0.0427$$

17.16 CONTINUED -

$$q_2 = 2(2.6)(0.08)^2(21)(T-300) \times \left[ 1 - 2 \frac{-0.0421}{23.45} \right]$$

$$= 0.096(T-300)$$

$$1000 = (17.19 + 0.096)(T-300)$$

$$T = 355.9 \text{ K}$$

17.17

$$q_{\text{LOSS}} = q_1 + q_2 + q_3$$

$$q_1 = 17.19(T-300) \quad \text{FROM PREV. PROB.}$$

$$q_2 = \text{SAME EXPRESSION}$$

$$A = (0.08)(0.08) - \frac{\pi}{4}(0.019)^2$$

$$= 0.00612 \quad \left\{ \begin{array}{l} \text{PREVIOUSLY} \\ 0.0064 \end{array} \right\}$$

FOR PEDESTAL MAT'L:

$$q_2 \approx 0.7(T-300)$$

$$q_3 = \text{CONDUCTION THROUGH BOLTS}$$

$$= \frac{kA}{L} \Delta T = 42.9 \frac{\pi}{4} \frac{(0.019)^2}{0.15} \Delta T$$

$$= 0.081(T-300)$$

$$1000 = [17.19 + 0.7 + 0.081](T-300)$$

$$T = 355.6 \text{ K}$$

17.18

$$q = \Delta T / \sum R$$

$$\Delta T = 292.7 - 70 = 222.7 \text{ F}$$

ROOM TEMP (ASSUMED)

FOR 2-IN SCHED 40 ID = 2.067 IN

OD = 2.375 "

$$R_{ST} = \frac{\ln D_o/D_i}{2\pi k L} = 1.475 \times 10^{-5}$$

$$R_{INS} = \frac{\ln D_o/D_i}{2\pi k L} = 0.0529$$

$$R_{AIRS} = 1/hA_o = 0.00537 \text{ w/o INSUL}$$

$$= 0.00237 \text{ w/ INSUL}$$

$$\sum R_{WITH} = 0.0553 \quad q = 4030 \text{ BTU/HR}$$

$$\sum R_{WITHOUT} = 0.00537 \quad q = 41500 "$$

$$\Delta q = 37470 \text{ BTU/HR}$$

$$\text{COST} = 37470 \left( \frac{\$0.68}{10^5} \right) = \$0.255/\text{HR}$$

$$\text{TIME} = \frac{(60 \text{ FT})(\$0.75/\text{FT})}{\$0.255/\text{HR}} = 177 \text{ HOURS}$$

17.19

267.25 F  $\xrightarrow{R_1} \xrightarrow{R_2} \xrightarrow{R_3} 80 \text{ F}$

$$R_1 = \frac{1}{\pi D_i h_i L} = \frac{1}{\pi (1.612)(1500)(10)} = 0.0015$$

$$R_2 = \frac{\ln D_o/D_i}{2\pi k L} = \frac{\ln 1.078}{2\pi (24)(10)} = 6.19 \times 10^{-5}$$

$$R_3 = \frac{1}{\pi (1.755/12)(12)} = 0.0725$$

$$\sum R = 0.072 \text{ HR F/BTU}$$

$$q = \frac{\Delta T}{\sum R} = \frac{187.25}{0.072} = 2530 \text{ BTU/HR (a)}$$

267.25 F  $\xrightarrow{R_1 R_2} \xrightarrow{R_3 R_4} 80 \text{ F}$

$$R_1 + R_2 = 0.00156 \text{ HR F/BTU}$$

$$R_3 = \frac{\ln 5.755/1.755}{2\pi (0.04)(10)} = 0.061 \text{ HR F/BTU}$$

17.19 CONTINUED -

$$R_4 = \frac{1}{\pi(5.755/12)(3)(10)} = 0.0224 \quad (b)$$

$$\sum R = 0.485 \quad \dot{q} = \frac{\Delta T}{\sum R} = \frac{380 \text{ Btu/hr}}{0.485}$$

for BASE PIPE:  $\dot{m}_{stm} = \dot{q}/h_{eg}$ 

$$= \frac{2530 \text{ Btu/hr}}{933.7 \text{ Btu/hr-ft}} = 2.71 \frac{\text{lbm}}{\text{hr}} \quad (c)$$

17.20

$$\dot{q} \frac{\pi D^2 L}{4} = h \pi D L \Delta T$$

$$\frac{I^2 R}{\pi D^2/4 L} = h \Delta T$$

$$I^2 R = 10 \text{ kW}$$

$$h = 850 \text{ W/m}^2 \cdot \text{K}$$

$$L = 0.6 \text{ m} \quad \Delta T = 1280 \text{ K}$$

$$D = \frac{4(10000 \text{ W})}{(1280 \text{ K})(\pi)(0.6 \text{ m})(850 \text{ W/m}^2 \cdot \text{K})}$$

$$= 0.0195 \text{ m} = 1.95 \text{ cm} \quad (a)$$

$$14 \text{ GAGE} = 0.004 \text{ m}, \text{ DIAM} = 1.626 \times 10^{-3} \text{ m}$$

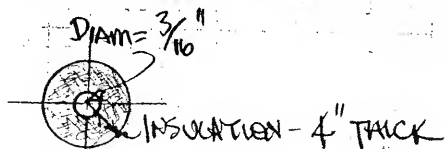
$$L = 7.2 \text{ m} \quad (b)$$

$$\text{for } h = 1150 \text{ W/m}^2 \cdot \text{K}$$

$$D = 1.44 \text{ cm}$$

$$L = 5.32 \text{ m} \quad (c)$$

17.21



$$\frac{\dot{q}}{L} = \frac{2\pi k}{\ln r_o/r_i} (T_i - T_o)$$

$$= \frac{2\pi(0.14)}{\ln \frac{8.094}{0.1875}} (120 - 70)$$

$$= 11.72 \frac{\text{Btu}}{\text{hr-ft}} = 3.43 \text{ W}$$

17.21 CONTINUED -

$$I^2 R = 3.43 \text{ W}$$

$$R = \rho \frac{L}{A} = 2.95 \times 10^{-4} \Omega$$

$$I^2 = \frac{3.43}{2.95 \times 10^{-4}} \quad I = 107.9 \text{ Amp}$$

17.22

$$\frac{\dot{q}}{L} = \frac{120 - 70}{\frac{\ln 8.094}{0.1875} + \frac{1}{\pi(131/16 \times 12)(4)}}$$

$$I = 100.1 \text{ Amp}$$

$$11.34 = 4 \left[ \frac{131}{16 \times 12} \right] \pi (T - 70)$$

$$T = 71.3 \text{ F}$$

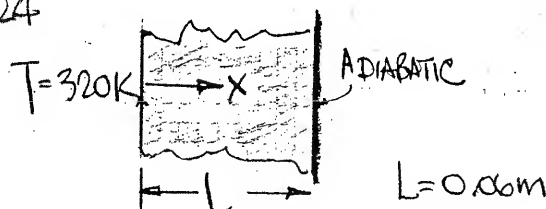
17.23

SAME AS PROB 17.21 EXCEPT  
WIRE IS ALUMINUM -

$$R = 4.85 \times 10^{-4} \Omega$$

$$I = 83.9 \text{ Amp}$$

17.24



$$\dot{q} = \dot{q}_0 \left[ 1 - \frac{x}{L} \right]$$

POISSON EQN APPLIES! (ENERGY BALANCE)

$$\frac{d^2 T}{dx^2} + \frac{\dot{q}}{k} = 0$$

$$\frac{d^2 T}{dx^2} = -\frac{\dot{q}_0}{k} \left[ 1 - \frac{x}{L} \right]$$

$$\frac{dT}{dx} = -\frac{\dot{q}_0}{k} \left[ x - \frac{x^2}{2L} \right] + C_1$$

17.24 CONTINUED

$$\frac{dT}{dx}(L) = 0 \quad C_1 = \frac{q_0 L}{k} \frac{1}{2}$$

$$\frac{dT}{dx} = \frac{q_0 L}{k} \left[ 1 - 2\frac{x}{L} + \frac{x^2}{L^2} \right]$$

$$T = \frac{q_0 L}{2k} \left[ x - \frac{x^2}{L} + \frac{x^3}{3L^2} \right] + C_2$$


$$T(0) = T_0 = C_2$$

$$T = T_0 + \frac{q_0 L}{2k} \left[ x - \frac{x^2}{L} + \frac{x^3}{3L^2} \right] \quad (a)$$

(b)  $T = T_{\max}$  where  $\frac{dT}{dx} = 0$  i.e. @  $x = L$

$$T(L) = T_0 + \frac{180 \left( \frac{0.06}{2} \right) \left( \frac{0.06}{3} \right) (1000)}{0.6} = 320 + 180 = 500 \text{ K} \quad (c)$$

17.25

$T(r_i) = T_i$   
 $T(r_o) = T_o$   
  
 $T_{\infty} = 298 \text{ K}$

IN WASTE MAT'L:  $q = \dot{q} (\pi r_i^2 L) \quad (1)$

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{\dot{q}}{k} = 0$$

$$r \frac{dT}{dr} + \frac{\dot{q} r^2}{2k} = C_1/r$$

$$\frac{dT}{dr}(0) = 0 \Rightarrow C_1 = 0$$

$$T + \frac{\dot{q} r^2}{4k} = C_2$$

$$T(r_i) = T_i \quad C_2 = T_i + \frac{\dot{q} r_i^2}{4k}$$

$$T = T_i + \frac{\dot{q}}{4k} (r_i^2 - r^2)$$

for ST. STEEL:

$$q = \frac{2\pi k L}{\ln(r_o/r_i)} (T_i - T_o) = 2\pi r_o L h (T_o - T_{\infty})$$

EQUATING WITH EQN (1)

$$\frac{2\pi k L}{\ln(r_o/r_i)} (T_i - T_o) = 2\pi r_o L h (T_o - T_{\infty}) = \dot{q} \pi r_i^2 L$$

17.25 CONTINUED -

PUTTING IN VALUES -  $T_o = 339.7 \text{ K} \quad (a)$

$$T_i = 339.7 + 303 = 642.7 \text{ K}$$

@ CENTER OF WASTE MAT'L:

$$T = 642.7 + \frac{\dot{q}}{4k} r_i^2$$

$$= 642.7 + 625 = 1268 \text{ K} \quad (b)$$

17.26

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{\dot{q}}{k} = 0$$

$$\frac{d}{dr} \left( r \frac{dT}{dr} \right) + \frac{\dot{q} r}{k} = 0$$

$$r \frac{dT}{dr} + \frac{\dot{q} r^2}{2k} = C_1/r$$

$$\frac{dT}{dr}(0) = 0 \Rightarrow C_1 = 0$$

$$T + \frac{\dot{q} r^2}{4k} = C_2$$

$$T(r) = T_R \quad C_2 = T_R + \frac{\dot{q} R^2}{4k}$$

$$T - T_R = \frac{\dot{q}}{4k} (R^2 - r^2) \quad (a)$$

$$T_{\max} = T @ r = 0$$

$$T_{\max} = T_R + \frac{\dot{q} R^2}{4k}$$

$$= T_R + \frac{(51.7 \times 10^6)(0.107)^2}{4(339)}$$

$$= T_R + 442$$

$$q = \dot{q} V = \dot{q} \frac{\pi D^2 L}{4}$$

$$= \frac{(51.7 \times 10^6)(\pi)(0.107)^2(0.106)}{4}$$

$$= h(\pi)(0.107)(0.106) \Delta T$$

$$\Delta T = 307 \text{ K}$$

$$T_{\text{surf}} = 332^\circ \text{C} \quad T_{\max} = 774^\circ \text{C}$$

(a) ↑

(b) ↑

17.27 ASSUME THIN-WALLED INNER VESSEL IS 77 K THROUGHOUT

$$R_i = \frac{1/r_i + 1/r_o}{4\pi k} = \frac{1/0.5 + 1/0.55}{4\pi(0.002)} = 7.234$$

$$R_o = \frac{1}{4\pi r_o^2 h} = \frac{1}{4\pi(0.55)^2(18)} = 0.046$$

$$q = \frac{\Delta T}{\sum R} = \dot{m} h_{fg}$$

$$\dot{m} = \frac{221/7.29}{2 \times 10^5} = 1.524 \times 10^{-4} \text{ kg/s}$$

17.28 FOR  $q = \frac{1}{2}$  OF VALUE IN 17.27

$$\sum R = 14.58 = R_{\text{conv}} + R_{\text{ins}}$$

$$R_{\text{ins}} = \frac{1/0.5 - 1/r_o}{4\pi(0.002)} \quad R_{\text{conv}} = \frac{1}{4\pi r_o^2(18)}$$

$$14.58 = \frac{1}{4\pi} \left[ 500 \left( \frac{1}{0.5} - \frac{1}{r_o} \right) + \frac{1}{18 r_o^2} \right]$$

$$r_o = 0.611 \text{ m} \quad \text{INS. THICKNESS} = 0.0555 \text{ m}$$

$$\text{ADDED THICKNESS} = 0.0055 \text{ m} \text{ OR } 5.5 \text{ mm}$$

17.29 PER FOOT OF BASE TUBE

$$q = h A \Delta T = (6 \frac{\text{Btu}}{\text{hr ft}^2 \text{F}}) (\frac{\pi}{12} \text{ ft})^2 (170 \text{ F}) = 267 \text{ Btu/hr}$$

FOR LONGITUDINAL FINS:

$$A_f = 12 \left( \frac{3}{4} \right) \left( \frac{1}{12} \right) (2) (1) = 1.5 \text{ ft}^2$$

(ENDS NEGLECTED)

$$A_o = \left( \frac{\pi}{12} \right) (1) - 12 \left( \frac{3}{32} \right) \left( \frac{1}{12} \right) = 0.168 \text{ ft}^2$$

$$L \left( \frac{h}{kt} \right)^{1/2} = \left( \frac{3/4}{12} \right) \left[ \frac{6}{24.8 \left( \frac{3}{64} \right) \left( \frac{1}{12} \right)} \right]^{1/2} = 0.539 \quad \eta_f \approx 0.92$$

17.29 CONTINUED

$$q = h (A_o + \eta_f A_f) \Delta T = 6 (0.168 + 0.92 \times 1.5) (170) = 1580 \frac{\text{Btu}}{\text{hr}}$$

$$\text{INCREASE} = 1580 - 267 = 1310 \frac{\text{Btu}}{\text{hr}}$$

$$\% \text{ INCR} = 491 \%$$

FOR CIRCULAR FINS:

$$\text{PER FIN: } A_f = 2 \frac{\pi}{4} \left[ \frac{2.5^2 - 1^2}{144} \right] = 0.0573 \text{ ft}^2$$

$$\text{PER FOOT } n = 1.5 / 0.0573 = 26 \text{ FINS}$$

$$A_o = \frac{\pi}{12} - \left( \frac{\pi}{12} \times \frac{3}{32} \times \frac{1}{12} \times 26 \right) = 0.209 \text{ ft}^2$$

$$(r_o - r_i) \left[ \frac{h}{kt} \right]^{1/2} = \frac{3/4}{12} \left[ \frac{6}{24.8 \left( \frac{3}{64} \right) \left( \frac{1}{12} \right)} \right]^{1/2} = 0.539 \quad \eta_f \approx 0.88$$

$$q = 6 [0.209 + (0.88)(26)(0.0573)] (170) = 1560 \frac{\text{Btu}}{\text{hr}}$$

$$\text{INCREASE} = 1560 - 267 = 1290 \frac{\text{Btu}}{\text{hr}}$$

$$\% \text{ INCR} = \frac{1290}{267} = 484 \%$$

17.30  $h = 60 \text{ Btu/hr ft}^2 \text{F}$

$$\text{LONGITUDINAL CASE } L \left( \frac{h}{kt} \right)^{1/2} = 1.70$$

$$\eta_f \approx 0.56$$

$$q = 10820 \frac{\text{Btu}}{\text{hr}}$$

$$\text{CIRCULAR CASE } (r_o - r_i) \left( \frac{h}{kt} \right)^{1/2} = 1.70$$

$$\eta_f \approx 0.52$$

$$q = 10180 \frac{\text{Btu}}{\text{hr}}$$

$$\% \text{ FINS } q = 2670 \frac{\text{Btu}}{\text{hr}}$$

INCREASE:

$$\text{LONG: } \frac{8150 \frac{\text{Btu}}{\text{hr}}}{2670 \frac{\text{Btu}}{\text{hr}}} = 305 \%$$

$$\text{CIRC: } \frac{7510 \text{ "}}{2670 \text{ "}} = 281 \%$$

17.31

SOLUTION for  $\theta = \frac{T - T_\infty}{T_0 - T_\infty}$   
 IS IN FORM  $\theta = C_1 e^{mx} + C_2 e^{-mx}$   
 $m = \left[ \frac{(17 \text{ W/m}^2 \cdot \text{K})(\pi)(0.03 \text{ m})^4}{k \pi (0.03 \text{ m})^2} \right]^{1/2} = \frac{47.6}{k^{1/2}}$

LONG FIN APPROXIMATION:  $\theta = C_2 e^{-mx}$ 

$$\theta_1 = 99 = C_2 e^{-mx}$$

$$\theta_2 = 65 = C_2 e^{-m(x_1 + 0.076)}$$

$$\frac{\theta_2}{\theta_1} = \frac{65}{99} = e^{-m(x_1 + 0.076 - x_1)}$$

$$m = 5.536 = \frac{47.6}{k^{1/2}}$$

$$k \approx 74 \text{ W/m}^2 \cdot \text{K}$$

17.32 CONTINUED

$$q = \frac{\Delta T_{OA}}{1/25 + 1/58} = 1747 \Delta T_{OA}$$

$$\% \text{ GAIN} = 549\%$$

TO WATER SIDE:

$$q = \frac{\Delta T_{OA}}{1/483 + 1/3} = 298 \Delta T_{OA}$$

$$\% \text{ GAIN} = 11.2\%$$

BOTH SIDES:

$$q = \frac{\Delta T_{OA}}{1/483 + 1/58} = 51.78 \Delta T_{OA}$$

$$\% \text{ GAIN} = 1832\%$$

17.32

$$\eta_f = \text{fn of } L \left[ \frac{h}{k t} \right]^{1/2}$$

AIRSIDE:  $= \frac{0.75}{12} \left[ \frac{2(3)}{(0.05/12)(229)} \right]^{1/2}$   
 $= 0.456 \quad \eta_f \approx 0.98$

WATERSIDE:  $= \frac{0.75}{12} \left[ \frac{2(25)}{(0.5/12)(229)} \right]^{1/2}$   
 $= 0.143 \quad \eta_f \approx 0.98$

FOR  $1 \text{ FT}^2 \quad A_0 = 1 - 150 \left( 1 \times \frac{0.05}{12} \right) = 0.375 \text{ FT}^2$

$$A_f = 150(2) \left( \frac{0.75}{12} \right) (1) + 0.625 = 19.35 \text{ FT}^2$$

W/O FINS:  $q = 25 \Delta T_w = 3 \Delta T_A = \frac{\Delta T_{OA}}{1/25 + 1/3}$

WITH FINS:

WATERSIDE:  $q = 25 \Delta T_w (0.375 + 18.96) = 483 \Delta T_w$

AIRSIDE:  $q = 3 \Delta T_A (0.375 + 18.96) = 58.0 \Delta T_A$

FINS ADDED TO AIRSIDE ONLY;

17.33

ONE-DIM. CONDUCTION WITH  
INTERNAL HT GENERATION

$$\frac{d^2 T}{dx^2} = -\frac{\dot{q}}{k}$$

$$\frac{dT}{dx} = -\frac{\dot{q}}{k} x + C_1$$

$$T = -\frac{\dot{q}}{2k} x^2 + C_1 x + C_2$$

$$T(0) = T_0 = C_2$$

$$T(L) = T_L = -\frac{\dot{q}}{2k} \frac{L^2}{2} + C_1 L + T_0$$

$$C_1 = \frac{T_L - T_0}{L} + \frac{\dot{q} L}{2k}$$

$$T = \frac{\dot{q}}{2k} \left[ \frac{Lx}{2} - \frac{x^2}{2} \right] + \frac{T_L - T_0}{L} x + T_0$$

OR  $T - T_0 = \frac{\dot{q} L^2}{2k} \left[ \frac{x}{L} - \left( \frac{x}{L} \right)^2 \right] + (T_L - T_0) \left( \frac{x}{L} \right)$

AT  $\frac{L}{2}$ :  $T - T_0 = \frac{\dot{q} L^2}{2k} \left( \frac{1}{4} \right) + \frac{T_L - T_0}{2}$

$$\dot{q} = \frac{I^2 R}{V} = \frac{I^2 \rho L}{A^2 L}$$

$$= \frac{(10)^2 (2 \times 10^{-5})}{\pi/4 (0.01)^2} = 2547 \text{ W/m}^3$$

17.33 CONTINUED

MID-PT. TEMP:

$$T_{m.p.} = \frac{15.47 \text{ W/m}^3 (0.04 \text{ m})^2}{8 (2 \text{ W/m}\cdot\text{K})} + 50$$

$$\approx 50.00^\circ\text{C}$$

$$q = -kA \frac{dT}{dx} = -kA \left[ -\frac{q}{k}x + C_1 \right]$$

$$= -kA \left[ -\frac{q}{k}x + \frac{T_L - T_0}{L} + \frac{q}{k} \frac{L}{2} \right]$$

$$@x=0 \quad q = -kA \left[ \frac{T_L - T_0}{L} + \frac{qL}{2k} \right]$$

$$= -0.393 \text{ W}$$

$$ATx=L \quad q = -kA \left[ -\frac{qL}{2k} + \frac{T_L - T_0}{L} \right]$$

$$= +0.393 \text{ W}$$

$$17.34 \quad m^2 = \frac{hP}{kA} = \frac{(740)(\pi)(0.019)(4)}{(54)(\pi)(0.019)^2}$$

$$= 2885 \text{ m}^{-2}$$

$$\theta = \theta_0 \frac{\cosh m x}{\cosh mL/2}$$

$$\frac{d\theta}{dx} = m\theta_0 \frac{\sinh m x}{\cosh mL/2} \Rightarrow \sinh m x = 0 @x=0$$

$$\theta_{max} = \frac{\theta_0}{\cosh mL/2} = \frac{145 \text{ K}}{\cosh 12.08}$$

$$T \approx 625 \text{ K}$$

17.35 ENERGY BALANCE:

$$\frac{d^2\theta}{dx^2} - m^2\theta = -\frac{W}{kA} \quad \left\{ W \text{ in } \frac{\text{Btu}}{\text{ft}} \right\}$$

$$\theta = C_1 e^{mx} + C_2 e^{-mx} + \frac{W}{hP}$$

$$\theta(0) = 0 \quad C_1 = \frac{-W/hP(1 - e^{-mL})}{e^{mL} + e^{-mL}}$$

$$\theta(L) = 0 \quad C_2 = \frac{-W/hP(e^{mL} - 1)}{e^{mL} - e^{-mL}}$$

17.35 CONTINUED.

PUTTING IN VALUES:

$$m = 2.28 \quad C_1 = -0.0017 \text{ W/hP}$$

$$C_2 = -0.999$$

$$\theta = \theta_{max} @ 1.5 \text{ ft}$$

$$W_{max} = I_{max}^2 R$$

$$R = \frac{8L}{A} = \frac{(172 \times 10^{-6})(3)}{\frac{\pi}{4} (1/48)^2 (30.48)}$$

$$= 4.97 \times 10^{-4} \Omega/\text{ft}$$

$$\theta_{max} = 90 \left[ -0.0017 e^{2.28(1.5)} - 0.999 e^{-0.228(1.5)} + 1 \right] \frac{\text{W}}{\text{hP}}$$

$$\frac{W}{hP} = 96.3 \quad W = 96.3 (6)(\pi)(0.25/12)$$

$$= 37.8 \text{ Btu/hr ft}$$

$$= 11.08 \text{ W/ft}$$

$$I_{max}^2 = \frac{11.08}{4.97 \times 10^{-4}} = 2.23 \times 10^4 \text{ A}^2$$

$$I_{max} = 150 \text{ A}$$

17.36

$$m^2 = \frac{hP}{kA} = \frac{45(2)}{42(0.0159)} = 1348 \text{ m}^{-2}$$

$$m = 11.61 \text{ m}^{-1}$$

$$q = kAm\theta_0 \frac{\sinh mL + \frac{h}{mk} \cosh mL}{\cosh mL + \frac{h}{mk} \sinh mL}$$

$$kAm\theta_0 = (42)(0.0159)(11.61)(300)$$

$$= 2330$$

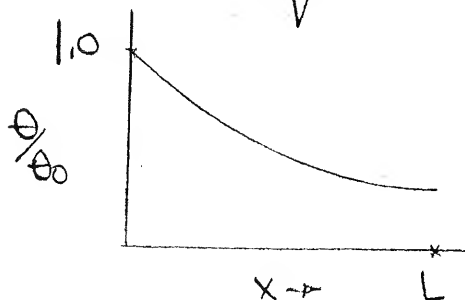
$$\sinh mL = 2.01 \quad \cosh mL = 1.75$$

$$\frac{h}{mk} = \frac{45}{11.61(42)} = 0.0923$$

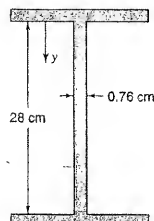
17.36 CONTINUED-

SUBSTITUTING!

$$\underline{\underline{q = 2.25 \text{ kW}}}$$



17.37



$$\frac{\theta}{\theta_0} = \left[ \frac{q}{\theta_0} - e^{-mL} \right] \left[ \frac{e^{mx} - e^{-mx}}{e^{mL} - e^{-mL}} \right] + e^{-mx}$$

$$\theta_0 = 400 \quad m = 19.6 \text{ m}^{-1}$$

$$mL = 5.69 \quad e^{mL} = 242 \quad e^{-mL} = 0.0041$$

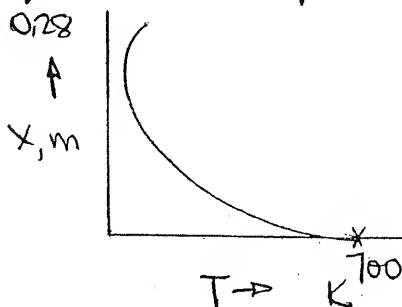
SUBSTITUTING!

$$\theta = 0.262 e^{19.6x} + 399.7 e^{-19.6x}$$

$$q = -kA \frac{dT}{dx} \quad \frac{dT}{dx}(0) = -1829$$

$$\frac{dT}{dx}(L) = 1305$$

$$\underline{\underline{q_0 = -2.32 \text{ kW}}} \quad \underline{\underline{q_L = 387 \text{ W}}}$$



17.38

FOR ALUMINUM I-BEAM!

SAME PROCEDURE AS PREVIOUS PROB.

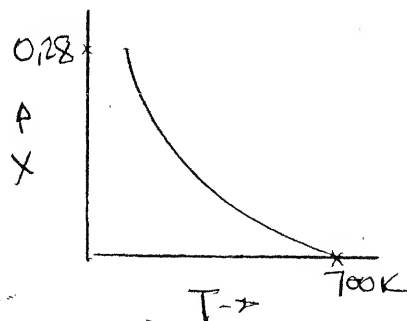
$$m = 8.08$$

$$e^{mL} = 9.61$$

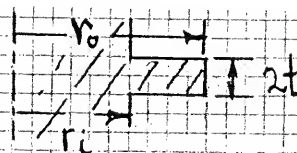
$$e^{-mL} = 0.104$$

$$\theta = 3 e^{8.08x} + 688 e^{-8.08x}$$

$$\underline{\underline{q_0 = 551 \text{ kW}}} \quad \underline{\underline{q_L = 170 \text{ W}}}$$



17.39



$$r_i = \frac{0.3}{2} = 0.15 \text{ m} \quad r_o = 0.15 + 0.02 = 0.17 \text{ m}$$

$$t = 0.003/2 = 0.0015 \text{ m}$$

$$\begin{aligned} A_{\text{PER}} &= 2\pi(r_o^2 - r_i^2) + A_{\text{END}} \\ &= 2\pi(0.17^2 - 0.15^2) + 2\pi(0.17)(0.003) \\ &= 0.0434 \text{ m}^2 \end{aligned}$$

$$(r_o - r_i) \sqrt{h/k} t = 0.02 \left[ \frac{12}{(46.4)(0.0015)} \right]^{1/2} = 0.163$$

$$r_o/r_i = 1.13$$

$$\text{FROM FIG 17.11} \quad \eta_f \approx 0.96$$

$$\begin{aligned} q &= A_f \eta_f h (T_0 - T_\infty) \\ &= (0.0434)(0.96)(12)(270) \\ &= \underline{\underline{135 \text{ W PER FIN}}} \end{aligned}$$

17.39 CONTINUED -

FOR A 30% 3 kW ENGINE

$$Q_{IN} = \frac{3 \text{ kW}}{0.3} = 10 \text{ kW}$$

$$Q_{OUT} = Q_{IN} - W = 7 \text{ kW}$$

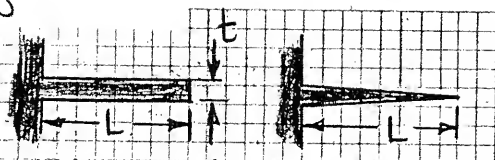
$$\text{AMOUNT Tx FROM FINS} = 0.5 (7) = 3.5 \text{ kW}$$

NO. OF FINS REQ'D:

$$n = \frac{3500 \text{ W}}{135 \text{ W/FIN}} = 25.92$$

26 FINS REQUIRED

17.40



a)  $L = 20 \text{ mm}$  b)

$t = 6 \text{ mm}$

FOR BOTH CASES:  $T_b = 120^\circ\text{C}$

$T_\infty = 20^\circ\text{C}$

$h = 60 \text{ W/m}^2\cdot\text{K}$   $k_{ss} = 15.3 \text{ W/m}\cdot\text{K}$

FOR CASE a) STRAIGHT FIN

$$q = \eta_F h A_F \theta_b$$

USING TEXT - FIG 17.11

$$\left(L + \frac{t}{2}\right)^{3/2} \left[ \frac{h}{k t} \left(L + \frac{t}{2}\right) \right]^{1/2} = (0.02 + 0.003)^{3/2} \left[ \frac{60}{(15.3)(0.006)(0.023)} \right]^{1/2}$$

$$= 0.588 \quad \eta_F \approx 0.80$$

FOR m OF WIDTH! (NEAREST ENDS)

$$q_F = 0.80 (60)(100)(2)(0.020)$$

$$= \underline{192 \text{ W/m}}$$

17.40 CONTINUED -

FOR CASE b) - TRIANGULAR

$$L^{3/2} \left[ \frac{h}{k t L} \right]^{1/2}$$

$$= 0.02^{3/2} \left[ \frac{60}{(15.3)(0.003)(0.02)} \right]^{1/2}$$

$$= 0.723 \quad \eta_F \approx 0.81$$

$$q = \eta_F A_F h \theta_b$$

$$= (0.81)(2)(0.02)(60)(100)$$

$$= \underline{194.4 \text{ W/m}}$$

17.41

w/out FINS:

$$q_0 = h A \pi \left(\frac{2}{12}\right) = 7.12 \text{ Btu/hr}$$

FOR LONGITUDINAL FINS:

$$L \left( \frac{h}{k t} \right)^{1/2} = \frac{1}{12} \left[ \frac{8}{10 \left( \frac{1/32}{12} \right)} \right] = 1.46$$

$$\eta_F \approx 0.6$$

$$A_0 = \pi D_0 - 16(2t) = 0.440 \text{ ft}^2$$

$$A_F = 16(2)(1/12) + 16(1/16)(1/12) \approx 2.67 \text{ ft}^2$$

$$q = h A \left[ A_0 + \eta_F A_F \right]$$

$$= 8(170) \left[ 0.440 + 0.6(2.67) \right] \approx 2780 \text{ Btu/hr}$$

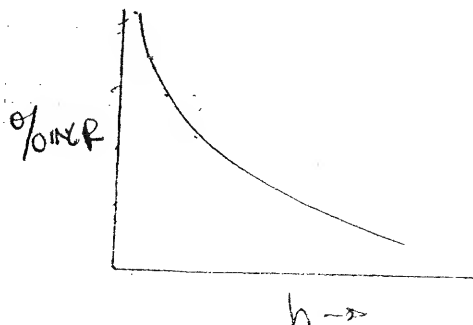
$$\text{INCREASE} = \frac{2068 \text{ Btu/hr}}{7.12 \text{ Btu/hr}} \approx 290\% \text{ a)}$$

FOR VARYING VALUES OF  $h$ :

$h$	$L \sqrt{h/k t}$	$\eta_F$	% INCR
2	0.731	0.84	412
5	1.156	0.71	346
8	1.462	0.60	290
15	2.00	0.48	229
50	3.65	0.27	122
100	5.17	0.19	81

142

17.41 CONTINUED

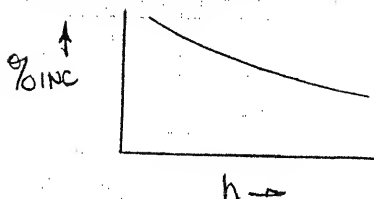


i.e., FINS ARE MOST EFFECTIVE  
WHEN  $h$  IS SMALL

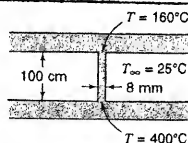
17.42 SAME AS 17.41 EXCEPT MAT'L  
IS ALUMINUM

W/O FINS  $q = 712 \text{ Btu/hr}$   
 $L \left[ \frac{h}{kt} \right]^{1/2} = 0.121 \quad \eta_F = 0.99$   
 $q = 4300 \text{ Btu/hr} \quad \% \text{ INC} = 503\%$

$h$	2	5	15	50	100
% INC	488	471	448	370	303



17.43



FOR KNOWN END TEMPS!

$$\frac{\theta}{\theta_0} = \frac{T - T_{\infty}}{T_b - T_{\infty}} = \left[ \frac{\theta_L}{\theta_0} - e^{-ML} \right] \left[ \frac{e^{MX} - e^{-MX}}{e^{ML} - e^{-ML}} \right] + e^{-MX}$$

$$\theta_0 = 160 - 25 = 135 \quad \frac{\theta_L}{\theta_0} = 2.78$$

$$\theta_L = 400 - 25 = 375$$

$$m = \left[ \frac{hP}{kA} \right]^{1/2} = \left[ \frac{300(2)}{(0.008)(229)} \right]^{1/2} = 18.1 \text{ m}^{-1}$$

$$e^{ML} = 6.11 \quad e^{-ML} = 0.164$$

17.43 CONTINUED

SUBSTITUTING!

$$\frac{\theta}{\theta_0} = 0.440 e^{18.1x} + 0.560 e^{-18.1x}$$

$$\frac{d\theta}{dx} = \theta_0 \left[ 7.964 e^{18.1x} - 10.14 e^{-18.1x} \right]$$

$$@x=0 \quad \frac{d\theta}{dx} = -2.176 \theta_0$$

$$@x=L \quad \frac{d\theta}{dx} = 47.0 \theta_0$$

$$q = -k \left[ \frac{d\theta}{dx} \right] \quad q_L(0) = -294 \text{ W/m}$$

$$q_L(L) = 6345 \text{ kW/m}$$

17.44 USING TABLE 17.1

$$S = \frac{2\pi}{\cosh^{-1} \left( \frac{1 + \beta^2 - \epsilon^2}{2\beta} \right)}$$

$$\beta = 0.5 \quad \epsilon = 1/6$$

$$S = \frac{2\pi}{\cosh^{-1} \left( \frac{1 + 1/4 - 1/36}{1} \right)} = \frac{2\pi}{\cosh^{-1} 11/9}$$

$$= 9.6$$

$$q_L = kSAT = (0.023)(9.6)(300) = 66.3 \text{ Btu/hr-ft}$$

17.45 TABLE 17.1

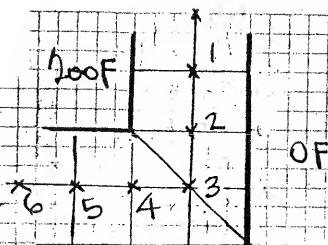
$$S = \frac{2\pi}{\cosh^{-1}(\beta/r)} = \frac{2\pi}{\cosh(2.5)} = 1.311$$

$$\frac{q}{L} = kSAT$$

$$= (0.341)(1.311)(60)$$

$$= 26.83 \text{ W/m}$$

17.46



$$200 + 0 + 2T_2 - 4T_1 = 0$$

$$200 + 0 + T_1 - T_3 - 4T_2 = 0$$

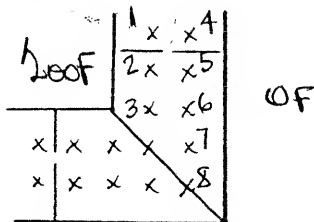
$$0 + 0 + 2T_2 - 4T_3 = 0$$

$$T_1 = 91.7F \quad T_2 = 83.3F \quad T_4 = 41.7F$$

$$\frac{q}{L} = 8k \left[ \frac{200 - 91.7}{2} + \frac{200 - 83.3}{2} \right]$$

$$= \underline{\underline{205 \text{ Btu/hr-ft}}}$$

17.47



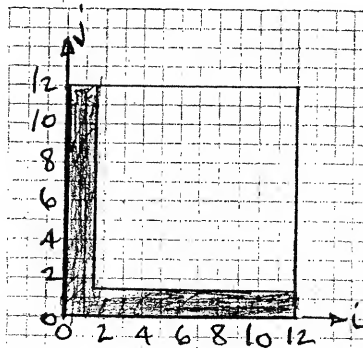
By Iteration:

Node	T, F
1	123
2	112
3	74
4	58
5	51.5
6	36
7	18

$$\frac{q}{L} = 8k \left[ (200 - 123) + (200 - 112) \right]$$

$$= \underline{\underline{198 \text{ Btu/hr-ft}}}$$

17.48



17.48 CONTINUED-

NUMERICAL SOLUTION USING A  
12 x 12 MESH YIELDS THE  
FOLLOWING:

$$\frac{q}{L} \cong \underline{\underline{1400 \text{ Btu/hr-ft}^2}} \text{ per ft.}$$

$$T_{\text{MIN}} \cong \underline{\underline{91.8 F}} \text{ AT } i, j = 12, 12$$

17.49

$$S = \frac{2\pi}{\ln(r_2/r_1) - 0.271}$$

$$= \frac{2\pi}{\ln(1.2/0.5) - 0.271}$$

$$= 8.17$$

$$q = kSL\Delta T$$

$$= (0.037)(8.17)(300)(50)$$

$$= 2180 \text{ Btu/hr}$$

$$\text{STEAM CONDENSED} = \frac{q}{h_{fg}}$$

$$= \frac{2180}{826} = \underline{\underline{2.65 \text{ Lbm/hr}}}$$

17.50

$$S = \frac{2\pi}{\cosh^{-1}(8/r)} = \frac{2\pi}{\cosh^{-1}(1.2/0.324)}$$

$$= 3.17$$

$$q = kSL\Delta T$$

$$= (0.166 \text{ W/m}\cdot\text{K})(3.17)(90\text{K})(1.5\text{m})$$

$$= \underline{\underline{273 \text{ W}}}$$

# CHAPTER 18

$$18.1 \quad Bi = \frac{hV/s}{k} = \frac{3}{10} \frac{3/488}{0.5} = 0.0369$$

∴ CAN USE LUMPED PARAMETERS

By 1<sup>st</sup> LAW:  $\dot{Q} - hA\theta = \rho c_p V \frac{d\theta}{dt}$

WHERE  $\theta = T - T_\infty$

$$\frac{d\theta}{dt} = \frac{\dot{Q}}{\rho c_p V} - \frac{hA\theta}{\rho c_p V} = a - b\theta$$

Solve:  $t = \frac{1}{b} \ln \frac{a}{a - b\theta} = \frac{1}{b} \ln \frac{1}{1 - \frac{b}{a}\theta}$

$$a = \frac{\dot{Q}}{\rho c_p V} = \frac{(500)(3413)}{3(0.11)} = 5170 \text{ F/hr}$$

$$b = \frac{hA}{\rho c_p V} = \frac{3(0.05)}{(0.11)(3)} = 4.54 \text{ hr}^{-1}$$

$$\Rightarrow t = \frac{1}{4.54} \ln \frac{1}{1 - 4.54/5170(160)} = 0.0333 \text{ hours} = 2.0 \text{ min.}$$

18.2

$$V = \frac{\pi D^2}{4} L = \frac{\pi}{4} (0.0001)^2 (0.005) = 3.93 \times 10^{-11} \text{ m}^3$$

$$A = \frac{2\pi D^2}{4} + \pi D L$$

$$= \frac{\pi}{2} (0.0001)^2 + \pi (0.0001)(0.005) = 1.587 \times 10^{-6} \text{ m}^2$$

$$\frac{hV}{kA} = \frac{10 (3.93 \times 10^{-11})}{20 (1.587 \times 10^{-6})} \approx 1.24 \times 10^{-5}$$

Clearly A LUMPED PARAMETER CASE  
ENERGY BALANCE:

$$\dot{Q} - hA(T - T_\infty) = \rho c_p V \frac{dT}{dt}$$

LET  $\theta = T - T_\infty$

$$\frac{d\theta}{dt} = \frac{\dot{Q}}{\rho c_p V} - \frac{hA}{\rho c_p V} \theta = A - B\theta$$

18.2 CONTINUED -

$$\int_0^\theta \frac{d\theta}{A - B\theta} = \int_0^t dt$$

$$-\frac{1}{B} \ln \frac{A - B\theta}{A} = t$$

$$\Rightarrow t = \frac{1}{B} \ln \frac{A}{A - B\theta}$$

$$A = \frac{\dot{Q}}{\rho c_p V} = \frac{9(0.2)}{k/\alpha(V)} = \frac{1.8}{(20/5 \times 10^{-5})(393 \times 10^{11})} = 1.145 \times 10^5$$

$$B = \frac{hA}{\rho c_p V} = \frac{10(1.587 \times 10^{-6})}{(20/5 \times 10^{-5})(393 \times 10^{11})} = 1.01$$

$$t = \frac{1}{1.01} \ln \frac{1.145 \times 10^5}{1.145 \times 10^5 - (1.01)(870)} = 7.63 \times 10^{-3} \text{ s} = 7.63 \text{ ms}$$

18.3 ALUMINUM WIRE:

$$D = 0.794 \text{ mm} \quad R = 0.0572 \Omega/\text{m}$$

$$k = 229 \text{ W/m}\cdot\text{K} \quad \rho = 2701 \text{ kg/m}^3$$

$$c_p = 938 \text{ J/kg}\cdot\text{K} \quad \alpha = 9.16 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Bi = \frac{hV}{kS} = \frac{h}{k} \frac{\pi R^2 L}{4 \pi R L} = \frac{(500)(0.794 \times 10^{-3})}{(229)(4)} = 4.33 \times 10^{-4}$$

- LUMPED PARAMETER SOLN IS OK.

STEADY STATE CASE - PER M

$$I^2 R = hA \Delta T$$

$$\Delta T = \frac{(100)^2 (0.0572) \text{ W}}{(550 \text{ W/m}^2\cdot\text{K}) \pi (0.794 \times 10^{-3})^2 \text{ m}^2} = 416.9 \text{ K}$$

$$T_{\max} = 25 + 416.9 = 441.9 \text{ C}$$

18.3 CONTINUED -

TRANSIENT CASE -

$$\rho C_p V \frac{dT}{dt} = I^2 R = hS(T - T_\infty)$$

$$\frac{d\theta}{dt} = \frac{I^2 R}{\rho C_p V} - \frac{hS}{\rho C_p V} \theta = A - B\theta$$

$$A = \frac{I^2 R}{\rho C_p V} = \frac{(100)^2 (0.0572)}{(2701)(938) \frac{\pi}{4} (0.714 \times 10^{-3})^2} = 456 \text{ K/s}$$

$$B = \frac{hS}{\rho C_p V} = \frac{(550)(\pi)(0.714 \times 10^{-3})}{(2701)(938) \frac{\pi}{4} (0.714 \times 10^{-3})^2} = 1.094 \text{ s}^{-1}$$

$$\frac{d\theta}{dt} = A - B\theta$$

$$\int_0^\theta \frac{d\theta}{\theta - B\theta} = \int_0^t dt$$

$$-\frac{1}{B} \ln \frac{A - B\theta}{A} = t$$

$$t = \frac{1}{B} \ln \frac{A}{A - B\theta}$$

$$= \frac{1}{1.094} \ln \frac{456}{456 - 1.094(419)} = 4.06 \text{ s}$$

$$18.4 \quad Bi = \frac{hV}{kS} = \frac{6}{0.151} \left[ \frac{(0.6)(0.3)(0.45)}{0.6(0.45)(2) + 0.6(0.3)(2) + 0.3(0.45)(1)} \right] = 2.75$$

A DISTRIBUTED PARAMETER PROB.

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{320 - 297}{420 - 297} = 0.187 = Y_A Y_B Y_C$$

$$M_x = \frac{0.151}{6(0.15)} = 0.168 \quad X_x = \frac{\alpha t}{0.15^2} = 2.75 \times 10^{-4}$$

$$M_y = 0.119 \quad X_y = 1.22 \times 10^{-4} t$$

$$M_z = 0.042 \quad X_z = 1.72 \times 10^{-4} t$$

18.4 CONTINUED -

By Trial & Error

$$t \approx 80 \text{ hours}$$

$$18.5 \quad Bi = \frac{hV}{kS} = \frac{16}{23} \left( \frac{6/12}{6} \right) = 0.058$$

Lumped Parameter!

$$Fo = \frac{\alpha t}{(V/S)^2} = \frac{23}{460(0.10) \text{ Hr}} \frac{t}{(6/12)^2} = 72 t$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{600}{2000} e^{-(0.058)(72t)}$$

$$t = 0.288 \text{ Hr} = 17.3 \text{ MIN}$$

18.6 LUMPED PARAMETER SOLN APPLIES

IF  $Bi = \frac{hV}{kS} < 0.1$  OR  $h < 0.1 \text{ kS/V}$

$$\frac{kS}{V} = \frac{k \pi D^2}{\pi D^3/6} = 0.47(6) = 2.82$$

So  $h$  MUST BE  $< 0.1(2.82) = 0.282 \text{ W/m}^2\text{K}$

BUT  $h = 15 \Rightarrow$  USE DISTRIBUTED PARAM. SOLN.

$$\frac{dt}{r_0^2} = \frac{(0.47)t}{(940)(3800)(0.005)^2} = 5.26 \times 10^{-5} t$$

$$\frac{T_s - T_\infty}{T_0 - T_\infty} = 0.5 \quad \frac{k}{hr_0} = \frac{0.47}{15(0.05)} = 0.627$$

$$X \approx 0.17 = 5.26 \times 10^{-5} t$$

$$t = 3230 \text{ s} = 53.9 \text{ MIN}$$

$$18.7 \quad Bi \approx 0.005$$

USE LUMPED PARAMETER SOLN.

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-h/S \frac{V}{k} t}$$

$$t = \frac{\rho C_p V}{h S} \ln \frac{T_0 - T_\infty}{T - T_\infty}$$

$$= \frac{51 \text{ V}}{4 \text{ A} \times 12} (0.62) = 0.658 \frac{\text{V}}{\text{A}}$$

18.7 CONTINUED -

$$\frac{V}{A} = \frac{\pi D^2 L / 4}{\pi D L + 2 \pi D^2 / 4} = \frac{D}{4 + 2D/L} = \frac{3}{4 + 6/L}$$

CASE	L (IN)	V/A	t (MIN)
a	3	0.5	19.7
b	6	0.6	23.6
c	12	0.67	26.3
d	24	0.706	27.9
e	60	0.732	28.9

$$18.8 \quad \frac{T_c - T_\infty}{T_o - T_\infty} = \frac{500 - 1000}{70 - 1000} = 0.538$$

$$\frac{V}{A} = \frac{D}{4 + 2D/L} = \frac{1}{17}$$

$$Bi = \frac{hV/s}{k} = \frac{4/17}{k}$$

a) Cu -  $Bi \leq 0.1$  - LUMPED

$$t = \frac{8 \rho V}{hA} \ln \frac{1}{0.538} = \underline{27.9 \text{ MIN.}}$$

b) Al  $Bi \leq 0.1$ 

$$t = 0.345 \text{ hr} = \underline{20.7 \text{ MIN}}$$

c) Zn  $Bi < 0.1$ 

$$t = 0.381 \text{ hr} = \underline{22.9 \text{ MIN}}$$

d) STEEL  $Bi < 0.1$ 

$$t = 0.502 \text{ hr} = \underline{30.2 \text{ MIN}}$$

18.9 WATER IS WELL-MIXED  
 $\therefore$  LUMPED  $\sim T = T(t)$  ONLY

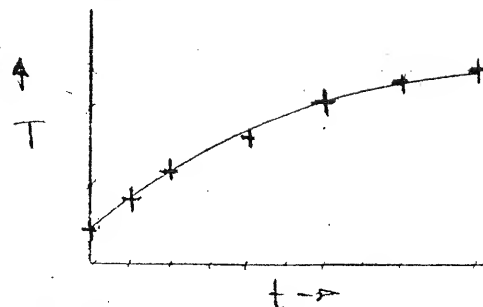
$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left(-\frac{hAt}{8 \rho V}\right)$$

$$= \exp\left[-\frac{40(\pi)(15)(2)t}{62.4(1)(\pi)(1.5^2/4)(2)}\right]$$

$$T = 300 - 260 e^{-1.71t}$$

t, hr	0	0.1	0.2	0.4	0.6	0.8	1.0
T, F	40	81	115	169	210	237	253

18.9 CONTINUED



$$18.10 \quad Bi = \frac{hV/s}{k} = \frac{h}{k} \frac{\pi D^2 L / 4}{\pi D L + 2 \pi D^2 / 4}$$

$$= \frac{hD}{4k(L + D/2)} = \frac{85(0.6)(0.6)}{(229)(4)(0.9)} = 0.0371$$

 $\sim$  LUMPED PARAMETER CASETEMP MAY BE CONSIDERED  
UNIFORM AT ANY TIME

$$Fo = \frac{\alpha t}{(V/s)^2} = \frac{(9.16 \times 10^{-5})(3600)}{(0.10)^2}$$

$$= 32.98$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{-Bi Fo}{e} = \frac{-(0.0371)(32.98)}{e}$$

$$= 0.294$$

$$T = 345 + 0.294(130)$$

$$= \underline{383.2 \text{ K}}$$

18.11

$$Bi = \frac{hV/s}{k} = \frac{15 \left[ \frac{\pi D^2}{4} L / \pi D L \right]}{12.4}$$

$$= 0.151$$

MUST USE DISTRIBUTED PARAMETER  
SOLN. FIG. F.8

$$\frac{T_b - T}{T_b - T_o} = \frac{2300 - 1500}{2300 - 200} = 0.381$$

$$\frac{\alpha t}{x^2} = \frac{0.15}{(0.25)^2} t = 2.4 t$$

$$\frac{\alpha t}{x^2} = \frac{0.15}{(0.25)^2} t = 2.4 t$$

18.11 CONTINUED -

$$\eta = \frac{x}{x_1} = 0 \quad m = \frac{k}{hx_1} = 3.31$$

$$X \approx 1.7 \Rightarrow t = \frac{1.7}{2.4} = 0.708 \text{ hr}$$

$$\text{Velocity} = \frac{70}{0.708} = 28.2 \frac{\text{ft}}{\text{hr}} = 0.47 \frac{\text{ft}}{\text{min}}$$

$$18.12 \quad \frac{T_c - T_\infty}{T_0 - T_\infty} = \frac{410 - 435}{295 - 435} = 0.179$$

$$\frac{\alpha t}{x_1^2} = \frac{(6.19 \times 10^{-8}) t}{(0.015)^2} = 2.75 \times 10^{-4} t$$

$$m \approx 0 \text{ from Chart } X \approx 0.8$$

$$t = \frac{0.8}{2.75 \times 10^{-4}} = 29125 = 48.5 \text{ min}$$

$$18.13 \quad B_i = \frac{hV}{kS} = \frac{40 \pi (0.3)^3 / 6}{19.3 \pi (0.2)} = 0.00575$$

LUMPED PARAMETER!

$$Fo = \frac{\alpha t}{(V/S)^2} = \frac{0.8 (15/3600)}{[0.2/2(6)]^2} = 432$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-Bi Fo} = 0.0834$$

$$T = 115.9 \text{ F}$$

$$18.14 \quad X_1 = 0.15 \text{ m} \quad X = 0.05 \text{ m}$$

$$x/x_1 = 1/3$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{100 - 380}{25 - 380} = 0.789$$

$$m = k/hx_1 = \frac{0.20}{140(0.15)} = 0.00952$$

$$\alpha t/x_1^2 = \frac{1.1 \times 10^{-7} t}{(0.15)^2} = 4.89 \times 10^{-6} t$$

18.14 CONTINUED -

{ USING CHART FOR CYL }

$$@ \frac{x}{x_1} = 0.2 \quad \frac{\alpha t}{x_1^2} = 0.10$$

$$@ \quad 0.4 \quad \quad \quad = 0.07$$

$$\text{INTERPOLATING} - @ 0.33 \quad \frac{\alpha t}{x_1^2} \approx 0.08$$

$$t \approx 0.08 \quad 489 \times 10^{-6} = 16360 \text{ S}$$

$$= 273 \text{ MIN} = 4.54 \text{ HR}$$

$$18.15 \quad B_i = \frac{hV/S}{k} = \frac{22.8 \pi \frac{D^2}{4} L}{0.19 \pi D L} = 3.9$$

~ DISTRIBUTED PARAMETER SOLN:

$$\frac{\alpha t}{x_1^2} = \frac{0.19 t}{(580)(1050)(0.065)^2} = 7.38 \times 10^{-5} t$$

$$\frac{k}{hx_1} = \frac{0.19}{22.8(0.065)} = 0.128$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{530 - 810}{295 - 810} = 0.544 \quad \eta = 0$$

$$\text{from Chart } X \approx 0.23 = 7.38 \times 10^{-5} t$$

$$t = 3114 \text{ S} = 51.9 \text{ MIN}$$

18.16 USING CHART SOLN!

$$\frac{T_s - T}{T_s - T_0} = \frac{100 - 250}{100 - 400} = 0.5$$

$$Y_A Y_B Y_C = 0.5 \quad \eta_A = \eta_B = \eta_C = 0$$

$$M_A = M_B = M_C = 0 \quad \bar{X}_A = 442 t$$

$$\bar{X}_B = 70.7 t$$

$$\bar{X}_C = 0.884 t$$

$$Y_C \approx 1$$

$$\text{By TRIAL \& ERROR, } t \approx 8.4 \times 10^{-4} \text{ hr}$$

$$\approx 3.05 \text{ S}$$

18.17

$$Bi = \frac{hV}{kS} = \frac{230}{0.151} \frac{(0.6)(0.3)(0.45)}{[see Prob 18.7]} \\ = 105 \quad \left\{ \begin{array}{l} \text{DISTRIBUTED} \\ \text{PARAMETER} \end{array} \right\}$$

$$\frac{T_s - T_\infty}{T_o - T_\infty} = 0.187 = Y_A Y_B Y_C$$

$$m_x = \frac{0.151}{230(0.15)} = 4.377 \times 10^{-3}$$

$$m_y = 2.92 \times 10^{-3}$$

$$m_z = 1.09 \times 10^{-3}$$

$$X_x = \frac{(6.19 \times 10^{-8})t}{(0.15)^2} = 2.75 \times 10^{-6}t$$

$$X_y = 1.22 \times 10^{-6}t$$

$$X_z = 1.09 \times 10^{-6}t$$

TRIAL & ERROR:  $t \approx \underline{\underline{62 \text{ HOURS}}}$

$$18.18 \quad Bi = \frac{hV}{kS} = \frac{(90 \text{ W/m}^2 \cdot \text{K})(\pi D^2 L/4)}{(0.5 \text{ W/m} \cdot \text{K})(\pi D L)}$$

$$= 0.9 \quad \left\{ \begin{array}{l} \text{DISTRIBUTED} \\ \text{PARAMETER} \end{array} \right\}$$

$$\frac{T_c - T_\infty}{T_o - T_\infty} = \frac{80 - 100}{5 - 100} = 0.211$$

$$m = \frac{k}{h x_1} = \frac{0.5}{90(0.01)} = 0.556$$

$$n=0 \quad X = \frac{x_1^2}{X_1^2} = \frac{0.5t}{880(3350)(0.01)^2} \\ = 1.70 \times 10^{-3}t$$

FROM CHART  $X \approx 0.76 = 1.70 \times 10^{-3}t$

$$t = \underline{\underline{447 \text{ S}}} = \underline{\underline{7.45 \text{ MIN.}}}$$

18.19



$$2L = 2r_o$$

$$V = \pi r_o^2 (2L)$$

$$= \frac{2.25}{991} = 2.27 \times 10^{-3} \text{ m}^3$$

$$L = r_o = \frac{2.27 \times 10^{-3}}{\pi} = 0.0712 \text{ m}$$

FOR A FINITE CYLINDER:

$$\frac{T(r_o, t) - T_\infty}{T_i - T_\infty} = P(r_o, t) X(r_o, t)$$

$$\frac{T(0, 0, t) - T_\infty}{T_i - T_\infty} = \frac{95 - 190}{5 - 190} = 0.514$$

FOR BOTH  $r$  &  $x$  DIRECTIONS:

$$h r_o = \frac{hL}{k} = \frac{15(0.0712)}{0.675} = 1.582$$

$$\frac{x_1^2}{r_o^2} = \frac{x_1^2}{L^2} = \frac{0.167 \times 10^{-6}t}{(0.0712)^2} = 3.29 \times 10^{-5}t$$

TRIAL & ERROR  $\frac{x_1^2}{L^2} = \frac{x_1^2}{r_o^2} \approx 0.34$   
{USING CHARTS}

$$t = \frac{0.34}{3.29 \times 10^{-5}} = 10330 \text{ S} \\ = \underline{\underline{172 \text{ MIN}}} = \underline{\underline{2.87 \text{ HR}}}$$

18.20 SAME CYL AS IN PROB 18.15  
BUT  $H$  VARIES -

$$\text{AS } H/D \rightarrow \infty \quad t = 3114 \text{ S} = 51.9 \text{ MIN.}$$

WITH ENDS CONSIDERED!  $D = \text{LENGTH} = 13 \text{ cm}$

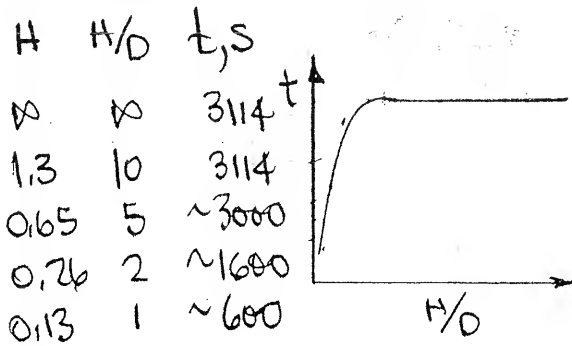
$$\left. \frac{x_1^2}{x_1^2} \right|_{\text{CYL}} = 7.38 \times 10^{-5}t \quad \left. \frac{k}{h x_1} \right|_{\text{CYL}} = 0.128$$

$$Y = 0.544 = Y_{\text{CYL}} Y_{\text{PL}}$$

FOR PLANE:  $\frac{x_1^2}{x_1^2} = \frac{1.25 \times 10^{-5}t}{H^2}$

$$k/h x_1 = 0.0167/H$$

18.20 CONTINUED -



18.21 USE SEMI-INFINITE WALL SOLN.

$$\frac{T_p - T}{T_p - T_0} = \text{erf } \eta + \exp\left(\beta + \frac{\beta^2}{4\eta^2}\right) \left[1 - \text{erf}\left(\frac{\beta}{2\eta} + \eta\right)\right]$$

$$\eta = x/\sqrt{2\alpha t} \quad \beta = \frac{h\sqrt{t}}{k} \quad \frac{\beta}{2\eta} = \frac{h\sqrt{t}}{k}$$

@ SURFACE ~  $x=0$

$$\frac{T_p - T}{T_p - T_0} = \exp\left(\frac{\beta^2}{4\eta^2}\right) \left(1 - \text{erf} \frac{\beta}{2\eta}\right)$$

$$\frac{h^2 \alpha t}{k^2} = \frac{(200)^2 (0.35)t}{(1.73)^2} = 4678 t$$

t IN HOURS

$$\frac{h\sqrt{\alpha t}}{k} = 68.4 t^{1/2}$$

$$0.1 = e^{68.4 t^{1/2}} (1 - \text{erf } 4678 t)$$

APPROXIMATION: USE 1ST TERM IN SERIES EXPANSION:

$$\frac{T_p - T_s}{T_p - T_0} = 0.1 = \frac{2\eta}{\sqrt{\pi} \beta} = \frac{1}{\sqrt{\pi}} \frac{k}{h\sqrt{\alpha t}}$$

$$t = 6.80 \times 10^{-3} \text{ hr} = \underline{24.5 \text{ s}}$$

AT THIS TIME  $\eta = 0.427$   $\beta = 4.81$

SOLVING FOR T:  $\underline{T = 143 \text{ F}}$

18.22

USE SEMI-INFINITE SOLN.

$$\frac{T - T_s}{T_0 - T_s} = \text{erf} \frac{y}{2\sqrt{\alpha t}} = \frac{2}{\sqrt{\pi}} \int_0^{y/2\sqrt{\alpha t}} e^{-\beta^2} d\beta$$

GIVEN:  $\alpha = 0.0456 \text{ ft}^2/\text{hr}$   $T_s = 0 \text{ F}$

$$\frac{\partial T}{\partial y}(0) = 0.02 \text{ F/ft} \quad T_0 = 7000 \text{ F}$$

$$\frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[ \frac{2}{\sqrt{\pi}} \int_0^{y/2\sqrt{\alpha t}} e^{-\beta^2} d\beta \right] (T_0 - T_s)$$

$$= \frac{2}{\sqrt{\pi}} \left[ e^{-y^2/4\alpha t} \frac{1}{2\sqrt{\alpha t}} \right] (T_0 - T_s)$$

for  $y=0$   $\frac{\partial T}{\partial y}(0) = \frac{T_0 - T_s}{\sqrt{\pi \alpha t}^2}$

$$t = \frac{(T_0 - T_s)^2}{\pi \alpha (\partial T / \partial y)_0} = \frac{(7000)^2}{\pi (0.0456) (4 \times 10^4)}$$

$$= \underline{9.75 \times 10^7 \text{ YEARS}}$$

18.23

USE SEMI-INF WALL SOLN.

$$\frac{T - T_0}{T_p - T_0} = \text{erf} \frac{x}{2\sqrt{\alpha t}} \left[ \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \times \left[ \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right) \right] \right]$$

@  $x=0$  THIS REDUCES TO

$$\frac{T - T_0}{T_p - T_0} = 1 - e^{-z^2} (1 - \text{erf } z)$$

WHERE  $z = \frac{h\sqrt{\alpha t}}{k}$

$$\alpha = \frac{k}{Sc_p} = \frac{0.17}{(545)(2385)} = 1.3 \times 10^{-7} \text{ m}^2/\text{s}$$

$$\frac{T - T_0}{T_p - T_0} = \frac{400 - 21}{900 - 21} = 0.431$$

$$z = \frac{h\sqrt{\alpha t}}{k} = \frac{30}{0.17} \sqrt{1.3 \times 10^{-7} t^{1/2}}$$

$$= 0.0636 t^{1/2}$$

$$z^2 = 0.00405 t$$

18.23 CONTINUED -

SUBSTITUTING  $z^2$

$$0.431 = 1 - e^{-z^2} (1 - \operatorname{erf} z)$$

$$e^{-z^2} (1 - \operatorname{erf} z) = 0.569$$

BY TRIAL & ERROR:  $z \approx 0.6$

$$t = \frac{z^2}{0.00405} = \underline{\underline{88.9 \text{ s}}} \\ = \underline{\underline{1.48 \text{ MIN}}}$$

$$18.24 \quad \frac{T - T_s}{T_o - T_s} = \frac{2}{\sqrt{\pi}} \int_0^{x/2\sqrt{\alpha t}} e^{-\beta^2} d\beta \\ \frac{\partial T}{\partial t} = (T_o - T_s) \frac{2}{\sqrt{\pi}} e^{-x^2/4\alpha t} \left[ -\frac{x}{4\sqrt{\alpha t}^{3/2}} \right] \\ = -\frac{(T_o - T_s)}{t\sqrt{\pi}} \frac{x}{2\sqrt{\alpha t}} e^{-x^2/4\alpha t}$$

$$\frac{\partial T}{\partial t} = -\frac{(T_o - T_s)}{t\sqrt{\pi}} z e^{-z^2} \quad \left\{ z = \frac{x}{2\sqrt{\alpha t}} \right\}$$

So  $\left| \frac{\partial T}{\partial t} \right|$  is MAX when  $z e^{-z^2}$  is MAX

$$\frac{d}{dz} (z e^{-z^2}) = e^{-z^2} - 2z^2 e^{-z^2} = 0$$

$$z = 1/\sqrt{2} \text{ OR } \infty$$

$$\Rightarrow \frac{x}{2\sqrt{\alpha t}} = \frac{1}{\sqrt{2}} \text{ OR } x = \sqrt{2\alpha t}$$

$$X = \left[ 2(0.0456)(9.8 \times 10^{-7})(24)(365) \right]^{1/2} \\ = \underline{\underline{2.8 \times 10^5 \text{ FT}}} = \underline{\underline{53 \text{ MILES}}}$$

18.25 THIS IS A SEM-INFINITE CASE

$$\frac{T_s - T}{T_s - T_o} = \operatorname{erf} \frac{x}{2\sqrt{\alpha t}}$$

$$\frac{x}{2\sqrt{\alpha t}} = \frac{0.25 \text{ m}}{2[(5.16 \times 10^{-7} \text{ m}^2/\text{s})(1800 \text{ s})]^{1/2}} \\ = 1.297 \quad \operatorname{erf} 1.297 \approx 0.934$$

$$\frac{100 - T}{100 - 280} = 0.934 \quad T = \underline{\underline{334 \text{ K}}}$$

18.26 SOLID IS AMENABLE TO EITHER NUMERICAL OR ANALYTICAL APPROACH

$$\frac{T_s - T_p}{T_o - T_p} = \exp\left(\frac{h^2 \alpha t}{k^2}\right) \left[ \operatorname{erfc} \frac{h\sqrt{\alpha t}}{k} \right]$$

$$\Delta T_s = T_s - T_p \quad h = 0.44 (T - T_p)^{1/3}$$

$$\Delta T_s = 900 \exp(0.00732 \Delta T_s^{2/3} t) \times \left[ \operatorname{erfc} (0.0856 \Delta T_s^{1/3} t^{1/2}) \right]$$

TRIAL & ERROR - AT EACH  $t$ :

$t$	1 hr	6 hr	24 hr
$T_p$	580	396	275

18.27

$$\frac{T - T_o}{T_p - T_o} = \operatorname{erfc} \frac{x}{2\sqrt{\alpha t}} \left[ \exp\left(\frac{h^2 \alpha t}{k^2}\right) + \frac{h^2 \alpha t}{k^2} \right] \times \left[ \operatorname{erfc} \left( \frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k} \right) \right]$$

$$\frac{T - T_o}{T_p - T_o} = \frac{400 - 25}{800 - 25} = 0.484$$

18.27 CONTINUED

@ SURFACE ( $x=0$ )

$$\frac{x}{2\sqrt{kt}} = 0 \quad \text{erf}(0) = 0 \quad \text{erfc}(0) = 1$$

$$\frac{h\sqrt{kt}}{k} = \frac{20}{0.21} (1.07 \times 10^{-7})^{1/2} t^{1/2}$$

$$= 0.0311 t^{1/2} = Z$$

GOVERNING EXPRESSION BECOMES:

$$0.484 = 1 - e^{-Z^2} (1 - \text{erf} Z)$$

TRIAL & ERROR:  $Z \approx 0.73 = 0.0311 t^{1/2}$

$$t = \underline{\underline{551 \text{ s}}} = \underline{\underline{9.18 \text{ MIN}}}$$

18.28

FOR GLASS:

$$\alpha = \frac{k}{\rho c_p} = \frac{0.45}{(170)(0.2)} = 0.0132 \text{ ft}^2/\text{hr}$$

USING SEMI-INFINITE WALL EXPRESSION:

$$\frac{T - T_0}{T_s - T_0} = \frac{32 - 30}{65 - 30} = 0.0571 = \text{erfc} \frac{x}{2\sqrt{kt}}$$

$$\frac{x}{2\sqrt{kt}} = 1.38$$

$$t = \underline{\underline{3.9 \text{ s}}}$$

18.29

CHARTS APPLY BUT ARE DIFFICULT TO READ -

CHECK VALIDITY OF INFINITE WALL SOLN

$$\frac{L}{2\sqrt{kt}} = \frac{1 \text{ ft}}{2(0.0231 t)^{1/2}} > 2$$

WORKS FOR  $t > 2.7 \text{ HOURS}$

$$\frac{x}{2\sqrt{kt}} = \frac{1}{2(0.0231 t)^{1/2}} = \frac{3.29}{t^{1/2}}$$

$$\frac{T_s - T}{T_s - T_0} = \text{erf} \frac{3.29}{t^{1/2}} \quad t \approx \underline{\underline{5.2 \text{ HR}}}$$

18.30 APPLICABLE EXPRESSION IS

$$\frac{T_\infty - T}{T_\infty - T_0} = \text{erf} A + \exp\left(\frac{hx}{k} + B^2\right) [1 - \text{erf}(A+B)]$$

$$A = \frac{x}{2\sqrt{kt}} = \frac{0.05}{2(0.444 \times 10^{-5} t)^{1/2}}$$

$$= 11.92 t^{-1/2}$$

$$\frac{hx}{k} = \frac{22(0.05)}{17.3} = 0.0636$$

$$B = \frac{h\sqrt{kt}}{k} = \frac{22}{17.3} (0.444 \times 10^{-5} t)^{1/2}$$

$$= 0.00267 t^{1/2}$$

$$B^2 = 7.11 \times 10^{-6} t$$

TRIAL & ERROR:  $t \approx 49000 \text{ s}$

$$\approx \underline{\underline{13.6 \text{ HOURS}}}$$

18.31

$$\frac{T_p - T}{T_p - T_0} = \text{erf} A + \exp\left(\frac{hx}{k} + B^2\right) [1 - \text{erf}(A+B)]$$

@  $x=0$ ,  $t=180 \text{ s}$

$$A = \frac{x}{2\sqrt{kt}} = 0 \quad \frac{hx}{k} = 0$$

$$B = \frac{h\sqrt{kt}}{k} = \frac{110}{17.3} \sqrt{180 \times} = 0.180$$

$$\frac{20 - T}{20 - 300} \approx 0.96$$

$$T = 20 + 268 = \underline{\underline{288 \text{ C}}}$$

AT  $x=50 \text{ mm}$

$$A = 0.89$$

$$\frac{hx}{k} = 0.318$$

$$B = 0.180 \quad B^2 = 0.0324$$

$$\frac{20 - T}{20 - 300} = 0.974 \quad T = \underline{\underline{293 \text{ C}}}$$

18.32. Ex 18-33

$$\frac{q_x}{A} = \frac{d}{dt} \int_0^{\delta} \rho c_p T dx - \rho c_p T_0 \frac{d\delta}{dt}$$

$$\text{For } \frac{T-T_0}{T_s-T_0} = \phi\left(\frac{x}{\delta}\right)$$

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = (T_s-T_0) \frac{1}{\delta} \left. \frac{\partial \phi}{\partial x} \right|_{x=0}$$

$$\left. \frac{\partial \phi}{\partial x} \right|_{x=0} = K \quad (\text{A CONSTANT})$$

$$\frac{q_x}{A} = -k \left. \frac{\partial T}{\partial x} \right|_{x=0} = F(t)$$

$$\therefore T_s - T_0 = \frac{F(t) \delta}{k K}$$

$$\frac{F(t)}{\rho c_p} = \frac{d}{dt} \int_0^{\delta} T dx - T_0 \frac{d\delta}{dt}$$

$$\begin{aligned} \frac{d}{dt} \int_0^{\delta} T dx &= T_0 \frac{d\delta}{dt} + \frac{d}{dt} (T_s - T_0) \int_0^{\delta} \phi dx \\ &= T_0 \frac{d\delta}{dt} + \frac{d}{dt} [(T_s - T_0) B \delta] \\ &\quad \{B \text{ A CONSTANT}\} \end{aligned}$$

$$\Rightarrow \frac{F(t)}{\rho c_p} = \frac{d}{dt} [(T_s - T_0) B \delta]$$

$$= \frac{d}{dt} \left[ \frac{B \delta^2 F(t)}{k K} \right]$$

$$\frac{k}{\rho c_p B} F(t) = \frac{d}{dt} [\delta^2 F(t)]$$

$$\delta^2 F(t) = \frac{k}{B} \times \int_0^t F(t) dt$$

$$\delta = (\text{CONSTANT}) \sqrt{\alpha \left[ \frac{\int_0^t F(t) dt}{F(t)} \right]^{1/2}}$$

18.33 NUMERICAL SOLN REQ'D

INITIAL TEMP PROFILE--

$$T = 35 + 0.5X \quad T \text{ IN } ^\circ\text{F}, X \text{ IN FT}$$

ALGORITHMS -

FOR ALL NODES EXCEPT SURFACE:

$$T_i^{t+1} = \frac{T_{i+1} + T_{i-1}}{2}$$

FOR SURFACE NODE:

$$T_0^{t+1} = T_1^t - \frac{h \Delta x}{k} T_0^t$$

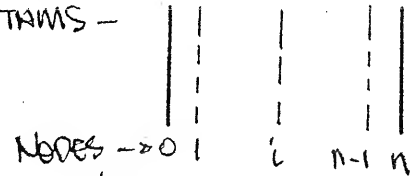
$$\sim \frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2} \quad \frac{2h \Delta t}{\rho c_p \Delta x} = \frac{h \Delta x}{k} \quad T_p = 0$$

RESULT - USING SPREADSHEET OR PROGRAM

$$\text{TIME} \approx 1800 \text{ HOURS}$$

18.34 NUMERICAL SOLN REQ'D

ALGORITHMS -



$$\text{NODE 1: } T_1^{t+1} = \frac{T_0 + T_2}{2}$$

$$T_{n-1}^{t+1} = \frac{T_n + T_{n-2}}{2}$$

$$T_i^{t+1} = \frac{T_{i+1} + T_{i-1}}{2}$$

$$\text{For } \frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2} \quad \text{IF } \Delta x = 0.25 \text{ FT}$$

$$\Delta t = 1.95 \text{ HR}$$

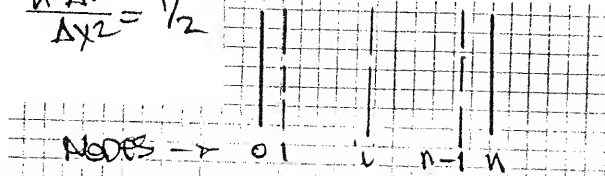
NO OF INCREMENTS  $\approx 7.4$

$$\text{TIME} = 7.4 (1.95) \approx 14.4 \text{ HR}$$

$$F_{B,35} \quad T = 520 + 330 \sin \frac{\pi x}{L}$$

$$\alpha = \frac{k}{\rho c_p} = \frac{0.66}{(1670)(8138)} = 4.72 \times 10^{-5} \frac{m^2}{s}$$

$$\frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2}$$



SAME ALGORITHMS AS PROB (F8-34)

$$\text{FOR } \frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2} \quad \text{IF } \Delta x = 0.225 \text{ m}$$

$$\Delta t = 536 \text{ s}$$

NO OF INCREMENTS  $\approx 2.4$

$$\text{Time} \approx 2.4(536) = 1286 \text{ s}$$

$$= 21.4 \text{ MIN}$$

$$\text{AT THIS TIME: } T_{\text{surf}} \approx \underline{\underline{360 \text{ K}}}$$

# CHAPTER 19

19.1 For A PLANE WALL:

VARIABLES	DIMENSIONS
$T$	$T$
$T_o$	$T$
$T_\infty$	$T$
$x$	$L$
$L$	$L$
$\alpha$	$L^2/t$
$k$	$Q/LtT$
$t$	$t$
$h$	$Q/L^2T$

$$i = n - r = 5$$

IF TEMPS ARE GROUPED AS

$$T - T_\infty, T_o - T_\infty \quad i = n - r = 4$$

$$\pi_1 = \Delta T^a L^b k^c \alpha^d (T - T_\infty)$$

$$\pi_2 = ( \quad ) (x)$$

$$\pi_3 = ( \quad ) (t)$$

$$\pi_4 = ( \quad ) (h)$$

$$\pi_1 = \frac{T - T_\infty}{T_o - T_\infty} \quad \pi_2 = \frac{x}{L} \quad \pi_3 = \frac{\alpha t}{L^2} \quad \pi_4 = \frac{hL}{k}$$

19.2 Air H<sub>2</sub>O Benz Hg GLYC

	<sup>-5</sup>	<sup>-5</sup>	<sup>-5</sup>	<sup>-6</sup>	<sup>-2</sup>
$\nu$	$1.9 \times 10^{-5}$	$0.474 \times 10^{-5}$	$0.473 \times 10^{-5}$	$1.06 \times 10^{-6}$	$0.18 \times 10^{-2}$
$c_p$	$1.008 \times 10^{-3}$	1.0	0.45	0.083	0.578
$k$	0.0243	0.383	0.0762	5.03	0.165
$Re$	$2.3 \times 10^5$	$1.02 \times 10^7$	$1.02 \times 10^7$	$4.57 \times 10^7$	37,800
$Pr$	0.699	2.72	5.21	0.021	13.1
$Nu$	348	15.4	77.3	1.17	35.7
$St$	$2.16 \times 10^{-3}$	$5.55 \times 10^{-7}$	$1.45 \times 10^{-6}$	$1.22 \times 10^{-6}$	$7.21 \times 10^{-5}$

19.3 ~ PLOTS ~

19.4 Air @ 310 K:  $Pr = 0.705$

$$k = 27 \times 10^{-2} \text{ W/m}\cdot\text{K}$$

$$\rho g / \nu^2 = 1.161 \times 10^8 / \text{m}^3 \cdot \text{K}$$

$$Gr = \frac{\rho g}{\nu^2} \Delta T = (1.161 \times 10^8) (110) x^3$$

$$\frac{\delta}{x} = 3.94 Pr^{-1/2} (Pr + 0.954)^{1/4} Gr_x^{-1/4}$$

$$x = 15 \text{ cm} \quad 30 \text{ cm} \quad 1.5 \text{ m}$$

$$Gr_x = 4.31 \times 10^7 \quad 3.45 \times 10^8 \quad 4.31 \times 10^{10}$$

$$\delta = 0.985 \text{ cm} \quad 1.17 \text{ cm} \quad 1.75 \text{ cm}$$

$$Nu_x = 30.5 \quad 51.2 \quad 171.3$$

$$h_x = 5.48 \text{ W/m}^2 \cdot \text{K} \quad 4.61 \quad 3.08$$

19.5

$$h_x = \frac{k}{x} 0.332 Re_x^{1/2} Pr^{1/3}$$

$$= 0.055 \frac{Re_x^{1/2}}{x} Pr^{1/3}$$

$$T = 30 \text{ F} \quad T_F = 55 \text{ F} \quad h_x = 0.4 \frac{Re_x^{1/2}}{x}$$

$x$	$Re_x^{1/2}/x$	$h_x$
0	$\infty$	$\infty$
0.5	10.92	8.74
1	15.46	6.19
1.5	18.92	5.05
2	21.85	4.37

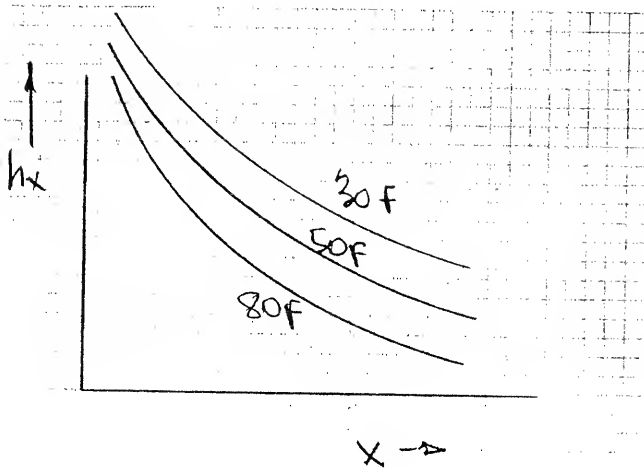
$$T = 50 \text{ F} \quad T_F = 75 \text{ F} \quad h_x = 0.256 \frac{Re_x^{1/2}}{x}$$

$x$	$Re_x^{1/2}/x$	$h_x$
0	$\infty$	$\infty$
0.5	22.2	11.37
1	31.45	8.06
1.5	38.5	6.57
2	44.5	5.70

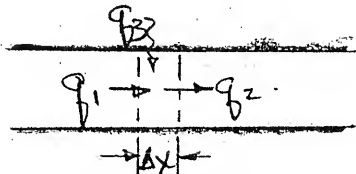
19.5 CONTINUED -

$$T = 80^{\circ}\text{F} \quad T_f = 105 \quad h_x = 0.119 \frac{\text{ft}^{1/2}}{\text{s}}$$

X	$Re_x^{1/2}/x$	$h_x$
0	$\infty$	$\infty$
0.5	67.5	16.06
1	95.2	11.32
1.5	117	9.29
2	135	8.03



19.6



As per Development in Text:

$$q_2 - q_1 - q_3 = 0$$

$$8V\phi z T|_{x=L} - 8V\phi z T|_{x=0} - \frac{q}{A}(L) = 0$$

$$\frac{8V\phi z}{2} \frac{T|_{x=L} - T|_{x=0}}{\Delta x} - \frac{q}{A} = 0$$

19.6 CONTINUED -

In Limit as  $\Delta x \rightarrow 0$

$$\frac{8V\phi z}{2} \frac{dT}{dx} - \frac{q}{A} = 0$$

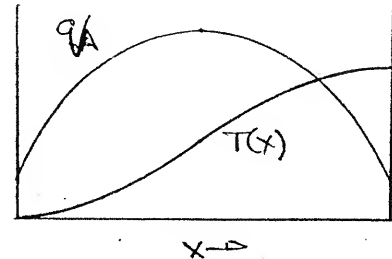
$$\int_{T_0}^T dT = \frac{2}{8V\phi z} \int_0^L \left( \alpha + \beta \sin \frac{\pi x}{L} \right) dx$$

$$T - T_0 = \frac{2}{8V\phi z} \left[ \alpha x + \frac{\beta L}{\pi} \left( 1 - \cos \frac{\pi x}{L} \right) \right]$$

$$\frac{2}{8V\phi z} = \frac{1}{30} \frac{\text{hr} \cdot \text{ft} \cdot \text{F}}{\text{Btu}}$$

$$\beta L / \pi = 1910 \text{ Btu/hr-ft}$$

X	$q/A$	$T - 120$
0	250	0
1	1310	27
2	1750	80.3
3	1310	134
4	250	161



19.7

For A Sinus 4-FT-LENGTH PLATE:

$$q = W \int_0^L \left( \alpha + \beta \sin \frac{\pi x}{L} \right) dx$$

$$= W \left( \alpha x + \frac{\beta L}{\pi} \cos \frac{\pi x}{L} \right) \Big|_0^L$$

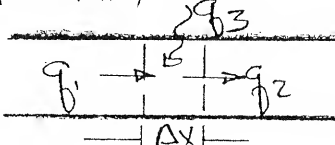
$$= WL \left( \alpha + 2 \frac{\beta}{\pi} \right)$$

$$= 16 \text{ ft}^2 \left( 250 + \frac{2(1500)}{\pi} \right)$$

$$= 19300 \text{ Btu/hr}$$

For Stack of Plates:

$$q = 19300 \frac{(640)}{16} = 772,000 \text{ Btu/hr}$$

19.8  $\frac{q}{A} = a + b \sin \frac{\pi x}{L} = 900 + 1500 \sin \frac{\pi x}{1.22}$   
 $\frac{q}{A}$  IN  $W/m^2$ ,  $x$  IN m.  


ENERGY BALANCE:  $q_2 - q_1 - q_3 = 0$

STANDARD PROCEDURE

RESULTING EXPRESSIONS

$$\frac{dT}{dx} = \frac{2q/A}{8Vc_p D(1)} = \frac{C}{A} \quad \left\{ \begin{array}{l} C \text{ A} \\ \text{CONSTANT} \end{array} \right\}$$

$$\int_{T_E}^T dT = c \int_0^x (a + b \sin \frac{\pi x}{L}) dx$$

$$T - T_E = c \left[ ax + \frac{Lb}{\pi} (1 - \cos \frac{\pi x}{L}) \right]$$

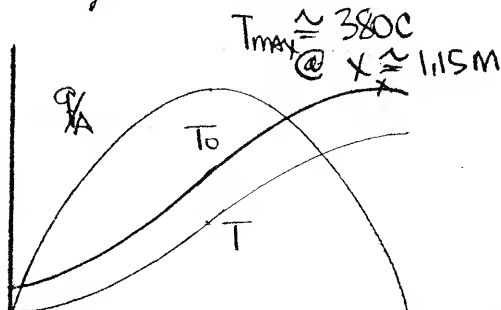
$$C = \frac{2}{8Vc_p D(1)} = \frac{2}{(7.5)(1034)(0.003)(1)} = 0.086 \text{ m} \cdot \text{kg} / \text{W}$$

$$T - T_E = 77.4 x + 83.5 (1 - \cos \frac{\pi x}{1.22})$$

FOR  $h = 56 \text{ W/m}^2 \cdot \text{K}$   $\frac{q}{A} = h(T_0 - T)$

$$T_0 = T + \frac{q/A}{56}$$

x	q/A	T	T <sub>0</sub>
0	900	100	116
0.4	3040	171	226
0.8	3110	285	340
1.2	1030	360	378
1.22	900	361	377



x →

19.9  $q = \int_0^L h_x \Delta T dx = \int_0^L Nu_x \frac{k}{x} \Delta T dx$   
 $= k \Delta T^{5/4} (0.508) Pr^{1/2} (Pr + 0.954)^{-1/4}$   
 $\times \left( \frac{Pr}{Pr_s} \right)^{1/4} \left( \frac{4}{3} L^{3/4} \right)$   
 $= 995 \text{ W PER m OF WIDTH}$

19.10 FOR  $V = a + by + cy^2$

B.C.  $V(0) = 0$

$V(\delta) = u_{\infty}$   $\frac{V}{u_{\infty}} = 2 \frac{y}{\delta} - \left( \frac{y}{\delta} \right)^2$

$\frac{dV}{dy}(\delta) = 0$

FOR  $T - T_s = x + \beta y + \gamma y^2$

B.C.  $(T - T_s)|_0 = 0$

$(T - T_s)_{\delta_t} = T_p - T_s$   $\frac{T - T_s}{T_p - T_s} = 2 \frac{y}{\delta_t} - \left( \frac{y}{\delta_t} \right)^2$

$\frac{dT}{dy}(T - T_s)|_{\delta_t} = 0$

INTO MOMENTUM EQN - TO GET  
 $\delta^2 = 30 \frac{Dx}{u_{\infty}} \quad (1)$

INTO ENERGY EQN:

$\delta \xi \left( \delta \xi^2 - \frac{1}{5} \delta \xi^3 \right) = 12 \frac{dx}{u_{\infty}} \quad (2)$

WHERE  $\xi = \delta_t / \delta$

SOLN GIVES  $\xi \approx Pr^{-1/3}$

$\therefore \delta_t = Pr^{-1/3} \delta$

SINCE  $\frac{q}{A} = -k \frac{dT}{dy}(0) = h(T_s - T_p)$

$\frac{h}{k} = \frac{2}{\delta_t} = \frac{2 Pr^{1/3}}{\delta} = 0.365 Pr^{1/3} \left( \frac{u_{\infty}}{Dx} \right)^{1/2}$

OR,  $Nu_x = \frac{hx}{k} = 0.365 Pr^{1/3} Re_x^{1/2}$

$$19.11 \quad \frac{q}{A} = \alpha + \beta \sin \frac{\pi x}{L}$$

$$= \pi D \int_0^L \left( \alpha + \beta \sin \frac{\pi x}{L} \right) dx$$

$$= \pi D \left[ \alpha L + 2 \frac{\beta L}{\pi} \right]$$

$$= \pi \left( \frac{15}{12} \right) \left[ 250 + \frac{3000}{\pi} \right] = 4730 \frac{\text{Btu}}{\text{hr}}$$

$$T_{\text{ext}} = T_i + \frac{q}{SAVC_p} = 60 + \frac{4730}{60(1)(0.02)(3600)}$$

$$= 60.3 \text{ F}$$

$$T_w = 60.3 + \frac{250}{976} \approx 60.6 \text{ F}$$

---


$$19.12 \quad T_0 = 300 - 240 \frac{-720 \text{ St}}{2}$$

$$q = \pi D L S_c V (T_s - T_b) St$$

$$= 471 (S_c V) St$$

For Air @ 180 F  $S = 0.0622$   
 $q_p = 0.241$

$$q = 471 (0.0622)(0.241)(15 \times 3600) St$$

$$= 3.82 \times 10^5 \text{ St}$$

$$v = 0.228 \times 10^{-3}$$

$$Re = \frac{Dv}{\nu} = 5480$$

$$f = 8.8 \times 10^{-3}$$

Reynolds!  $St = 0.00444$

$$T_0 = 290 \text{ F} \quad q = 937 \frac{\text{Btu}}{\text{hr}}$$

Corbuen!

$$St = 0.00444 Pr^{2/3} = 0.00467$$

$$T_0 = 298 \text{ F} \quad q = 959 \frac{\text{Btu}}{\text{hr}}$$

19.13

N <sub>2</sub> AT	100 F	200 F	150 F
$S_{\infty}$	0.009	0.0583	
$\nu_{\infty}$	$1.71 \times 10^{-3}$	$0.236 \times 10^{-3}$	$0.209 \times 10^{-3}$
$k$	0.0154	0.0174	0.0164
$Pr$	0.71	0.71	0.71

$$Re = \frac{uV}{\nu} = \frac{4 \text{ ft}(10 \text{ ft/s})}{0.209 \times 10^{-3} \text{ ft}^2/\text{s}} = 1.91 \times 10^5$$

a)  $\delta = 5x = \frac{5(4)}{(1.91 \times 10^5)^{1/2}} = 0.0457 \text{ ft}$   
 $= 0.549 \text{ in}$

b)  $\delta_t = \frac{\delta}{Pr^{1/3}} = \frac{\delta}{(0.71)^{1/3}} = 0.615 \text{ in}$

c)  $C_{fx} = 0.664 Re_x^{-1/2} = 0.00152$

d)  $C_{fL} = 1.328 Re_L^{-1/2} = 0.00304$

e)  $h_x = 0.332 \frac{k}{x} Re_x^{1/2} Pr^{1/3} = 0.531 \frac{\text{Btu}}{\text{hr ft}^2 \text{ F}}$

f)  $h = 0.664 \frac{k}{L} Re_L^{1/2} Pr^{1/3} = 1.06 \text{ "}$

g)  $f_d = A C_f \frac{\rho V^2}{2} = \frac{2(0.00304)(0.003)(10)^2}{2(32.2)}$   
 $= 5.95 \times 10^{-4} \text{ lbf}$

h)  $q = hA\Delta T = 1.06(2)(100)$   
 $= 212 \text{ Btu/hr}$

19.14

For Air AT  $T_f = 325 \text{ K}$ :  
 $\rho = 1.087 \text{ kg/m}^3$   $\nu = 1.807 \times 10^{-5} \text{ m}^2/\text{s}$   
 $c_p = 1.008 \text{ kJ/kg} \cdot \text{K}$   $Pr = 0.702$   
 $k = 2.816 \text{ W/m} \cdot \text{K}$

(a)  $C_{fL} = 1.328 Re_L^{-1/2}$

$$Re = \frac{LV}{\nu} = \frac{(1 \text{ m})(2.8 \text{ m/s})}{1.807 \times 10^{-5} \text{ m}^2/\text{s}} = 1.55 \times 10^5$$

$$C_{fL} = (1.328)(1.55 \times 10^5)^{-1/2} = 0.00337$$

19.14 CONTINUED-

$$(b) F_D = C_{FL} A \frac{\rho V^2}{2}$$

$$= \frac{(0.00337)(0.25)(1)(1.087)(2.8)^2}{2}$$

$$= 3.59 \times 10^{-3} \text{ N}$$

$$(c) q = h A \Delta T$$

USING Colburn Analogy:

$$St_L = \frac{C_{FL}}{2} Pr^{-2/3}$$

$$= \frac{(0.00337)}{2} (0.702)^{-2/3}$$

$$= 2.133 \times 10^{-3}$$

$$= h / \rho c_p V$$

$$h = (2.133 \times 10^{-3})(1.087)(1008)(2.8)$$

$$= 6.54 \text{ W/m}^2 \cdot \text{K}$$

$$q = 6.54 (1)(0.25)(55)$$

$$= \underline{90.0 \text{ W}}$$

19.15

MOMENTUM THEOREM ~ X DIR.

$$\sum F_x = \iint_{c.s.} u_x \rho (\vec{V} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{c.v.} u_x \rho dV$$

AT Low Velocity:  $\rho \approx \text{const}$ ,  $V_w = 0$   
 & STEADY STATE

$$\text{LHS: } \sum F_x = (\text{BOUOYANT FORCE}) - (\text{VISCIOUS FORCE})$$

BOUOYANT FORCE (B.F.)  $\rho g \Delta S$  { PER UNIT VOLUME }

$$\beta \equiv \frac{1}{T} \left( \frac{\partial \rho}{\partial T} \right)_p = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p \approx \frac{1}{\rho} \frac{\Delta \rho}{\Delta T}$$

$$\therefore \Delta \rho \approx -\rho \beta \Delta T$$

$$\text{B.F.} = \Delta x \rho g \int_0^{\delta_t} (T - T_w) dy$$

19.15 CONTINUED-

$$\text{VISCIOUS FORCE (V.F.)} = \Delta x \mu \frac{\partial u}{\partial y} (0)$$

$$\text{RHS } \iint_{c.s.} u_x \rho (\vec{V} \cdot \vec{n}) dA = \int_0^{\delta} \rho u_x^2 dy \Big|_{x+\Delta x}$$

$$- \int_0^{\delta} \rho u_x^2 dy \Big|_x - \underbrace{u_x \rho}_{\text{on side}}$$

EQUATING: (LHS) = (RHS) & DIV. BY  $\Delta x$ :

$$\rho g \int_0^{\delta_t} (T - T_w) dy - \mu \frac{\partial u}{\partial y} (0)$$

$$= \frac{\int_0^{\delta} \rho u_x^2 dy \Big|_{x+\Delta x} - \int_0^{\delta} \rho u_x^2 dy \Big|_x}{\Delta x}$$

IN LIMIT AS  $\Delta x \rightarrow 0$  &  $\rho = \text{CONSTANT}$

$$\rho g \int_0^{\delta_t} (T - T_w) dy - \mu \frac{\partial u}{\partial y} (0) = \frac{d}{dx} \int_0^{\delta} u_x^2 dy$$

ENERGY EQN: SAME FOR BOTH  
 NATURAL & FORCED CONVECTION

$$x \frac{\partial T}{\partial y} (0) = \frac{d}{dx} \int_0^{\delta_t} (T_w - T) u_x dy$$

19.16

$$Pe_L = \frac{(2 \text{ FT})(10 \text{ FT/K})}{\nu} = 20/\omega$$

LAMINAR FLOW FOR ALL VALUES OF  $\omega$

$$C_{FL} = \frac{1.328}{Re_L^{1/2}} \quad F_D = A C_F \rho V^2 / 2$$

$$F_D = \frac{4(1.328)(3)(100)}{Re_L^{1/2}} = 8.25 \frac{3}{Re_L^{1/2}}$$

$$Nu_L = 0.664 Re_L^{1/2} Pr^{1/3}$$

$$h = \frac{k}{L} (0.664) Re_L^{1/2} Pr^{1/3}$$

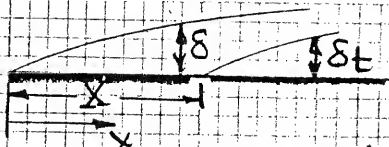
19.16 CONTINUED -

T, F	S	N	Re	F <sub>0</sub>
30	79.5	$6.61 \times 10^{-2}$	303	37.6 U <sub>∞</sub>
50	79.0	$1.52 \times 10^{-2}$	1320	17.9 "
80	78.2	$0.13 \times 10^{-2}$	15400	5.2 "

T <sub>∞</sub>	T <sub>f</sub>	T <sub>f</sub>	N	Re <sup>1/2</sup>	Pr <sup>1/3</sup>
30	80	55	0.0419	21.9	7.25
50	100	75	0.0101	44.6	4.64
80	130	105	0.0011	134.8	2.16

T	h <sub>c</sub>	q/A
30	874 Btu/ft <sup>2</sup> F	437 Btu/hr ft <sup>2</sup>
50	11.4 "	570 "
80	16.0 "	800 "

19.17



Assuming  $T - T_s = \alpha + \beta y + \gamma y^2 + \delta y^3$

B.C.  $T(0) = T_s$   $\frac{\partial T}{\partial y}(\delta) = 0$   
 $T(\delta) = T_\infty$   $\frac{\partial T}{\partial y^2}(0) = 0$

Temp profile becomes

$$\frac{T - T_s}{T_\infty - T_s} = \frac{3}{2} \frac{y}{\delta_t} - \frac{1}{2} \left( \frac{y}{\delta_t} \right)^3 \quad (1)$$

Similarly for velocity:

$$\frac{u}{u_\infty} = \frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \quad (2)$$

INTO INTEGRAL EXPRESSION!

$$\alpha \frac{\partial T}{\partial y}(0) = \frac{d}{dx} \int_0^{\delta_t} (T_\infty - T) u dy$$

(LHS) ~ LEFT-HAND SIDE:

$$\alpha \frac{\partial T}{\partial y}(0) = \alpha (T_\infty - T_s) \left( \frac{3}{2\delta_t} \right)$$

19.17 CONTINUED

$$(RHS) \int_0^{\delta_t} (T_\infty - T_s) u dy = (T_\infty - T_s) u_\infty \int_0^{\delta_t} \left( 1 - \frac{T - T_s}{T_\infty - T_s} \right) dy$$

SUBST EONS (1) & (2):

$$= (T_\infty - T_s) u_\infty \left[ \frac{3}{20} \frac{\delta_t^2}{\delta} - \frac{3}{80} \frac{\delta_t^4}{\delta^3} \right]$$

NEGLECT

$$\text{GIVING: } (RHS) = (T_\infty - T_s) u_\infty \frac{d}{dx} \left( \frac{3}{20} \frac{\delta_t^2}{\delta} \right)$$

EQUATING: (LHS) = (RHS)

$$\delta_t \frac{d}{dx} \frac{\delta_t^2}{\delta} = 10 \frac{x}{u_\infty}$$

LETTING  $\xi = \delta_t / \delta$

$$\delta \xi d(\xi^2 \delta) = 10 \frac{x}{u_\infty} dx$$

$$\delta = \frac{4.64}{\sqrt{u_\infty / \nu}} x^{1/2}$$

SUBSTITUTION & SOME ALGEBRA GIVE  
 $4.31 x \xi^2 d\xi = \left( \frac{x}{u_\infty} - 1.077 \xi^3 \right) dx$

SEPARATING VARIABLES:

$$\frac{4.31 \xi^2 d\xi}{\frac{1}{Pr} - 1.077 \xi^3} = \frac{dx}{x}$$

$$\xi^3 = \frac{1}{1.077} \frac{1}{Pr} \left[ 1 - \left( \frac{x}{x_0} \right)^{3/4} \right]$$

$$\xi = \frac{\delta_t}{\delta} \approx \frac{1}{Pr^{1/3}} \left[ 1 - \left( \frac{x}{x_0} \right)^{3/4} \right]^{1/3}$$

19.17 CONTINUED -

NUSSELT No. "

$$\begin{aligned} q &= -k \frac{\partial T}{\partial y}(0) = -\frac{3}{2} k \frac{T_p - T_s}{\delta_t} \\ &= \frac{3}{2} k \frac{T_s - T_p}{\delta} Re_x^{1/2} \left[ \frac{Pr}{1 - (x/x)^{3/4}} \right]^{1/3} \\ &= h(T_s - T_p) \\ \therefore Nu_x &= \frac{h_x x}{k} = 0.323 Re_x^{1/2} \left[ \frac{Pr}{1 - (x/x)^{3/4}} \right]^{1/3} \end{aligned}$$

19.18  $Nu_x = \frac{h_x x}{k} = 0.508 Pr^{1/2} (Pr + 0.954)^{-1/4} Gr_x^{1/4}$

For Air:  $Pr = 0.72$   $k = 0.015$   $Btu/hr \cdot ft \cdot F$

$$h = k (0.38) \left( 2 \times 10^6 \frac{\Delta T}{x} \right)^{1/4} = 0.245 \left( \frac{\Delta T}{x} \right)^{1/4}$$

$$q = h_L A \Delta T = \int_0^L h_x A \Delta T dx$$

$$h_L = \frac{1}{L} \int_0^L h_x dx = 0.286 \left( \frac{\Delta T}{L} \right)^{1/4} = \alpha \left( \frac{\Delta T}{L} \right)^{\beta}$$

$$\Rightarrow \underline{\alpha = 0.286} \quad \underline{\beta = 1/4}$$

19.19  $\frac{v}{u_x} = \frac{y}{\delta} \left( 1 - \frac{y}{\delta} \right)^2 \quad \frac{T - T_p}{T_s - T_p} = \left( 1 - \frac{y}{\delta} \right)^2$

INTO ENERGY EQN:

$$\alpha \frac{\partial T}{\partial y}(0) = \frac{d}{dx} \int_0^{\delta_t} u_x (T_p - T) dy$$

LEFT:  $LHS = -\frac{2\alpha (T_s - T_p)}{\delta}$

RHS =  $(T_p - T_s) \frac{d}{dx} \left( \frac{8u_x}{30} \right)$

EQUATING:  $\frac{2\alpha}{\delta} = \frac{d}{dx} \frac{8u_x}{30}$

19.19 CONTINUED -

INTO MOMENTUM EQN:

$$\beta g \int_0^{\delta_t} (T_s - T_p) dy - \nu \frac{\partial^2 u}{\partial y^2}(0) = \frac{d}{dx} \int_0^{\delta} u_x^2 dy$$

$$LHS = \beta g \frac{(T_s - T_p) \delta}{3} - \frac{\nu u_x}{\delta}$$

$$RHS = \frac{d}{dx} \left( \frac{8u_x^2}{105} \right)$$

EQUATING:

$$\frac{\beta g (T_s - T_p) \delta}{3} - \frac{\nu u_x}{\delta} = \frac{d}{dx} \int_0^{\delta} u_x^2 dy$$

LETTING  $\delta = Ax^a \quad u_x = Bx^b$

PREVIOUS TWO EQNS BECOME:

$$\frac{2\alpha}{A} x^{-a} = \frac{AB(a+b)}{30} x^{a+b-1}$$

$$-\frac{\nu B}{A} x^{-a+b} + \beta g \Delta T \frac{A}{3} x^a = \frac{AB^2(a+b)}{105} x^{a+b-1}$$

EXPONENTS ON X MUST AGREE

$$\Rightarrow -a = a+b-1$$

$$-a+b = a = a+b-1$$

GIVING  $a = 1/4 \quad b = 1/2$

SO EQNS FOR A & B BECOME

$$\frac{2\alpha}{A} = \frac{AB}{30} \left( \frac{3}{4} \right)$$

$$\frac{\nu B}{A} + \beta g \Delta T \frac{A}{3} = \frac{AB^2}{105} \left( \frac{5}{4} \right)$$

SO WE HAVE

$$A = \left[ 240 \left( \frac{\nu^3}{\beta g \Delta T} \right) \left( \frac{\alpha^2}{\nu^2} \right) \left( \frac{20}{21} + \frac{\nu}{\alpha} \right) \right]^{1/4}$$

$$B = \frac{80\alpha}{A}$$

19.19 CONTINUED -

§ FINALLY - OPEN SUBSTITUTION!  
 $\delta = Ax^{1/4} = 3.94 Pr^{-1/2} (Pr + 0.954)^{1/4} Gr_x^{-1/4} x$

$$\frac{q}{A} = -k \frac{\partial T}{\partial y}(0) = \frac{2k}{\delta} (T_s - T_\infty) = h (T_s - T_\infty)$$

$$\Rightarrow Nu_x = 0.508 Pr^{1/2} (Pr + 0.954)^{1/4} Gr_x^{1/4}$$

19.20

$$\frac{\delta_T}{\delta} = \frac{1}{Pr^{1/3}} \left[ 1 - \left( \frac{x}{x'} \right)^{3/4} \right]^{1/3}$$

$$Nu_x = 0.33 \left[ \frac{Pr}{1 - (x/x')^{3/4}} \right]^{1/3} Re_x^{1/2}$$

$$Re_x = \frac{0.4(5)}{1.569 \times 10^{-5}} = 127,500$$

$$\delta = \frac{54}{Re_x^{1/2}} = \frac{5(40)}{(1.275 \times 10^5)^{1/2}} = 0.56 \text{ cm}$$

$$\delta_t = \frac{0.56}{0.708^{1/3}} \left[ 1 - \left( \frac{1}{2} \right)^{3/4} \right]^{1/3} = 0.465 \text{ cm}$$

$$C_{fx} = \frac{0.664}{Re_x^{1/2}} = \frac{1.86 \times 10^{-6}}{(1.275 \times 10^5)^{1/2}}$$

$$Nu_x = 0.33 \left[ \frac{0.708}{1 - (1/2)^{3/4}} \right]^{1/3} (1.275 \times 10^5)^{1/2}$$

$$= 143$$

$$h_x = k/x (143) = 9.38 \text{ W/m}^2 \cdot \text{K}$$

19.21  $U = a + by$  BC,  $U(0) = 0$   
 $U(\delta) = U_\infty$

$$1. \frac{U}{U_\infty} = \frac{y}{\delta}$$

$$T - T_s = x + \beta y \quad \text{B.C. } (T - T_s)_0 = 0$$

$$(T - T_s)_{\delta_t} = T_p - T_s$$

$$\therefore \frac{T - T_s}{T_p - T_s} = \frac{y}{\delta_t}$$

19.21 CONTINUED

INTO MOMENTUM EQN!

$$\rho \frac{\partial U}{\partial y}(0) = \frac{\partial}{\partial x} \int_0^\delta (U_\infty - U) U dy$$

$$\text{LHS} = \rho U_\infty / \delta$$

$$\text{RHS} = -U_\infty^2 \frac{\partial}{\partial x} \int_0^\delta \left( 1 - \frac{y}{\delta} \right) \left( \frac{y}{\delta} \right) dy$$

EQUATING & SOLVING!

$$\delta^2 = \frac{12 \nu x}{U_\infty} \quad (1)$$

ENERGY EQN!

$$\alpha \frac{\partial T}{\partial y}(0) = \frac{\partial}{\partial x} \int_0^{\delta_t} (T_p - T) U dy$$

SUBSTITUTING & SOLVING!

$$\frac{6\alpha}{U_\infty \delta \xi} = \frac{\partial}{\partial x} (\delta \xi^2) \quad \left\{ \begin{array}{l} \xi = \frac{\delta_t}{\delta} \end{array} \right. \quad (2)$$

$$\delta_t = 0 \quad \text{for } x = x'$$

$$\xi^3 = \frac{x}{x'} \left[ 1 - \left( \frac{x}{x'} \right)^{3/4} \right]$$

$$\frac{\delta_t}{\delta} = Pr^{-1/3} \left[ 1 - \left( \frac{x}{x'} \right)^{3/4} \right]^{1/3}$$

$$\frac{q}{A} = -k \frac{\partial T}{\partial y}(0) = -\frac{k(T_s - T_\infty)}{\delta_t} = h(T_s - T_\infty)$$

GIVING!

$$Nu_x = \frac{h x}{k} = 0.288 \left[ \frac{Pr}{1 - (x/x')^{3/4}} \right]^{1/3} Re_x^{1/2}$$

IF  $x = 0$

$$Nu_x = 0.288 Pr^{1/3} Re_x^{1/2}$$

19.22  $v = a \sin by$   $T - T_s = \alpha \sin by$

BC,  $v(0) = 0$   $(T - T_s)|_0 = 0$

$v(\delta) = v_{\infty}$   $(T - T_s)|_{\delta_t} = T_p - T_s$

$\Rightarrow \frac{v}{v_{\infty}} = \sin \frac{\pi y}{2\delta}$   $\frac{T - T_s}{T_p - T_s} = \sin \frac{\pi y}{2\delta_t}$

INTO ENERGY EQN.

$\delta_t \frac{d\delta_t}{dx} = \frac{\alpha \pi}{v_{\infty}} \left( \frac{\pi}{4 - \pi} \right)$

{ PRESUMES  $\delta = \delta_t$  FOR INTEGRATION }

$\frac{q}{A} = -k \frac{dT}{dy}(0) = h(T_s - T_{\infty})$

$\frac{k\pi}{2\delta_t} = h$  OR  $\frac{h}{k} = \frac{\pi}{2\delta_t}$

$\Rightarrow Nu_x = \frac{hx}{k} = 0.327 Pr^{1/3} Re_x^{1/2}$

19.23

$\frac{v}{v_{\infty}} = \left( \frac{y}{\delta} \right)^{1/4}$   $\frac{T - T_s}{T_p - T_s} = \left( \frac{y}{\delta_t} \right)^{1/4}$

ENERGY EQN.

$\alpha \frac{dT}{dy}(0) = v_{\infty}(T_p - T_s) \frac{d}{dx} \int_0^{\delta_t} \frac{v}{v_{\infty}} \left( 1 - \frac{T - T_s}{T_p - T_s} \right) dy$

LHS =  $\frac{0.0225(T_p - T_s)v_{\infty}}{\alpha/\alpha} \left( \frac{\alpha}{\delta v_{\infty}} \right)^{1/4}$

RHS =  $v_{\infty}(T_p - T_s) \frac{7}{12} \frac{d\delta}{dx}$

{ ASSUMES  $\delta = \delta_t$  FOR INTEGRATION }

EQUATING & SOME ALGEBRA:

$\frac{\delta}{x} = 0.371 Pr^{4/5} Re_x^{-1/5}$

19.23 CONTINUED -

$\frac{q}{A} = -k \frac{dT}{dy}(0) = -\frac{k(0.0225)(\Delta T)v_{\infty}}{\alpha} \left( \frac{\alpha}{\delta v_{\infty}} \right)^{1/4}$   
 $= h\Delta T$

$Nu_x = \frac{hx}{k} = 0.0288 Re_x^{1/2} Pr^{1/5}$

19.24  $q = hA\Delta T$

$\frac{q}{A} = 284 - 95 = 189 \text{ W/m}^2$

$\Delta T = 8 \text{ K}$   $A = (1)(18.3) = 18.3 \text{ m}^2$

$h = \frac{189}{8} = 23.63 \text{ W/m}^2 \cdot \text{K}$

FOR CONDITIONS SPECIFIED:

$Re_L = \frac{(18.3 \text{ m})v}{1.5689 \times 10^{-5} \text{ m}^2/\text{s}} = 1.166 \times 10^6 v$

PROBABLY TURBULENT B.L.

USE COULLEN ANALOGY:  $St = \frac{C_f}{2} Pr^{-2/3}$

FROM CH 13 - FOR TURB. B.L.

$C_{fx} = 0.0576 Re_x^{-1/5}$

$C_{fL} = \frac{1}{L} \int_0^L C_{fx} dx$

$= 0.072 Re_L^{-1/5}$  { ASSUMING ALL SURFACE EXPOSED TO TURB. B.L. }

$St = \frac{h}{\rho c_p v} = \frac{0.072}{2} Re_L^{-1/5} Pr^{-2/3}$   
 $= 0.036 [1.166 \times 10^6 v]^{-1/5} (0.708)^{-2/3}$

$= 0.00277 v^{-1/5}$

$h = \rho c_p v (0.00277 v^{-1/5})$

$= (1.177)(1006)(0.00277) v^{4/5} \text{ W/m}^2 \cdot \text{K}$

$= 3.280 v^{4/5} = 23.63$

$v = 11.8 \text{ m/s}$

$$19.25 \quad \frac{T-T_s}{T_o-T_s} = \exp\left(-St \frac{4L}{D}\right)$$

$$\frac{T-300}{60-300} = \exp\left[-St \frac{4(15)}{12}\right] = \exp(-720 St)$$

$$U = 12.25 \text{ FT/s}$$

REYNOLDS ANALOGY: ASSUME  $T_L = 240 \text{ F}$

$$T_{avg} = 150 \text{ F} \quad T_f = 225 \text{ F}$$

$$Re = \frac{1/12 (12.25)}{3.07 \times 10^{-5}} = 3.32 \times 10^5$$

$$f = 0.0036 \quad St = 0.0018$$

$$T = 300 - (240) e^{-(0.0018)(720)}$$

$$= 234.5 \text{ F}$$

CLOSE ENOUGH - DOING OVER  
WITH  $T_L = 234.5$  WILL YIELD  
 $T_L \approx 234.5$  AS A RESULT.

COLBURN ANALOGY: ASSUME  $T_L = 200 \text{ F}$

$$T_{avg} = 130 \quad T_f = 215$$

$$Re = \frac{1/12 (12.25)}{0.321 \times 10^{-5}} = 3.18 \times 10^5$$

$$f = 0.0036 \quad St = 0.0018 (1.79)^{-2/3}$$

$$= 0.00122$$

$$T = 300 - (240) e^{-(0.00122)(720)}$$

$$= 201 \text{ F}$$

$$q = \dot{m} c_p \Delta T = \frac{30}{7.48} (62.3)(0.999) \Delta T$$

$$= 250 \Delta T \text{ BTU/min}$$

SUMMARY	$\Delta T, \text{ F}$	$q, \text{ BTU/min}$
REYNOLDS	174	43,500
COLBURN	141	35,300

$$19.26 \quad q = \frac{500 \text{ BTU}}{\text{HR FT}^2} (\pi)(15/12) \text{ FT}^2$$

$$= 1960 \text{ BTU/HR}$$

$$q = \dot{m} c_p \Delta T = 1960$$

$$\Delta T = \frac{1960}{(30/7.48)(62.3)(60)(0.999)}$$

$$= 0.131 \text{ F}$$

$$T_{w \text{ EXIT}} = 60.13 \text{ F}$$

FROM COLBURN ANALOGY:  $T_L \approx 60 \text{ F}$

$$Re = \frac{(1/12)(30/7.48)(144 \times 4)}{0.76 \times 10^{-3}} \left(\frac{1}{60}\right)$$

$$= 1344 \quad \{\text{LAMINAR}\}$$

$$f = \frac{16}{1344} = 0.0119$$

$$St = \frac{0.0119}{2} (8.07)^{-2/3} = 0.00148$$

$$h = 62.3 \left( \frac{30 \times 144 \times 4}{7.48 \times \pi \times 60} \right) (0.24)(3600)(St)$$

$$= 976 \text{ BTU/HR FT}^2 \text{ F}$$

$$T_{wall} = 60.13 + \frac{500}{976} = 60.6 \text{ F}$$

19.27

$$q = \text{SAME AS IN PROB 19.26}$$

$$= 1960 \text{ BTU/HR}$$

$$T = T_o + \frac{q}{SAUC_p}$$

$$= 60 + \frac{1960}{(0.0764)(\frac{\pi}{4} \times \frac{1}{144})(15 \times 3600)(0.24)}$$

$$= 423 \text{ F}$$

19.28

$$T_L = T_0 - \Delta T e^{-St 4/D}$$

$$= 300 - 100 e^{-St 4/D}$$

$$V = \frac{30(144)}{7.48(60)(\pi/4)} = 12.25 \text{ ft/s}$$

$$Re = \frac{DV}{\nu} = \frac{(1/2)(12.25)}{7.43 \times 10^{-6}} = 137,100$$

$$f_r = 0.0118 \quad f_f = 0.0044$$

$$\text{REYNOLDS} \quad St = 0.0022$$

ANALOGY:

$$T_L = 300 - 100 e^{-1.585} = \underline{179.5 \text{ F}}$$

$$q = 8A5cp \Delta T = \underline{100.5 \text{ Btu/s}}$$

$$\text{CARBON} \quad St = 0.0022 (0.0118)^{2/3}$$

$$\text{ANALOGY:} \quad = 0.0424$$

$$T_L = 300 - 100 e^{-30.1} \approx 300 \text{ F}$$

$$q = \underline{126.4 \text{ Btu/s}}$$

19.29

for constant  $q/A$ 

$$T = T_0 + \frac{q/A}{50cp}$$

$$= 200 + \frac{500}{(58.1)(12.25 \times 3600)(0.332)}$$

$$\approx \underline{200 \text{ F}}$$

## CHAPTER 20

20.1  $\frac{q}{A} = \frac{750(3443)}{\pi(3/48)(1/2)} = 26100 \frac{\text{Btu}}{\text{hr ft}^2}$

ENDS ARE NEGLECTED

$$h = \frac{k}{L} Nu = \frac{k}{L} \left[ 0.825 + \frac{0.387 Ra^{1/4}}{\left\{ 1 + \left( \frac{0.492}{Pr} \right)^{9/16} \right\}^{8/27}} \right]^2$$

FOR VERTICAL ORIENTATION

BY TRIAL & ERROR!  $\Delta T \approx 103 \text{ F}$

HTR SURFACE TEMP = 198 F

HORIZONTAL ORIENTATION!

$$h = \frac{k}{D} \left[ 0.60 + \frac{0.387 Ra^{1/4}}{\left\{ 1 + \left( \frac{0.559}{Pr} \right)^{9/16} \right\}^{8/27}} \right]^2$$

TRIAL & ERROR!  $\Delta T = 99 \text{ F}$

HTR SURFACE TEMP = 194 F

20.2 Bismuth  $T_p = 700 \text{ F}$

AS IN PROBLEM 20.1  $\frac{q}{A} = 26100 \frac{\text{Btu}}{\text{hr ft}^2}$

VERTICAL - SAME FORMULA AS ABOVE

TRIAL & ERROR!  $\Delta T \approx 57 \text{ F}$

$T_{\text{surf}} \approx \underline{757 \text{ F}}$

HORIZONTAL - SAME FORMULA AS ABOVE

TRIAL & ERROR!  $\Delta T \approx 44 \text{ F}$

$T_{\text{surf}} \approx \underline{744 \text{ F}}$

## 20.2 CONT. HYDRAULIC FLUID

SAME FORMULAS & PROCEDURES

VERTICAL  $\Delta T \approx 630 \text{ F}$   $T_{\text{surf}} \approx \underline{630 \text{ F}}$

PROPERTIES USED AT  
100 F - HIGHEST  
TEMP IN TABLES

HORIZONTAL  $\Delta T \approx 580 \text{ F}$   $T_{\text{surf}} \approx \underline{580 \text{ F}}$

20.3  $\frac{q}{A} = \frac{3413}{0.344} = 9900 \frac{\text{Btu}}{\text{hr ft}^2}$

16 cm (0.525 ft) - VERTICAL AT

$T_p = 71 \text{ F}$

$$h = \frac{k}{L} \left[ 0.825 + \frac{0.387 Ra^{1/4}}{\left\{ 1 + \left( \frac{0.492}{Pr} \right)^{9/16} \right\}^{8/27}} \right]^2$$

TRIAL & ERROR!  $\Delta T = 62 \text{ F}$   $T_s = \underline{133 \text{ F}}$

FOR 10 cm (0.328 ft) - HEIGHT

TRIAL & ERROR!  $\Delta T = 60 \text{ F}$   $T_s = \underline{131 \text{ F}}$

ENGLISH UNITS USED - TABLES  
EASIER TO USE

20.4 for  $T_f = 100 \text{ F}$

$$Gr_L = (107 \times 10^6) \left( \frac{1}{2} \right)^3 (100) = 1.337 \times 10^9$$

$$Pr = 4.51 \quad Ra = 6.03 \times 10^9$$

$$h_{\text{series}} = \frac{k}{L} \left[ 1 + \frac{0.387 Ra^{1/4}}{\left\{ 1 + \left( \frac{0.492}{Pr} \right)^{9/16} \right\}^{8/27}} \right]^2$$

$$= 190 \frac{\text{Btu}}{\text{hr ft}^2 \text{ F}}$$

$$R_i = \frac{hV/A}{k} = \frac{190}{2.20} (0.0357) = 0.0308$$

USE LUMPED PARAMETER

20.4 CONT. for  $T_{\text{avg}} = 150^\circ\text{F}$

$$\frac{T - T_p}{T_o - T_p} = \frac{50}{150} = \frac{1}{3} = e^{-BiFo}$$

$$Fo = \frac{\alpha t}{(V/A)^2} = \frac{398 t}{(0.0857)^2} = 3120 t$$

t in hours

$$-BiFo = \ln \frac{1}{3}$$

$$t = 0.0114 \text{ hr} = 0.686 \text{ min} = 41.15$$

SINCE LUMPED PARAMETER SOLUTION IS VALID - ANSWERS TO PARTS (a) & (b) ARE THE SAME

WHEN  $T_c = 100^\circ\text{F}$   $T_{\text{surf}} \approx 100^\circ\text{F}$

20.5  $T_s = 140^\circ\text{C}$   $T_p = 25^\circ\text{C}$   $T_f = 825^\circ\text{C}$

AIR @ 355 K:  $\rho_g = 0.625 \times 10^{-8} (\text{m}^3 \cdot \text{K})^{-1}$

$$\rho_a = (0.625 \times 10^{-8}) (0.035)^3 (115) = 3.08 \times 10^{-5}$$

HORIZ. CYLINDER:

$$Nu = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{9/16} \right]^{8/27}} \right\}^2$$

- For  $Pr = 0.696$   $Nu_D = 10.47$

$$q = h A \Delta T = \frac{k}{D} (\pi D L) (\Delta T)$$

$$= (0.0304) (\pi) (0.8) (115) (10.47)$$

$$= 92.0 \text{ W}$$

REMAINDER OF 100 W INPUT GOES TO ELECTRICAL & CONDUCTION LOSSES & TO ILLUMINATION

20.6 for A HORIZ. CYLINDER

$$Nu = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{9/16} \right]^{8/27}} \right\}^2$$

$$q = 27 \text{ W/m} = h A \Delta T = \frac{k}{D} Nu A \Delta T$$

$$= \frac{k}{D} Nu (\pi D) \Delta T$$

$$27 = \pi k Nu \Delta T$$

TRIAL & ERROR  $\Delta T \approx 9.8 \text{ K}$

$$T_{\text{surf}} \approx 39.8^\circ\text{C}$$

20.7 for Cu CYLINDER WITH

$H_T = 20.3 \text{ cm}$ ,  $\text{DIAM} = 2.54 \text{ cm}$

$$\frac{V}{A} = \frac{(\pi D^2/4) L}{\pi D L + \pi D^2/2} = \frac{DL/4}{L + D/2} = 0.598 \text{ cm}$$

for  $Bi = \frac{h V/A}{k} = 0.1$   $T_f \approx 16^\circ\text{C}$

$$h = \frac{0.1 (379)}{0.00598} = 6340 \text{ W/m}^2 \cdot \text{K}$$

FOR AN  $h$  VALUE  $< 6340$

$Bi < 0.1$   $\therefore$  LUMPED PARAM.

$$\frac{T - T_p}{T_o - T_p} = e^{-BiFo}$$

$$Fo = \frac{\alpha t}{(V/A)^2} = \frac{(10.27 \times 10^{-5}) (180)}{(0.00598)^2}$$

$$= 516.9$$

$$\frac{4.8 - (-1)}{32.5 - (-1)} = 0.173 = e^{-BiFo}$$

$$BiFo = 1.754 \quad Bi = 3.393 \times 10^{-3}$$

$$h = \frac{Bi (379)}{0.598} = 2115 \text{ W/m}^2 \cdot \text{K}$$

20.8 FOR A SPHERE:  $Nu_D = 2 + 0.43 Ra_D^{1/4}$   
 $T_{S, AVG} = 340 K$   $T_f = 320 K$   $k = 2.78 \times 10^{-2}$   
 $T_\infty = 295 K$   $Pr = 0.703$

$$Ra_D = (0.994 \times 10^8) D^3 (45)(0.703)$$

$$= 3144 \times 10^9 D^3$$

D, cm	h	k/hx,
7.5	615	0.0104
5	710	0.0135
1.5	1180	0.0271

LE, SURFACE RESISTANCE IS VERY SMALL:  $T_s = T_\infty$  & FALLS TO THIS VALUE ALMOST INSTANTANEOUSLY  
 $\sim \text{TIME} \approx 0$

20.9 FOR  $T_c$  TO REACH 320 K - USE VALUES CALCULATED IN PROB 20.8

$$\alpha = k/\rho c_p = 2.1 \times 10^{-7} m^2/s$$

D, cm	h	$\alpha t/x^2$	t
7.5	615	0.16	1.19 HR
5	710	0.16	31.7 MIN
1.5	1180	0.16	2.86 "

$T_{SURF} \approx 295 K$  AT ALL TIMES

20.10  $T_s = 240 F$   $T_\infty = 60 F$   $T_f = 150 F$   
 $\Delta T = 180 F$   $\frac{\rho g}{\nu^2} = 1.22 \times 10^6$   $Pr = 0.698$   
 $k = 0.0167$

a) HORIZONTAL:

$$h = \frac{k}{D} \left[ 0.6 + \frac{0.387 Ra_D^{1/6}}{\left\{ 1 + \left( \frac{0.559}{Pr} \right)^{1/6} \right\}^{8/27}} \right]^2$$

20.10 CONT.

$$h = 1.51 \frac{Btu}{HR FT^2}$$

$$\frac{q}{A} = h \Delta T = 271 \frac{Btu}{HR FT^2}$$

VERTICAL:

$$h = \frac{k}{L} \left[ 0.825 + \frac{0.387 Ra_D^{1/6}}{\left\{ 1 + \left( \frac{0.492}{Pr} \right)^{1/6} \right\}^{8/27}} \right]^2$$

$$h = 1.0 \quad \frac{q}{A} = 180 \frac{Btu}{HR FT^2}$$

20.11 SAME CONDITIONS AS PROB 20.10 EXCEPT FLUID IS  $H_2O @ 60 F$

$$\frac{\rho g}{\nu^2} = 403 \times 10^6$$
  $Pr = 2.72$   $k = 0.383$

HORIZ:  $h = 316 \frac{Btu}{HR FT^2}$   $\frac{q}{A} = 57000 \frac{Btu}{HR FT^2}$

VERT:  $h = 281$  "  $\frac{q}{A} = 50,600 \frac{Btu}{HR FT^2}$

20.12

SPHERICAL TANK  $D = 0.6 m$

$$T_c = 78 K$$
  $T_p = 278 K$   $q = \frac{\Delta T}{\Delta R}$

FOR SPHERE:

$$h = \frac{k}{D} Nu = \frac{k}{D} \left[ 2 + 0.43 Ra_D^{1/4} \right]$$

$$h = \frac{0.0247}{0.6} \left[ 2 + 0.43 (2.57 \times 10^8) (0.7)^3 (8)(0.72) \right]^{1/4}$$

$$= 2.86 W/m^2 \cdot K$$
 - PROPERTIES @ 260 K

$$R_{CONV} = \frac{1}{2.86 (4\pi) (0.35)^2} = 0.227$$

20.12 CONTINUED

$$R_{\text{cond}} = \frac{r_o - r_i}{4\pi k r_o r_i} = \frac{0.05}{4\pi(0.04)(0.3)(0.35)} = 0.947$$

$$\Sigma R = 1.174 \quad \dot{q} = \frac{200}{1.174} = 170 \text{ W}$$

$$\left\{ \begin{array}{l} \Delta T_{\text{conv}} \approx 39 \text{ K} \\ T_{\text{surf}} \approx 239 \text{ K} \\ T_f \approx 259 \text{ K} \end{array} \right\} \text{O.K.}$$

20.13

Assuming EACH PLATE IS INDEPENDENT

$$h = \frac{k}{L} \text{Nu} = \frac{k}{L} \left[ 0.825 + \frac{0.387 \text{Ra}_L^{1/4}}{\left[ 1 + \left( \frac{0.492}{\text{Pr}} \right)^{9/16} \right]^{4/7}} \right]^{1/2}$$

$$T_{\text{plate}} = 200 \text{ F} \quad T_p = 80 \text{ F} \quad T_f = 140 \text{ F} \\ \text{Pr} = 3.08 \quad \text{Ra} = (540 \times 10^6)(3)^3(20)(3.08) = 5.39 \times 10^{12}$$

$$h = 287 \text{ Btu/hr-ft}^2 \text{F}$$

$$\dot{q} = hA\Delta T = 287(30 \times 1 \times 3 \times 2)(20) = 6.19 \times 10^6 \text{ Btu/hr} = 1.81 \text{ kW}$$

20.14

$$\dot{q}_A = hA\Delta T$$

$$h = \frac{k}{L} [0.14 \text{Ra}_L^{1/3}] \text{ if } \text{Ra}_L > 2 \times 10^7$$

$$T_{\text{surf}} = 150 \text{ F} \quad T_p = 50 \text{ F} \quad T_f = 100 \text{ F}$$

$$\text{Pr} = 0.703 \quad \text{Ra} = (1.76 \times 10^6)(20)^3(100)(0.703) = 9.90 \times 10^{11}$$

$$h = \frac{0.0156}{20} (0.14)(9.90 \times 10^{11})^{1/3} = 1.09 \frac{\text{Btu}}{\text{hr-ft}^2 \text{F}}$$

$$\dot{q}_A = 1.09(100) = 109 \text{ Btu/hr-ft}^2$$

20.14 CONTINUED -

$$\text{FRACTION OF TOTAL} = \frac{109}{200} = 0.54$$

WITH 1 FT X 1 FT RIDGES

$$\text{Ra} = (1.76 \times 10^6)(1)^3(100)(0.703) = 1.237 \times 10^9$$

$$h = \frac{0.0156}{1} (0.14)(1.237 \times 10^9)^{1/3} = 1.09 \frac{\text{Btu}}{\text{hr-ft}^2 \text{F}}$$

$$\text{SAME AS IN PART (a) FRACT} = 0.54$$

20.15 FOR FORCED CONVECTION

$$\text{Re} = \frac{(20 \text{ FT})(6.1 \times 3,281 \text{ FT/S})}{0.181 \times 10^{-3} \text{ FT}^2/\text{S}} = 2.21 \times 10^6$$

Flow is  $\left\{ \begin{array}{l} \text{LAMINAR up to } \text{Re} = 2 \times 10^5 \\ \text{TURBULENT past } 3 \times 10^6 \end{array} \right.$

PLATE IS MOSTLY IN TRANSITION REGIME  
ASSUME LAMINAR B.L.

$$h = \frac{k}{L} \text{Nu} = \frac{0.0156}{20} (0.664 \text{Re}_L^{1/2} \text{Pr}^{1/3}) = 0.685 \text{ Btu/hr-ft}^2 \text{F}$$

$$\dot{q}_A = hA\Delta T = 68.5 \text{ Btu/hr-ft}^2$$

$$\text{FRACTION DUE TO F.C.} = 0.34$$

IF B.L. IS TURBULENT

$$h = \frac{0.0156}{20} [0.0296 \text{Re}_L^{0.8} \text{Pr}^{1/3}]$$

$$= 2.97 \text{ Btu/hr-ft}^2 \text{F}$$

$$\dot{q}_A = 2.97 \text{ Btu/hr-ft}^2 \quad \text{FRACT} = 1.49$$

FOR THIS CASE THERE IS MORE CAPACITY TO TRANSFER HEAT THAN THERE IS SOLAR ENERGY SUPPLIED. SURFACE TEMP WILL BE  $< 150 \text{ F}$

20.16

$$T_{\text{surf}} = 1300 \text{ K} \quad T_f = 785 \text{ K}$$

$$T_p = 270 \text{ K}$$

$$\frac{q}{A} = h \Delta T \quad h = \frac{k}{D} (2 + 0.43 \text{ Ra}^{1/4})$$

$$k = 5.69 \times 10^{-2} \text{ W/m} \cdot \text{K} \quad Pr = 0.688$$

$$\text{Ra} = (2.015 \times 10^6) (0.15)^3 (1030) (0.688)$$

$$= 4.82 \times 10^6$$

$$h = 8.4 \text{ W/m}^2 \cdot \text{K}$$

$$\frac{q}{A} = 8.4 (1030) = \underline{850 \text{ W/m}^2}$$

20.17

From prob 20.16

$$h = \frac{k}{D} \text{Nu} = \frac{k}{D} [2 + 0.43 \text{ Ra}^{1/4}]$$

$T_{\text{surf}}$	$T - T_{\infty}$	$T_f$	$\text{Ra}^{1/4}$	$h$
1300	1030	785	46.8	8.40
1000	730	685	50.3	7.65
700	430	485	63.1	8.10
420	150	345	57.2	6.69

$$h_{\text{avg}} \approx 7.71 \text{ W/m}^2 \cdot \text{K}$$

$$Bi = \frac{h V A}{k} = \frac{7.71 (0.15/6)}{39.8} = 0.00484$$

{Lumped parameter is OK}

$$\frac{T - T_{\infty}}{T_0 - T_{\infty}} = \frac{600 - 270}{1300 - 270} = 0.32 = e^{-Bi Fo}$$

$$Bi Fo = 1.1394 \quad Fo = 235.4 = \frac{t}{(V/A)^2}$$

$$t = \frac{(0.15/6)^2 (235.4)}{0.001125} = \underline{130.8 \text{ s}}$$

20.18

$$q = I^2 R = (400)^2 8 \text{ L/A}$$

$$R = \frac{(1.72 \times 10^6)(100)}{\pi/4 (0.5)^2} = 876 \times 10^{-4} \Omega/\text{m}$$

$$\frac{q}{A} = \frac{400^2 (876 \times 10^{-4})}{\pi (0.018)(1)} = 2480 \text{ W/m}$$

$$= h \Delta T \quad q = 140 \text{ W/m}$$

$$h = \frac{k}{D} C \text{ Ra}^n$$

$$q = \frac{k}{D} C \left[ \left( \frac{Bg}{\beta^2} \right) D^3 Pr \right]^n \Delta T^{1+n}$$

By TRIAL & ERROR:  $\Delta T \approx 220 \text{ K}$ 

$$h = \underline{11.0 \text{ W/m}^2 \cdot \text{K}}$$

$$T_{\text{surf}} = 290 + 220 = \underline{510 \text{ K}}$$

RESISTANCE OF INSULATION

$$= \frac{\ln D_o/D_i}{2\pi k} = \frac{\ln \frac{0.018}{0.005}}{2\pi (0.242)} = 0.842$$

$$q = \frac{\Delta T}{R} \quad \Delta T = \frac{140}{0.842} = 166 \text{ K}$$

$$T_{\text{INTERFACE}} = 510 + 166 = \underline{676 \text{ K}}$$

20.19

$$R_M = \frac{2.83 \times 10^6}{1.72 \times 10^6} (876 \times 10^{-4})$$

$$= 1.44 \times 10^{-3} \Omega/\text{m}$$

$$q = (400)^2 (1.44 \times 10^{-3}) = 231 \text{ W/m}$$

$$\frac{q}{A} = \frac{231}{\pi (0.018)} = 4080 \text{ W/m}^2$$

$$= \frac{k}{D} C \left[ \frac{Bg}{\beta^2} D^3 Pr \right]^n \Delta T^{1+n}$$

20.19 CONTINUED -

TRIAL & ERROR:  $\Delta T \approx 336 \text{ K}$

$h = 12.1 \text{ W/m}^2 \cdot \text{K}$

$T_{\text{SURF}} = 290 + 336 = 626 \text{ K}$

$R_{\text{INSUL}} = 0.842 \left\{ \text{PROB 20A3} \right\}$

$\Delta T = Q/R = \frac{231}{0.842} = 274 \text{ K}$

$T_{\text{INTERFACE}} = 626 + 274 = 900 \text{ K}$

20.20  $q_{\text{TOTAL}} = q_{\text{CONV}} + q_{\text{RAD}}$

ASSUME  $T_{\text{INSIDE}} = T_{\text{SURFACE}}$

$q = h_i A_i (T_{\text{STM}} - T) = h_o A_o (T - T_{\infty})$   
 $+ \sigma A_o \left[ \left( \frac{T}{100} \right)^4 - \left( \frac{T_{\infty}}{100} \right)^4 \right]$

T IN R

TRIAL & ERROR:  $T = 1147 \text{ R}$   
 $= 687 \text{ F}$

$h_o = \frac{k}{D} \left[ 0.6 + \frac{0.387 \text{ Ra}^{1/4}}{\left\{ 1 + \left( \frac{0.559}{\text{Pr}} \right)^{1/4} \right\}^{4/3}} \right]^2$

$\frac{Q}{A} = 33(1260 - 1147) = 3730 \text{ Btu/hr ft}^2$

$q = 3730 (\pi) \left( \frac{8.625}{12} \right) (20)$   
 $= 168,000 \text{ Btu/hr}$

20.21 FORCED CONVECTION OUTSIDE

$\frac{Q}{A} = 33(1260 - T) = h_o (T - 520)$   
 $+ \sigma \left[ \left( \frac{T}{100} \right)^4 - 5.34 \right]$   
 $h_o = \frac{k}{D} B \text{ Re}^n \text{ Pr}^{1/3} \quad \left\{ \begin{array}{l} B, n \text{ - functions} \\ \text{of Re} \end{array} \right\}$

ASSUME  $T = 650 \text{ F} = 1110 \text{ R}$   $T_{\infty} = 360 \text{ F}$

$\text{Re} = \frac{\left( \frac{8.625}{12} \right) (6.5) (3.281)}{0.348 \times 10^{-3}} = 4.40 \times 10^4$

TABLE 20.3  $B = 0.021$   $n = 0.805$

@ THIS TEMP  $h_o = 4.31$

$\text{LHS} = 4950$   $\text{RHS} = 4923 \left\{ \begin{array}{l} \text{Pretty} \\ \text{GOOD} \end{array} \right\}$

$q \approx 4950 \text{ Btu/hr ft}^2 \left( \frac{8.625}{12} \right) (\pi) (20)$   
 $= 224,000 \text{ Btu/hr}$   
 $= 65.5 \text{ kW}$

20.22 INSULATION ON OUTSIDE w/  
 NATURAL CONV. ON SURFACE

$R_{\text{INSULATION}} = \frac{\ln(r_o/r_i)}{2\pi k} = 1.401 \text{ Per FT}$

$\Sigma R = \frac{1}{A_i h_i} + 1.401 + \frac{1}{A_o h_o}$   
 $= \frac{1}{33\pi \left( \frac{8.625}{12} \right)^2} + 1.401 + \frac{1}{h_o \pi \left( \frac{14.625}{12} \right)^2}$   
 $= 1.414 + 0.261/h_o$

$\frac{Q}{L} = \frac{\Delta T}{\Sigma R} = \frac{730}{1.414 + 0.261/h_o} = \frac{800 - T}{1.414}$

WITH  $h_o = \frac{k}{D} \left[ 0.6 + \frac{0.387 \text{ Ra}^{1/4}}{\left\{ 1 + \left( \frac{0.559}{\text{Pr}} \right)^{1/4} \right\}^{4/3}} \right]^2$

20.22 CONTINUED

TRIAL 3 ERROR:  $T \approx 190^\circ\text{F}$

$$q = \frac{800 - 190}{1.414} \frac{\text{Btu}}{\text{hr ft}} (20 \text{ ft})$$

$$= 8630 \text{ Btu/hr} = \underline{\underline{253 \text{ kW}}}$$

20.23  $q = \Delta T / R$

$$= \frac{800 - T_i}{\frac{1}{\pi D_i (33)} \ln \frac{r_o}{r_i}} (T_i - 250) = \frac{800 - 250}{\Sigma R}$$

$$R_{\text{stm}} = \frac{1}{33(\pi)(8.625)} = 0.0134$$

$$R_{\text{insul}} = \frac{\ln D_o / 8.625}{2\pi(0.06)} = 2.65 \ln D_o / 8.625$$

EQUATIONS TO BE SOLVED ARE:

$$q = \frac{800 - T}{0.0134} = \frac{T - 250}{2.65 \ln D_o / 8.625} = \frac{550}{\Sigma R}$$

TRIAL 1 ERROR:  $T \approx 750^\circ\text{F}$

$$D_o \approx 9.08 \text{ in.}$$

INSULATION THICKNESS

$$= \frac{9.08 - 8.625}{2} = \underline{\underline{0.228 \text{ in.}}}$$

20.24

$$q = \frac{800 - T_1}{R_{\text{ins}}} = \frac{T_1 - T_2}{\frac{\ln D_o / D_i}{2\pi k}} = \pi D_o h_o (T_2 - T_o) (1/0.85)$$

$$\frac{800 - T_1}{0.0134} = (T_1 - T_2) \frac{0.377}{\ln D_o / D_i} = 3.70 D_o h_o (T_2 - T_o)$$

$$= \frac{730}{0.0134 + 2.65 \ln D_o / 8.625 + \frac{0.27}{D_o h_o}}$$

TRIAL 1 ERROR PROBLEM (LENGTHY)

20.24 CONTINUED -

ANSWER - APPROXIMATELY

$$T_1 = 793.8^\circ\text{F} \quad T_2 = 178.4^\circ\text{F}$$

$$q = 465 \frac{\text{Btu}}{\text{hr ft}} (20 \text{ ft}) = 9290 \text{ Btu/hr}$$

$$= \underline{\underline{2.72 \text{ kW}}}$$

$$\frac{800 - 793.8}{0.0134} = \frac{(793.8 - 178.4) 0.377}{\ln D_o / D_i}$$

$$\ln D_o / D_i = 0.50 \quad \frac{D_o}{D_i} = 1.651$$

$$D_o = 1.651(8.625) = 14.24 \text{ in}$$

$$\text{THICKNESS} = \frac{14.24 - 8.625}{2}$$

$$= \underline{\underline{2.81 \text{ INCHES}}}$$

20.25 FOR NATURAL CONVECTION CASE

PLANE UPWARD - FACING HOT SURFACE

$$Nu_L = 0.14 Ra_L^{1/3} \text{ IF } 1 \times 10^3 < Ra_L < 10^{10}$$

ASSUME TOP SURFACE IS SQUARE

$$\sim A = L^2$$

$$q = hA\Delta T = hL^2\Delta T$$

$$= \frac{k}{L} Nu_L L^2 \Delta T$$

$$= k [0.14 Ra_L^{1/3}] L \Delta T$$

$$Ra_L = \frac{\beta g L^3 \Delta T}{\nu^2} Pr$$

$$q = k \left[ 0.14 \left( \frac{\beta g \Delta T}{\nu^2} \right)^{1/3} Pr^{1/3} L \Delta T \right]$$

$$@ T_s = 45^\circ\text{C} \quad T_p = 20^\circ\text{C} \quad T_f = 32.5^\circ\text{C}$$

$$k = 0.02663 \frac{\text{W}}{\text{m} \cdot \text{K}} \quad \frac{\beta g}{\nu^2} = 1.24 \times 10^{-8} (\text{m}^3/\text{K})$$

20.25 CONTINUED

$$Re = (1.244 \times 10^8)(25)(0.707) L^3$$

$$= 2.199 \times 10^9 L^3$$

$$40 W = 0.0243 \left[ 0.14 (1.244 \times 10^8)^{1/3} (0.707)^{1/3} L^2 (25) \right]$$

$$L^2 = 0.965 \quad L \approx 0.982 m$$

$$Re = 2.08 \times 10^9 \sim \infty$$

Now - for SAME HT LOSS &  $L = 0.982 m$

$$q = hA\Delta T = \frac{k}{L} Nu_L A \Delta T \quad \left\{ \begin{array}{l} \text{FORCED} \\ \text{CONV.} \end{array} \right\}$$

$$\text{ASSUME } T_{\text{surf}} \approx 20^\circ C$$

$$k = 2.569 \times 10^{-2}$$

$$\nu = 1.506 \times 10^{-5}$$

$$Re = \frac{LV}{\nu} = \frac{0.965(20)}{1.506 \times 10^{-5}} = 1.28 \times 10^6$$

TRANSITION REGIME -

ASSUME LAMINAR B.L. -

$$Nu = 0.664 Re_L^{1/2} Pr^{1/3}$$

$$40 W = \frac{0.02569}{0.965} (0.965)^2 (0.14) (1.28 \times 10^6)^{1/2} \times (0.71)^{1/3} \Delta T$$

$$\Delta T = 2.4^\circ F \quad \underline{T_{\text{surf}} \approx 22.4^\circ C}$$

20.26  $q = \frac{\Delta T}{\sum R} = \frac{319 - 301}{\sum R} K$

$$R_o = \frac{1}{h_o \pi D_o L} = \frac{1}{(6800) \pi (0.009)} = 2.464 \times 10^{-3}$$

$$R_i = \frac{1}{h_i \pi D_i L} = \frac{1}{(5200) \pi (0.0165)} = 3.71 \times 10^{-3}$$

$$R_{\text{air}} = \frac{ln(r_o/r_i)}{2\pi k L} = \frac{ln(1.9/1.15)}{2\pi (385)} = 5.83 \times 10^{-5}$$

20.26 CONTINUED

$$\sum R = 6.232 \times 10^{-3}$$

$$\dot{m} = \frac{18}{(\sum R)(h_{\text{eq}})} = \frac{18}{6.232 \times 10^{-3} (2390)} = 1.21 \text{ kg/s}$$

20.27  $\dot{m}_{\text{PER TUBE}} = 0.49 \text{ kg/s}$

$$Re = \frac{0.49(4)}{\pi(0.0209)(79 \times 10^{-3})} = 3780$$

USE ANALOGY OR ASSUME TURBULENT

$$\ln \frac{T_c - T_s}{T_o - T_s} = -4 \frac{L}{D} St \quad St = \frac{f}{2} Pr^{-2/3}$$

$$\text{ASSUME } T_{\text{EXIT}} = 314 K \quad T_{\text{B AVG}} = 307 K$$

$$Pr \approx 121 \quad St = \frac{0.01}{2} (121)^{-2/3} = 2.04 \times 10^{-4}$$

$$f \approx 0.01 \quad -2.04 \times 10^{-4} (4)(5) / 0.0209$$

$$T_L = 372 - 72 \approx$$

$$= 313 K \sim \text{CHECK}$$

$$q = 1.47 (2000)(43) = 38.2 \text{ kW}$$

20.28 ASSUME  $T_L = 235^\circ F$   $T_{\text{AVG}} = 148^\circ F$

$$Re = \frac{Dv}{\nu} = \frac{(0.87/12)(40)}{0.209 \times 10^{-3}} = 1.39 \times 10^4$$

{TURBULENT}

$$St = 0.023 Re^{-0.8} Pr^{-0.2} = 4.33 \times 10^{-3}$$

$$\frac{T_L - T_s}{T_o - T_s} = e^{-4 \frac{L}{D} St}$$

$$T_L = 240 - 180(0.0083)$$

$$= 239^\circ F \quad \left\{ \begin{array}{l} \text{CLOSE} \\ \text{ENOUGH} \end{array} \right\}$$

$$20.29 \quad \dot{Q} = h \Delta T = 180 h$$

a) flow parallel To Tube

$$Re = \frac{LV}{D} = \frac{L(40)}{0.201 \times 10^{-3}} = 1.91 \times 10^5 L$$

If  $X \leq 1.5$  B.L. IS LAMINAR

If  $X = 10$  B.L. IS IN TRANSITION

If LAMINAR OVER TOTAL LENGTH

$$h = \frac{k}{L} (0.664) Re_L^{1/2} Pr^{1/3}$$

$$= \frac{0.167}{10} (0.664) (1.91 \times 10^5)^{1/2} (0.72)^{1/3}$$

$$= 13.74 \text{ Btu/hr ft}^2 \text{ F}$$

$$\frac{\dot{Q}}{A} = 13.74 (180) = \underline{2470 \text{ Btu/hr ft}^2}$$

b) CROSSFLOW CASE

$$Re = \frac{DV}{D} = 1.59 \times 10^4$$

$$h = \frac{k}{D} \left[ 0.193 (1.59 \times 10^4)^{0.618} (0.72)^{1/3} \right]$$

$$= 137 \text{ Btu/hr ft}^2 \text{ F}$$

$$\frac{\dot{Q}}{A} = 137 (180) = \underline{24,600 \text{ Btu/hr ft}^2}$$

20.30 WATER:

a) PARALLEL TO TUBE  $T_f = 150 \text{ F}$

$$Re = \frac{10(40)}{0.40 \times 10^{-5}} = 8.44 \times 10^7 \quad \{ \text{TURBULENT} \}$$

$$h = \frac{k}{L} (0.036) Re_L^{4/5} Pr^{1/3}$$

$$= \frac{0.383}{10} (0.036) (8.44 \times 10^7)^{4/5} (2.72)^{1/3}$$

$$= 4220 \text{ Btu/hr ft}^2 \text{ F}$$

$$\frac{\dot{Q}}{A} = 4220 (180) = \underline{7.6 \times 10^5 \text{ Btu/hr ft}^2}$$

20.30 CONTINUED -

b) CROSSFLOW

$$Re = 7.03 \times 10^5$$

$$h = \frac{k}{D} (0.027) (7.03 \times 10^5)^{0.805} (2.72)^{1/3}$$

$\{ \text{TABLE 20.3 VALUES @ HIGHEST } Re \}$

$$h = 8820 \text{ Btu/hr ft}^2 \text{ F}$$

$$\frac{\dot{Q}}{A} = 8820 (180) = \underline{1.59 \times 10^6 \text{ Btu/hr ft}^2}$$

20.31 a) PARALLEL

$$Re = \frac{10(40)}{5.45 \times 10^{-5}} = 7.34 \times 10^6 \quad \{ \text{TURB} \}$$

$$h = \frac{k}{L} (0.036) (7.34 \times 10^6)^{4/5} (80.5)^{1/3}$$

$$= 312 \text{ Btu/hr ft}^2 \text{ F}$$

$$\frac{\dot{Q}}{A} = 312 (180) = \underline{56,100 \text{ Btu/hr ft}^2}$$

b) CROSSFLOW  $Re = 61,000$

$$h = \frac{k}{D} (0.027) (61,000)^{0.805} (80.5)^{1/3}$$

$$= 642 \text{ Btu/hr ft}^2 \text{ F}$$

$$\frac{\dot{Q}}{A} = 642 (180) = \underline{115,600 \text{ Btu/hr ft}^2}$$

20.32  $Re = \frac{GD}{\mu}$   $T_f = 186 \text{ F}$

$$Re = \frac{(0.385/12)(20)}{0.379 \times 10^{-5}} = 169,000$$

FROM FIGURE 20.13  $j = 10^{-3}$   
 $\{ \text{EXTRAPOLATION} \}$

20.32 CONTINUED

$$\frac{h}{\text{Eq 5}} = 10^{-3} \text{Pr}^{-1/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

$$= 10^{-3} (2.14)^{-1/3} \left( \frac{0.273}{0.15} \right)^{0.14} = 0.631 \times 10^{-3}$$

$$h = (0.631 \times 10^{-3}) (1.01) (60.4) (3600)$$

$$= \underline{135 \text{ Btu/hr ft}^2 \text{ F}}$$

20.33  $q = h A \Delta T$

$$= 135 (48) (\pi) \left( \frac{0.387}{12} \right) (5)$$

$$= \underline{3280 \text{ Btu/hr}}$$

20.34  $T_{\text{surf}} = 380\text{K}$   
 $T_{\infty} = 295\text{K}$   $T_f = 337.5\text{K}$

$$\rho = 980.6 \text{ kg/m}^3 \quad \nu = 0.453 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = 0.661 \text{ W/m}\cdot\text{K} \quad \text{Pr} = 2.90$$

a) HORIZONTAL NATURAL CONV.

$$h = \frac{k}{D} \left\{ 0.60 + \frac{0.387 \text{Ra}_D^{1/6}}{\left[ 1 + \left( \frac{0.559}{\text{Pr}} \right)^{1/4} \right]^{1/4} \left[ 1 + \left( \frac{\text{Ra}_D}{282000} \right)^{5/8} \right]^{4/5}} \right\}^2$$

$$\text{Ra}_D = (27.54 \times 10^9) (0.0126)^3 (85) (2.9)$$

$$= 1.358 \times 10^7$$

$$h = 1898 \text{ W/m}^2 \cdot \text{K}$$

$$q = h A \Delta T = (1898) \pi (0.0126) (0.075) (85)$$

$$= \underline{479 \text{ W}}$$

20.34 CONTINUED -

b) VERTICAL NATURAL CONV.

$$h = \frac{k}{L} \left\{ 0.825 + \frac{0.387 \text{Ra}_L^{1/6}}{\left[ 1 + \left( \frac{0.492}{\text{Pr}} \right)^{1/4} \right]^{1/4} \left[ 1 + \left( \frac{\text{Ra}_L}{282000} \right)^{5/8} \right]^{4/5}} \right\}^2$$

$$\text{Ra}_L = (27.54 \times 10^9) (0.075)^3 (85) (2.9)$$

$$= 2.864 \times 10^9$$

$$h = 1351 \text{ W/m}^2 \cdot \text{K}$$

$$q = (1351) \pi (0.0126) (0.075) (85)$$

$$= \underline{341 \text{ W}}$$

c) CROSSFLOW

$$\text{Re} = \frac{D V}{\nu} = \frac{(0.0126) (1.5)}{0.453 \times 10^{-6}}$$

$$= 41700$$

$$h = \frac{k}{D} \left\{ 0.3 + \frac{0.62 \text{Re}^{1/2} \text{Pr}^{1/3}}{\left[ 1 + \left( \frac{0.4}{\text{Pr}} \right)^{1/4} \right]^{1/4} \left[ 1 + \left( \frac{\text{Re}}{282000} \right)^{5/8} \right]^{4/5}} \right\}$$

$$= 11030 \text{ W/m}^2 \cdot \text{K}$$

$$q = (11030) \pi (0.0126) (0.075) (85) = \underline{2.78 \text{ kW}}$$

20.35  $\text{Re} = \frac{0.15 (150)}{7.98 \times 10^{-5}} = 282,000$

$$h = \frac{k}{D} \text{Nu} = \frac{0.0566 (400)}{0.15}$$

↑  
from Fig 20.11

$$= 151 \text{ W/m}^2 \cdot \text{K}$$

$$\frac{q}{A} = 151 (1030) = \underline{155 \text{ kW/m}^2}$$

$$20.36 \quad q = 140 \text{ W/m} \quad \left\{ \begin{array}{l} \text{From Prob} \\ q_A = 2480 \text{ W/m}^2 \end{array} \right. \quad 20.18$$

$$R = \frac{0.018(9)}{1.569 \times 10^{-5}} = 10300 \quad \{T_f \approx 300\text{K}\}$$

$$R = 0.708 \quad k = 0.0262$$

$$h = \frac{k}{D} \left[ 0.193 (10300)^{0.48} (0.708)^{1/3} \right]$$

$$= 75.6 \text{ W/m}^2 \cdot \text{K}$$

$$\Delta T = 2480 / 75.6 = 32.8$$

$$\{T_s = 323, T_f = 300, \text{close enough}\}$$

$$h = 75.6 \text{ W/m}^2 \cdot \text{K}$$

$$\Delta T = 32.8 \quad T_{\text{surf}} = 323 \text{ K}$$

$$\text{INSUL. RESISTANCE} = 0.842 \quad \{ \text{Prob 20.43} \}$$

$$\Delta T = 140 / 0.842 = 166 \text{ K}$$

$$T_{\text{int. surf.}} = 323 + 166 = 489 \text{ K}$$

$$20.37 \quad \text{Sphere: } D = 0.075 \text{ m}$$

$$T_b = 25^\circ\text{C} \quad T_s = 145^\circ\text{C}$$

$$\eta = 1.59 \times 10^{-5} \quad \mu_p = 1.837 \times 10^{-5}$$

$$k = 0.0261 \quad \mu_s = 2.429 \times 10^{-5}$$

$$Pr = 0.708$$

$$q = hA\Delta T = h(\pi)(0.075)^2(120)$$

$$Re = \frac{Dv}{\nu} = \frac{(0.075)(0.5)}{1.551 \times 10^{-5}} = 2418$$

$$h = \frac{k}{D} \left[ 2 + \left( 0.4 Re^{1/2} + 0.06 Re^{2/3} \right) Pr^{0.4} \left( \frac{\mu_p}{\mu_s} \right)^{1/4} \right]$$

$$= \frac{0.0261}{0.075} \left\{ 2 + \left[ 0.4(2418)^{1/2} + 0.06(2418)^{2/3} \right] (0.708)^{0.4} \left( \frac{1.837}{2.429} \right)^{1/4} \right\}$$

$$20.37 \quad \text{CONTINUED -}$$

$$= 8.99 \text{ W/m}^2 \cdot \text{K}$$

$$q = 8.99(\pi)(0.075)^2(120)$$

$$= 19.07 \text{ W}$$

$$20.38 \quad G = \dot{m}/A = 3.64 \text{ lbm/s} \cdot \text{ft}^2$$

$$Re = \frac{GD}{\mu} = \frac{3.64(0.622/12)}{0.29 \times 10^{-3}} = 650$$

(LAMINAR)

USE SIEDER-TATE EQN. ASSUME  $T_{b, \text{avg}} = 150^\circ\text{F}$

$$Nu = 1.86 \left( Re Pr \frac{D}{L} \right)^{1/3} \left( \frac{\mu_b}{\mu_s} \right)^{0.14}$$

$$h = \frac{k}{D} Nu = \frac{0.383(1.86)}{0.622/12} \left[ (650)(2.72) \frac{0.622}{12(5)} \right]^{1/3} \times \left( \frac{0.29}{0.578} \right)^{0.14}$$

$$= 32.9 \text{ Btu/hr} \cdot \text{ft}^2 \cdot \text{F}$$

$$St = Nu / Re Pr = 0.00252$$

$$\frac{T_s - T_b}{T_o - T_b} = e^{-4 \frac{L}{D} St} = e^{-0.972} = 0.378$$

$$T = 80 + 0.378(100) = 117.8^\circ\text{F}$$

$$T_{b, \text{avg}} = \frac{117.8 + 180}{2} = 149^\circ\text{F} - \text{OK}$$

$$q = \dot{m} c_p \Delta T = 3.64(0.0021)(497 - 86.7)$$

↑  
FROM STEAM TABLES

$$= 1710 \text{ Btu/hr}$$

$$20.39 \quad G = 60.6(35) = 2120 \text{ lbm/s} \cdot \text{ft}^2$$

$$Re = 307,000 \quad \{ \text{TURBULENT} \}$$

$$T_f \approx 130^\circ\text{F} - \text{USE COLBURN EQ.}$$

$$St = 0.023(307,000)^{-0.2} (3.44)^{-2/3}$$

$$= 0.000807$$

20.39 CONTINUED

$$T = 80 + 100 e^{-St(4)(5)/0.022/12}$$

$$= 153.3 \text{ F} \quad \text{--- FIRST GUESS}$$

$$T_f = \left[ 80 + \frac{154}{2} + 180 \right] / 2 \approx 149 \text{ F}$$

AT THIS TEMP:  $Re = 443,000$   $Pr = 4.51$

$$St = 0.000625 \quad T \approx 155 \text{ F}$$

20.40  $GA = 10,000 \text{ lbm/hr}$

$$G = \frac{10,000}{0.276} = 37400 \text{ lbm/hr ft}^2$$

$$Re = \frac{37400 \left( \frac{7.001}{12} \right)}{1.63 \times 10^{-5} (3600)} = 3.72 \times 10^5$$

{TURBULENT}

USE DITUS-BOECKER EQN:

$$h = \frac{k}{D} (0.023) Re^{0.8} Pr^{0.3}$$

$$= \frac{0.0321}{(7.001/12)} (0.023) (3.72 \times 10^5)^{0.8} (0.912)^{0.3}$$

$$= 35.2 \text{ Btu/hr ft}^2 \text{ F}$$

20.41  $\dot{m}_{TOTAL} = 1.47 \frac{\text{kg}}{\text{s}} \sim 0.245 \frac{\text{kg}}{\text{s}}$

PER TUBE

$$Re = \frac{DGS}{\mu} = \frac{\dot{m}}{\pi D \mu} = \frac{0.245 (4)}{\pi (0.0209) (7.9 \times 10^{-3})}$$

$$= 1890 \quad \{ \text{LAMINAR} \}$$

$$\ln \frac{T_i - T_s}{T_o - T_s} = - \frac{4L}{D} St$$

$$St = 1.86 \left( \frac{D}{L} \right)^{1/3} (Re Pr)^{-2/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

20.41 CONTINUED -

ASSUME  $T_{OFT} = 305 \text{ K}$   $T_{AVG} \approx 302 \text{ K}$

$$St = 1.86 \left( \frac{0.029}{2.5} \right)^{1/3} (1890 \times 12)^{-2/3} \left( \frac{0.0414}{3.72 \times 10^{-5}} \right)^{0.14}$$

$$= 1.414 \times 10^{-3}$$

$$T_L = 372 - 72 e^{-(1.414 \times 10^{-3})(4)(25)/0.0209}$$

$$= 304.7 \quad \sim \text{GOOD AGREEMENT}$$

$$q = \dot{m} c_p \Delta T = 1.47 (1.84 \times 10^3) (4.7)$$

$$= 12740 \text{ J/s} = 12.74 \text{ kW}$$

20.42  $\frac{T - T_s}{T_o - T_s} = e^{-4 \frac{L}{D} St}$

$T_o = 160 \text{ C}$  ASSUME  $T \approx 140 \text{ C}$

$T_s = 100 \text{ C}$   $T_{BULK} = 150 \text{ C}$

$$Re = \frac{4 \dot{m}}{\pi D \mu} = \frac{4 (136)}{(3600) (0.015) \pi \mu}$$

@ 423 K  $\mu = (0.0068 \times 10^{-3}) (812)$

$$Re = 116 \quad \sim \text{LAMINAR}$$

$$St = \frac{Nu}{Re Pr} = \frac{1.86}{(Re Pr)^{1/3}} \left( \frac{D}{L} \right)^{1/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

$$= \frac{1.86}{[(116)(600)]^{1/3}} \left( \frac{0.015}{15} \right)^{1/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

$$= 0.00053$$

$$e^{-4 \frac{L}{D} St} = 0.654$$

$$T = 100 + 0.654 (60) = 139.3 \text{ C}$$

GOOD!

$$T_{OFT} \approx 139 \text{ C}$$

20.43 For THIS CASE

$$\frac{T - T_{\infty}}{T_0 - T_p} = e^{-4 \frac{L}{D} \frac{U}{8 \nu c_p}}$$

$$\frac{T - T_p}{T_0 - T_p} = \frac{30}{40} = 0.75$$

$$0.2877 = 4 \frac{L}{D} \frac{U}{8 \nu c_p}$$

$$U = \frac{1}{\frac{1}{h_i A_i} + \frac{1}{h_o A_o}} = \frac{\pi D L}{\frac{1}{h_i} + \frac{1}{h_o}}$$

$$h_o = 500 \text{ W/m}^2 \cdot \text{K} \quad h_i = \frac{k}{D} \text{Nu}_i$$

$$\text{Re}_D = \frac{4 \dot{V}}{\pi D V} = \frac{4(0.006)}{\pi(0.0025)(7 \times 10^{-7})(3600)}$$

$$= 1213 \quad \left\{ \text{LAMINAR} \right\}$$

$$\text{Pr} = \frac{\mu c_p}{k} = \frac{(1000)(7 \times 10^{-7})(4000)}{0.5} = 5.6$$

$$h_i = \frac{k}{D} (1.86) \left( \text{Re}_D \text{Pr} \frac{D}{L} \right)^{1/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

$$= \frac{0.5}{0.0025} (1.86) \left[ (1213)(5.6) \left( \frac{0.0025}{L} \right) \right]^{1/3}$$

$$= 956 L^{-1/3}$$

$$U = \frac{\pi D L}{\frac{1}{500} + \frac{L^{1/3}}{956}} = \frac{L}{0.255 + 0.133 L^{1/3}}$$

PUTTING EVERYTHING TOGETHER:

$$0.2877 = \frac{0.001178 L^2}{0.255 + 0.133 L^{1/3}}$$

By TRIAL & ERROR:

$$L \hat{=} 11.7 \text{ m}$$

20.44  $T_0 = 320 \text{ K}$

$$q = \frac{\Delta T}{\Sigma R} = \frac{\Delta T}{\frac{1}{h_i A_i} + \frac{\ln r_o/r_i}{2\pi k}}$$

{ Assume  $T_0$  IS OUTSIDE TUBE TEMP }

$$\frac{1}{h_i A_i} = \frac{1}{(1700)(\pi)(0.01656)} = 1.13 \times 10^{-2}$$

$$\frac{\ln r_o/r_i}{2\pi k} = \frac{\ln \frac{1.905}{1.656}}{2\pi(110)} = 2.03 \times 10^{-4}$$

$$\Sigma R = 1.15 \times 10^{-2}$$

$$\frac{q}{L} = \frac{320 - 290}{1.15 \times 10^{-2}} = 2.606 \text{ kW/m}$$

$$\dot{m}_{\text{cond}} = \frac{2.606 \text{ kW/m}}{2393 \text{ kJ/kg}} = 1.09 \text{ g/s}$$

$$= \frac{3920 \text{ g/hr}}{3.92 \text{ kg/hr}}$$

20.45 OUTSIDE OF TUBE INSULATED

$\therefore$  ALL HEAT GENERATED GOES INTO  $\text{H}_2\text{O}$

$$q = \dot{m} c_p \Delta T = (0.12 \text{ kg/s})(4177 \text{ J/kg} \cdot \text{K})(70 - 25) \text{ K}$$

$$= 22.56 \text{ kW}$$

$$\{ q = \dot{q} V = 1.5 \times 10^6 \text{ W/m}^3 \left[ \frac{\pi}{4} (0.045^2 - 0.025^2) \right] L$$

$$= 1.649 L \text{ kW}$$

$$L = 13.68 \text{ m}$$

$$\text{CONSTANT AT FLUX} = \frac{22560 \text{ W}}{\pi(0.025)(13.68) \text{ m}^2}$$

$$= 21000 \text{ W/m}^2$$

$$h = \frac{21000}{110 - 70} = 525 \text{ W/m}^2 \cdot \text{K}$$

$$20.46 \quad \frac{T - T_s}{T_o - T_s} = e^{-\frac{4L}{D} St}$$

$$St = \frac{D}{4L} \left( \frac{100 - 120}{100 - 120} \right) = \frac{0.0117}{L}$$

a)  $U = 15 \text{ ft/s}$

$$Re = \frac{(0.015/2)(25)}{0.181 \times 10^{-3}} = 8640$$

USE GILBERT ANALOGY:  $St = \frac{C_f}{2} Pr^{-2/3}$

$$St = \frac{0.0078}{2} (1,257) = 0.0049$$

$$L = \frac{0.0117}{0.0049} = \underline{\underline{3.5 \text{ ft}}}$$

b)  $U = 15 \text{ ft/s} \quad Re = 5190$

$St = 0.00566 \quad L = \underline{\underline{3.03 \text{ ft}}}$

20.47  $Re_{crit} = \frac{4(2)(4)}{(2)(6)} = \frac{16}{6} \text{ ft}$

$$Re = \frac{(16/6)(6)}{1.28 \times 10^{-5}} = 1.25 \times 10^6$$

$$St = 0.023 Re^{-0.2} Pr^{-2/3}$$

$$= 0.023 (1.25 \times 10^6)^{-0.2} (0.703)^{-2/3}$$

$$= 0.00176$$

FOR THE SHORT DISTANCE INVOLVED:

$$h = St (800) = (0.00176)(6)(124)$$

$$= 2.53 \times 10^{-3} \frac{Btu}{s \cdot ft^2 \cdot F}$$

$$q = h A \Delta T = (2.53 \times 10^{-3})(12)(40)$$

$$= 1.215 \frac{Btu}{s} \text{ per ft} = 4370 \frac{Btu}{hr} \text{ per ft}$$

$$\Delta T = \frac{q}{\dot{m} c_p} = \frac{1.215}{(0.24)(6)(8)} = \underline{\underline{0.105 \text{ F}}}$$

20.48

$T_{in} = 290 \text{ K}$  ASSUME  $T_{out} = 350$

$T_{surf} = 370 \text{ K} \quad T_{bulk} = 320 \text{ K}$

$N = 0.596 \times 10^{-6} \text{ m}^2/s \quad Pr = 3.87$

$$\frac{T - T_s}{T_o - T_s} = e^{-\frac{4L}{D} St}$$

$$Re = \frac{(0.0254)(1.5)}{0.596 \times 10^{-6}} = 63900$$

USE DITUS-BOECOR EQUATION:

$$St = 0.023 Re^{-0.2} Pr^{-0.6}$$

$$= 0.023 (63900)^{-0.2} (3.87)^{-0.6}$$

$$= 0.00112$$

$$e^{-4L/D(St)} = 0.414$$

$$T_{out} = 370 - (0.414)(80)$$

$$\approx 337$$

SECOND TRY -  $T_{out} = 337$

$T_{bulk} = 313.5$

$N = 0.663 \times 10^{-6} \quad Pr = 4.33$

$Re = 57460 \quad St = 0.00107$

$$e^{-4L/D(St)} = 0.432$$

$$T_{out} = 370 - (0.432)(80) = \underline{\underline{335 \text{ K}}}$$

$$q = \dot{m} c_p \Delta T$$

$$= (992) \frac{\pi}{4} (0.0254)^2 (1.5)(4175)(45)$$

$$= \underline{\underline{141.6 \text{ kW}}}$$

20.49 Rect. DUCT  $0.61 \text{ m} \times 1.22 \text{ m}$

$$D_{\text{equiv}} = \frac{4(0.61 \times 1.22)}{2(0.61 + 1.22)} = 0.813 \text{ m}$$

$$q = hA\Delta T$$

USE DITUS FÖRSTER EQN.

$$Re = \frac{DG}{\mu} = \frac{(0.813)(29.4)}{1.948 \times 10^{-5}} = 1.227 \times 10^6$$

$$Pr = 0.703$$

$$h = \frac{k}{D} (0.023) Re^{0.8} Pr^{0.3}$$

$$= \frac{0.0279}{0.813} (0.023) (1.227 \times 10^6)^{0.8} (0.703)^{0.3}$$

$$= 52.8 \text{ W/m}^2 \cdot \text{K}$$

$$q = hA\Delta T = 52.8(2)(0.61 \times 1.22)(22)$$

$$= 4250 \text{ W/m}$$

$$q = \dot{m} c_p \Delta T = G A c_p \Delta T$$

$$\Delta T = \frac{4250}{29.4(0.61)(1.22)(1000)}$$

$$= 0.193 \text{ K per m}$$

20.50 DUCT:  $7.5 \text{ cm} \times 15 \text{ cm}$

$$\frac{T - T_s}{T_o - T_s} = e^{-\frac{4L}{D} St} \quad L = 6 \text{ m}$$

$$D_{\text{equiv}} = 10 \text{ cm}$$

$$\frac{T - T_s}{T_o - T_s} = \frac{30 - 70}{10 - 70} = 0.667$$

$$\frac{4L}{D} St = 0.405$$

$$St = \frac{0.405(0.10)}{4(6)} = 0.00169$$

20.50 CONTINUED -

USE Colburn Eq.

$$St = 0.023 Re^{-0.2} Pr^{-2/3}$$

$$Re = \frac{DV}{\mu} = \frac{(0.10)(5)}{1.739 \times 10^{-5}} = 5750$$

$$Pr = 0.704$$

$$0.00169 = 0.023 (5750)^{-0.2} (0.704)^{-2/3}$$

$$V = 261 \text{ m/s}$$

UNREALISTIC BUT MATHEMATICALLY CORRECT

20.51 FILMS 20.12 & 20.13 APPLY STRICTLY FOR LIQUIDS FLOWING THROUGH TUBE BANKS BUT THEY WILL BE USED FOR LACK OF OTHER RESOURCES. -

USE FILM 20.13 -

$$Re = \frac{D_t V}{\mu} = \frac{(0.018)(60)}{1.505 \times 10^{-5}}$$

$$= 7.17 \times 10^3$$

AT THIS  $Re$ :  $j \approx 0.01$

$$h = 0.01 c_p G Pr^{-2/3} \left( \frac{\mu_w}{\mu_b} \right)^{0.14}$$

$$= 0.01 (1005.5) (1.205)(6) (0.707)^{-2/3} \left( \frac{2.117}{1.813} \right)^{0.14}$$

$$= 89.6 \text{ W/m}^2 \cdot \text{K}$$

FOR A BANK OF 10 TUBES, 10 ROWS DEEP

$$A = 100(\pi)(0.018)(1.8) = 10.18 \text{ m}^2$$

$$= (89.6)(10.18)(65) = 593 \text{ kW}$$

20.52 FOR SAME CONDITIONS AS  
PROB. 20.51 EXCEPT FOR  
STAGGERED TUBE ARRANGEMENT —

$$Re = 7.17 \times 10^3$$

∴ BOTH ARRANGEMENTS GIVE  
SAME VALUE FOR  $j$

$$\therefore \underline{q = 59.3 \text{ kW}}$$

20.53 USING FIG 20.12

SAME QUESTIONS AS FOR PROB  
20.51 —

$$D_{equiv} = \frac{4}{\pi(0.013)} \left[ (0.032)(0.032) - \frac{\pi}{4} (0.013)^2 \right]$$

$$= 0.0873 \text{ m}$$

$$Re = \frac{(0.0873)(1.25)}{1.569 \times 10^{-5}} = 6.95 \times 10^3$$

— OUT OF LAMINAR RANGE —  
MUST USE FIG 20.13

$$Re = \frac{0.013(1.25)}{1.569 \times 10^{-5}} = 1.04 \times 10^3$$

FOR IN-LINE CONFIGURATION —

$$j \approx 0.017$$

$$h = 0.017(1006.3)(1.177)(1.25) \times (0.708)^{-2/3} \left( \frac{2.143}{1.813} \right)^{-0.14}$$

$$= 30.95 \text{ W/m}^2 \cdot \text{K}$$

20.53 CONTINUED —

$$A = 64(\pi)(0.013)(18) = 4.70 \text{ m}^2$$

$$q = hA\Delta T$$

$$= 30.95(4.70)(63)$$

$$= \underline{\underline{9.164 \text{ kW}}}$$

20.54

SAME CONDITIONS AS PROB  
20.53 EXCEPT TUBES ARE IN  
STAGGERED CONFIGURATION.

ALL CALCULATIONS THE SAME AS  
IN PROB 20.53 EXCEPT  $j = 0.035$

$$\text{GIVEN } h = 63.7 \text{ W/m}^2 \cdot \text{K}$$

$$\therefore q = 63.7(4.70)(63) = \underline{\underline{18.87 \text{ kW}}}$$

## CHAPTER 21

21.1 PLATE IS ASSUMED TO BE COPPER

For  $H_2O$  @ 323 K  $L = 0.565$  FT

$\rho_2 = 1.26 \times 10^3$  (F.FT)<sup>-3</sup>  $Pr = 1.81$

$\mu_L = 0.702$  LB<sub>m</sub>/HR.FT  $C_L = 1.01$  BTU/LB<sub>m</sub>.F

$k = 0.393$  BTU/HR.FT<sup>2</sup>.F  $h_{fg} = 970$  BTU/LB<sub>m</sub>

$S_L - S_v \approx S_L = 60$  LB<sub>m</sub>/FT<sup>3</sup>

NATURAL CONVECTION:  $q = h \Delta T$

$$\frac{q}{A} = \frac{k}{L} \left[ 0.68 + \frac{0.67 Ra^{1/4}}{1 + (0.492/Pr)^{1/4}} \right] \Delta T$$

$$= 60 \left[ 0.68 + 89.6 \Delta T^{1/4} \right] \Delta T \quad (1)$$

NUCLEATE BOILING:

$$\frac{C_L \Delta T}{h_{fg} Pr^{1.7}} = C_{sf} \left[ \frac{q/A}{\mu_L h_{fg}} \left( \frac{g \sigma}{g [S_L - S_v]} \right)^{1/2} \right]^{1/3}$$

$\sigma = 3.79 \times 10^{-3}$  LB<sub>f</sub>/FT  $C_{sf} = 0.013$

LHS:  $\frac{C_L \Delta T}{h_{fg} Pr^{1.7}} = 3.80 \times 10^{-4} \Delta T$

RHS:  $C_{sf} \left[ \frac{q/A}{\mu_L h_{fg}} \left( \frac{g \sigma}{g [S_L - S_v]} \right)^{1/2} \right]^{1/3}$

$$= 0.013 \left[ \frac{q/A}{0.702(970)} \sqrt{\frac{3.79 \times 10^{-3}}{60}} \right]^{1/3}$$

$$= 2.94 \times 10^{-4} (q/A)^{1/3}$$

$$\frac{q}{A} = 214 \Delta T^3 \quad (2)$$

EQUATING: (1) = (2)

$$214 \Delta T^3 = 0.6 \left[ 0.68 + 89.6 \Delta T^{1/4} \right]$$

$$\Delta T \approx 6.3 F$$

PART (b): Plot  $q/A$  FROM (1),  
 $q/A$  FROM (2), & THEIR SUM

$$21.2 \frac{C_L \Delta T}{h_{fg} Pr^{1.7}} = C_{sf} \left[ \frac{q/A}{\mu_L h_{fg}} \left( \frac{\sigma}{g \Delta S} \right)^{1/2} \right]^{1/3}$$

IN ENGLISH UNITS:

$C_L = 1.03$   $Pr^{1.7} = 2.74$

$h_{fg} = 970$

$\mu_L = 0.195 \times 10^{-3}$

$T_{SAT} = 212$

$\Delta S = 60.2$

$$A = \frac{C_L \Delta T}{h_{fg}}$$

$$B = \frac{1}{\mu_L h_{fg}} \sqrt{\frac{\sigma}{g \Delta S}}$$

$$\frac{q}{A} = \left[ \frac{h_{fg}}{A} \right]^3 \left[ C_{sf} B (2.74) \right]$$

FOR Ni & BRASS  $C_{sf} = 0.006$

" Cu & Pt  $C_{sf} = 0.013$

$T_s$  (K)  $\Delta T$  (K)  $\Delta T$  (F) A  $\sigma \times 10^3$  B

390 17 31 0.033 5.04 0.364

420 47 85 0.090 4.67 0.360

450 77 139 0.148 3.79 0.355

$T_s$   $q/A$   $h$  W/m<sup>2</sup>.K  $q/A$   $h$

390 168  $2 \times 10^5$  165  $0.533 \times 10^5$

420 3516 85 " 346  $4.07 \times 10^5$

450 16340 24 " 1610  $1.610 \times 10^5$

Ni, BRASS

Cu, Pt

$$21.3 \frac{C_L \Delta T}{h_{fg}} = 0.0709$$

$$\frac{q}{A} = \left( \frac{0.0709}{0.01235} \right)^3 = 190 \text{ BTU/s.ft}^2$$

$$= 680,000 \text{ BTU/HR.FT}^2$$

$$q = 680,000 (\pi \times 1/24)^2 = 178,000 \text{ BTU/HR}$$

$$h = \frac{680,000}{68} = 10,000 \text{ BTU/HR.FT}^2$$

21.4 BOILING  $H_2O$  @ 1 ATM; BURNOUT  
POINT IS  $\Delta T \approx 100^\circ F$ ,  $T_s = 312^\circ F$   
AS IT COOLS THE CYLINDER IS IN

FILM BOILING  $500 < T_s < 312$

NUCLEATE "  $312 < T_s < 240$

FILM BOILING PART:

$$h = 0.62 \left[ \frac{k_f^3 g_r (\Delta T) g (h_{fg} + 0.4 c_{p,v} \Delta T_s)}{D \mu_v \Delta T_s} \right]^{1/4}$$

$$k_f = 0.0145 \text{ Btu/hr ft }^\circ F$$

$$g_r = 0.0372 \text{ lbm/ft}^3 \quad h_{fg} = 970 \text{ Btu/lbm}$$

$$g_L = 60.0 \quad c_{p,v} = 0.451 \text{ Btu/lbm }^\circ F$$

$$\mu_v = 3.12 \times 10^{-3} \text{ lbm/hr ft}$$

SUBSTITUTING INTO FORMULA:

$$h = 35.9 \text{ Btu/hr ft}^2 \text{ }^\circ F \quad \text{Avg } \Delta T \approx 194^\circ F$$

$$\frac{Q}{A} = h(194) = 35.9(194) = 6960 \text{ Btu/hr ft}^2$$

NUCLEATE BOILING PART:

$$\frac{C \Delta T}{h_{fg}} = C_{sf} \left[ \frac{g/A}{\mu_L h_{fg}} \left( \frac{5}{g \Delta T} \right)^{1/2} \right]^{1/3} Pr^{1/4}$$

$$\Rightarrow \frac{Q}{A} = \left( \frac{0.0168 \Delta T}{C_{sf}} \right)^3 = 4.8 \times 10^{-6} (\Delta T)^3$$

$$\text{WITH } \Delta T = 64^\circ F \quad C_{sf} = 0.013 \sim \text{Cu} \\ = 0.006 \text{ Br, Ni}$$

$$\frac{Q}{A} = 5.72 \times 10^5 \sim \text{Cu}$$

$$= 5.82 \times 10^6 \sim \text{Br, Ni}$$

$$\frac{Q}{A} = \frac{SVC_p \Delta T}{A \Delta T}$$

$$t = S \left( \frac{V}{A} \right) c_p \left[ \frac{\int_{312}^{500} \Delta T}{g/A|_f} + \frac{\int_{240}^{312} \Delta T}{g/A|_{nuc}} \right]$$

21.4 CONT.

$$t = S \frac{D}{4} c_p \left( \frac{188}{g/A|_f} + \frac{72}{g/A|_{nuc}} \right)$$

$$S \frac{D}{4} c_p = 555 \left( \frac{1}{96} \right) (0.092) = 0.532 \text{ Cu}$$

$$= 532 \left( \frac{1}{96} \right) (0.091) = 0.503 \text{ Br}$$

$$= 556 \left( \frac{1}{96} \right) (0.111) = 0.643 \text{ Ni}$$

$$\text{Copper } t = 0.532 \left[ \frac{188}{6960} + \frac{72}{5.72 \times 10^5} \right] = 52 \text{ S}$$

$$\text{Br: } t = 0.503 \left[ \frac{188}{6960} + \frac{72}{5.8 \times 10^5} \right] = 48.9 \text{ S}$$

$$\text{Ni: } t = 0.643 \left[ \frac{188}{6960} + \frac{72}{5.8 \times 10^5} \right] = 62.6 \text{ S}$$

21.5 USING ENGLISH UNITS:

$$A = \pi D L + 2 \frac{\pi D^2}{4} = \pi (0.02)(0.15) + \frac{\pi}{4} (2)(0.02)^2$$

$$= 0.1082 \text{ ft}^2$$

$$\frac{Q}{A} = \frac{500(3.413)}{0.1082} = 15800 \text{ Btu/hr ft}^2$$

ASSUME NUCLEATE BOILING:

$$\frac{1}{(1.8)^{1.7} (970)} = 0.006 \left[ \frac{15800}{(0.195 \times 10^{-3}) (970) (3600)} \times (3.79 \times 10^{-3})^{1/2} \right]^{1/3}$$

$$\Delta T = 9.0^\circ F$$

$$\text{SURFACE Temp.} = 221^\circ F$$

$$h = \frac{15800}{9} = 1760 \text{ Btu/hr ft}^2 \text{ }^\circ F$$

$$21.6 \quad h = 0.62 \left[ \frac{k_V^3 \rho_V \Delta T g (h_{fg} + 0.4 C_{pV} \Delta T)}{D \mu_V (T_s - T_{SAT})} \right]^{1/4}$$

$$= 0.62 \left[ \frac{(0.0153)^3 (0.0341 \times 40 / 14.7) (584)}{1/12 (0.914 \times 10^{-5}) (933)} \right]^{1/4}$$

$$\times 32.2 (3600) (934 + 0.4 \times 0.481 \times 933)$$

$$= 26.9 \text{ Btu/hr ft}^2 \text{ F}$$

$$\dot{Q} = h A \Delta T = 26.9 (1200 - 267) = 25,000 \text{ Btu/hr ft}^2$$

$$21.7 \quad \Delta T = 2200 - 240 = 1960 \text{ F} \left\{ \begin{array}{l} \text{FILM} \\ \text{BOILING} \end{array} \right\}$$

$$h = 0.62 \left[ \frac{(0.0155)^3 (0.035) (58.9) (32.2) (3600)}{(0.02/12) (1.53 \times 10^{-5}) (1960)} \right]^{1/4}$$

$$\times (952 + 0.4 \times 0.483 \times 1960)$$

$$= 43.3 \text{ Btu/hr ft}^2 \text{ F}$$

$$\dot{Q} = h A \Delta T = 43.3 (\pi) \left( \frac{0.2}{12} \right) (1) (1960)$$

$$= 444 \frac{\text{Btu}}{\text{hr}} \text{ PER FT}$$

$$21.8 \quad 2000 \text{ W} = 6826 \text{ Btu/hr}$$

PER PLATE:  $A = 2(0.05)(0.1) = 0.01 \text{ m}^2$

$$= 0.1076 \text{ ft}^2$$

$$\frac{\dot{Q}}{A} = 20,000 \text{ W/m}^2 = 63400 \text{ Btu/hr ft}^2$$

IT APPEARS THAT NUCLEATE BOILING ON ONE PLATE CAN ACHIEVE THIS.

$$T_{SAT} = 242 \text{ F}$$

$$\frac{C_{L\Delta T}}{h_{fg} Pr^{1.7}} = C_{sf} \left[ \frac{\dot{Q}/A}{\mu_L h_{fg}} \left( \frac{\sigma}{g \Delta \rho} \right)^{1/2} \right]^{1/3}$$

21.8 CONT.

$$\frac{\Delta T}{945(2.12)} = 0.013 \left[ \frac{63400}{0.167 \times 10^{-3} (970) (3600)} \right. \\ \left. \times \left( \frac{3.79 \times 10^{-3}}{59.1} \right)^{1/2} \right]^{1/3}$$

$$\Delta T = 24.9 \text{ F} \quad \left\{ \begin{array}{l} \text{OK} \\ \text{1 PLATE} \\ \text{WILL DO IT} \end{array} \right\}$$

21.9 PLOTS REQUIRED

PROCEDURE:

$T_{SURF} (K) \quad \Delta T (K)$

600	227	FILM BOILING *
500	127	" "
400	27	NUCLEATE **

\* USE EQN (21-7)

\*\* USE EQN (21-5)

$$\dot{Q} = h A \Delta T$$

$h_{\text{FILM B.}} \text{ VARIES AS } \Delta T^{-1/4}$

$h_{\text{NUC. B.}} \text{ " " } \Delta T^2$

$\frac{\dot{Q}}{A} \text{ FILM VARIES AS } \Delta T^{3/4}$

$\frac{\dot{Q}}{A} \text{ NUC " " } \Delta T^3$

PLATE IS LUMPED

$$\frac{\dot{Q}}{A} = Sc_p \frac{V}{A} \frac{\Delta T}{\Delta t} \quad \Delta T = \frac{\dot{Q}/A}{Sc_p V} \Delta t$$

21.10 IDEA HERE IS TO RETARD BOILING SUCH THAT INTERNAL PRESSURE WILL NOT BE TOO LARGE

$$q = \frac{(3 \times 10^6 \text{ Btu/hrft}^2)}{3600} \pi \left(\frac{3}{48}\right) (4) = 655 \text{ Btu/s}$$

for  $P = 1 \text{ ATM}$   $h_{fg} = 970 \text{ Btu/lbm}$

$$\dot{m}_{\text{EVAPORATION}} = \frac{655}{970} = 0.676 \text{ lbm/s}$$

$$= \frac{0.676}{0.0372} = 18.2 \text{ FT}^3/\text{s}$$

VOLUME OF PIPE { ALSO VOLUME OF FLUID }

$$= \pi \left(\frac{3}{48}\right)^2 (4) = 0.0123 \text{ FT}^3$$

$$\text{FLOW AREA} = \frac{\pi}{4} \left(\frac{3}{48}\right)^2 = 0.0031 \text{ FT}^2$$

FOR  $\dot{V}_W \left\{ \frac{\text{FT}^3}{\text{s}} \right\}$  OF FLUID ENTERING

$\dot{V}_W + 18.2$  TOTAL FLUID EXITS

$$\dot{V}_W = 18.2 \left( \frac{0.0372}{59.5} \right) \approx 0.0144 \text{ FT}^3/\text{s}$$

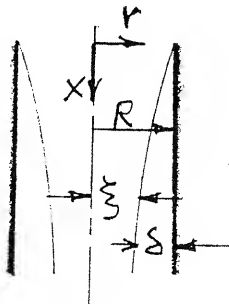
$$\dot{V}_W + 18.2 \approx 18.2 \text{ FT}^3/\text{s} \text{ OUT}$$

$$V_{\text{EXIT}} = \frac{18.2}{0.0031} = 5870 \text{ FT/s}$$

SUPERSONIC!

~ INCREASE PIPE DIAMETER, OR ADD ADDITIONAL PIPES IN PARALLEL, OR DECREASE  $q$  ~

21.11



{ NUSSELT FILM MODEL FOR FLOW INSIDE A CYLINDER }

21.11 CONT.

AT EQUILIBRIUM  $\sum F_x = 0$

UPWARD FORCE =  $T \Delta x 2\pi r = -\mu \frac{dv}{dr} 2\pi r \Delta x$  (VISCOS)

DOWNWARD FORCE =  $sg \pi (r^2 - s^2) \Delta x$

( $s < r < R$ )

$$\Rightarrow sg(r^2 - s^2) = -\mu \frac{dv}{dr} 2r$$

SEPARATING VARIABLES:  $dv = -\frac{sg}{2\mu} \left(r - \frac{s^2}{r}\right) dr$

$$v = -\frac{sg}{2\mu} \left(\frac{r^2}{2} - s^2 \ln r\right) + C$$

B.C.  $v(R) = 0 \Rightarrow C = \frac{sg}{\mu} \left(\frac{R^2}{2} - s^2 \ln R\right)$

$$v = \frac{sg}{2\mu} \left[ \frac{R^2 - r^2}{2} - s^2 \ln \frac{R}{r} \right]$$

AT ANY  $x$ , MASS FLOW RATE IS

$$\begin{aligned} \Gamma &= \int_s^R sv(2\pi r) dr \\ &= \frac{\pi s^2 g}{\mu} \int_s^R \left[ \frac{R^2}{2} - \frac{r^2}{2} - s^2 \ln \frac{R}{r} \right] dr \\ &= \frac{\pi s^2 g}{\mu} \left[ \frac{R^4}{8} - \frac{s^2 R^2}{2} + \frac{3s^4}{8} + \frac{s^2}{2} \ln R \right. \\ &\quad \left. - \frac{s^4}{2} \ln s \right] \end{aligned}$$

RATE OF HEAT FLOW TO WALL ~ THROUGH CONDENSATE ~

$$= \frac{2\pi k}{\ln r/s} \Delta x (T_v - T_w)$$

AMOUNT OF CONDENSATE IN DISTANCE  $\Delta x$

$$= \frac{\partial}{\partial x} \Gamma \Delta x = \frac{\partial}{\partial s} \Gamma \frac{ds}{dx} \Delta x = \frac{\partial \Gamma}{\partial s} ds$$

21.11 CONT

$$\frac{d\Gamma}{dx} = \frac{2\pi s^2 g}{\mu} \ln \eta \left[ -\xi R^2 + \xi^3 + 2\xi^3 \ln R - 2\xi^2 \ln \xi \right]$$

$$\therefore \text{RATE OF HEAT FLOW TO COOL WATER} \\ = \frac{2\pi s^2 g}{\mu} \ln \eta \left[ -\xi R^2 + \xi^3 + 2\xi^3 \ln R - 2\xi^2 \ln \xi \right] d\xi$$

EQUATING HEAT FLOW RATES -

$$\frac{k dx}{\ln r/g} (T_o - T_w) = \frac{s^2 g \ln \eta}{\mu} \left[ -\xi R^2 + \xi^3 + 2\xi^3 \ln R - 2\xi^2 \ln \xi \right] d\xi$$

SEPARATING VARIABLES & INTEGRATING

$$A \int_0^x dx = \int_R^\xi [ ] d\xi$$

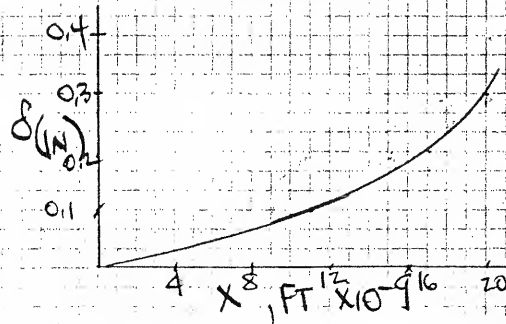
$$Ax = f(\xi/R) = f(\eta)$$

SOLVING {messy} WE GET, FOR  $X(\eta)$

$$X(\text{FT}) = (2.42 \times 10^9) \left[ (123 - 3.14 \ln \eta) \eta^4 + (184 - 0.5 \ln \eta) \eta^2 - 9.07 \right]$$

$\eta$	$\ln \eta$	$\eta^2$	$\eta^4$	$X \times 10^9$ (FT)	$\delta$ (IN)
0.9	-0.1	0.81	0.654	6.3	0.05
0.8	-0.22	0.64	0.409	11.2	0.10
0.6	-0.508	0.36	0.1294	17.3	0.20
0.4	-0.913	0.16	0.0256	20.2	0.30
0.2	-1.607	0.04	0.0016	21.6	0.40
0				22.0	0.50

21.11 CONT



21.12 a) VERTICAL TUBE

$$h = 0.943 \left\{ \frac{s_L g \Delta T k^3 \left[ \ln \eta + \frac{3}{8} C_{PL} \Delta T \right]}{L \mu \Delta T} \right\}^{1/4} \\ = 0.943 \left\{ \frac{961.2 (9.81) (0.679)^3 (0.961)}{1 (297 \times 10^6) (9)} \times \right. \\ \left. \times \left[ 2.25 \times 10^6 + \frac{3}{8} (4206) (9) \right] \right\}^{1/4}$$

$$= 6600 \text{ W/m}^2 \cdot \text{K}$$

$$\dot{m}_{\text{COND.}} = \frac{6600 (9) (\pi) (15) (1)}{225 \times 10^6} = \underline{\underline{0.0125 \text{ kg/s}}}$$

b) HORIZONTAL

$$h = 0.725 \left\{ \frac{s_L g \Delta T k^3 \left[ \ln \eta + \frac{3}{8} C_{PL} \Delta T \right]}{\mu \Delta T} \right\}^{1/4} \\ = 7100 \text{ W/m}^2 \cdot \text{K} \quad \dot{m} = \underline{\underline{0.0146 \text{ kg/s}}}$$

21.13 NEGLECT  $\Delta T$  ACROSS TUBE

$$h_i = \frac{k}{D_i} 0.023 \text{Re}_D^{0.8} \text{Pr}^{1/3}$$

$$h_o = 0.725 \left\{ \frac{s_L g \Delta T k^3 \left[ \ln \eta + \frac{3}{8} C_{PL} \Delta T \right]}{\mu \Delta T} \right\}^{1/4}$$

$$\text{For } h_i \quad T_{\text{in}} = 20^\circ \text{C} \quad T_w = ? \quad T_b = \frac{20 + T_{\text{out}}}{2}$$

$$\text{For } h_o \quad \text{PROPERTIES EVALUATED AT } T_f = \frac{100 + T_w}{2}$$

2.1.3 CONT.

$$q = \frac{\Delta T_{\text{OVERALL}}}{\sum R} = \frac{\Delta T_i}{R_i} = \frac{\Delta T_o}{R_o}$$

A MESSY TRIAL & ERROR PROBLEM  
- AFTER QUOTE A BIT OF WORK -

ASSUMING  $T_{i, \text{OUT}} = 36^\circ\text{C} = 309\text{ K}$

"  $T_{w, \text{AVG}} = 58^\circ\text{C} = 331\text{ K}$

GIVING  $T_{b, \text{AVG}} = 28^\circ\text{C} = 301\text{ K}$

$$R_{e_i} = \frac{D \nu S}{\mu} = \frac{4 \dot{m}}{\pi D \mu} = \frac{4(4000)}{\pi(0.0165)(863 \times 10^{-6}) \times (3600)}$$

$$= 99000$$

$$Pr = 5.95 \quad k = 0.611$$

$$h_i = \frac{0.611}{0.0165} (0.023)(99000)^{0.8} (5.95)^{0.4}$$

$$= 17200 \text{ W/m}^2 \cdot \text{K}$$

$$h_o = 0.725 \left\{ \frac{971.8(9.81)(971.8)(0.673)^3}{(352 \times 10^{-6})(0.019)(42)} \times \left[ 2.25 \times 10^6 + \frac{3}{8} (4194)(42) \right] \right\}^{1/4}$$

$$= 8960 \text{ W/m}^2 \cdot \text{K}$$

$$R_i = \frac{1}{17200(\pi)(0.0165)(2)} = 5.61 \times 10^{-4}$$

$$R_o = \frac{1}{8690(\pi)(0.019)(2)} = 9.35 \times 10^{-4}$$

$$\sum R = 14.96 \times 10^{-4}$$

$$\Delta T_{\text{TOTAL}} = 72\text{ K} \quad \Delta T_i \approx 27\text{ K} \quad \Delta T_o \approx 45\text{ K}$$

$$q = \frac{27}{R_i} = \frac{45}{R_o} = 48000 \text{ W}$$

2.1.3 CONT.

$$q = \dot{m} c_p \Delta T = \frac{4000}{3600} (4180) \Delta T$$

$$\Delta T \approx 10.4\text{ K}$$

$$\Rightarrow T_{w, \text{OUT}} \approx 303.5\text{ K} \quad T_{b, \text{AVG}} \approx 298\text{ K}$$

~ CLOSE TO ORIGINAL ASSUMPTION

FINALLY!  $h_{\text{H}_2\text{O}} = 17200 \text{ W/m}^2 \cdot \text{K}$  (a)

$h_{\text{CONDENS}} = 8690$  " (b)

$T_{w, \text{OUT}} = 303.5\text{ K}$  (c)

$\dot{m}_{\text{COND}} = \frac{48000}{2.25 \times 10^6} = 0.0213 \text{ kg/s}$  (d)

2.1.4 Flow Rate =  $\frac{0.042}{0.586} = 0.0717 \text{ m}^3/\text{s}$

Per m

ALLOWABLE WIDTH =  $\frac{0.0717}{(15)(1)} = 0.00478 \text{ m}$   
= 0.478 cm

$$0.478 + 2.8 = 3.278 \text{ cm}$$

$$\delta = 0.261 \text{ cm} = 0.00261 \text{ m}$$

Film Model:

$$\delta^4 = \left[ \frac{4 k \mu \times \Delta T}{g_L g \Delta \rho \left( h_{fg} + \frac{3}{8} c_p \Delta T \right) \right]$$

$$= \frac{4(0.392)(0.195 \times 10^{-3})(59) \times}{(59.5)(322)(59.5)(993.5)(3600)}$$

~ {ALL ENGLISH UNITS}

$$\delta^4 = 4.425 \times 10^{-14}$$

$$X = \frac{121,500 \text{ FT}}{37000 \text{ m}}$$

$$= 3.28 \text{ m}$$

$$21.15 \quad \frac{q}{A} = k_L \frac{\Delta T}{y} = 8 h_{fg} \frac{dy}{dt}$$

{AT A GIVEN DEPTH, y}

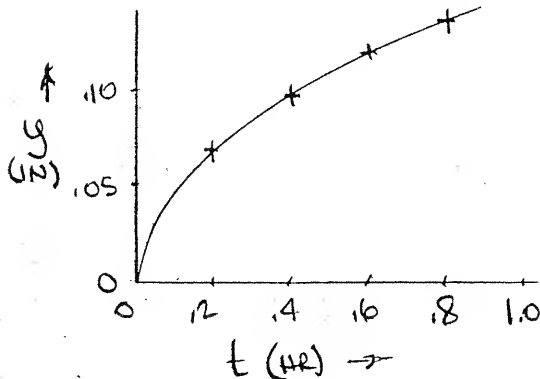
$$\int_0^y y dy = \frac{k_L \Delta T}{8 h_{fg}} \int_0^t dt$$

$$\frac{y^2}{2} = \frac{k_L \Delta T}{8 h_{fg}} t$$

$$y^2 = \frac{2(0.392)(12)}{60.1(970)} t = 1.614 \times 10^{-4} t$$

t, Hours      y  
ft  $\times 10^2$       INCHES

0	0	0
0.2	0.586	0.0681
0.4	0.804	0.0964
0.6	1.0	0.12
0.8	1.14	0.137
1.0	1.27	0.152



21.16 HORIZONTAL CYLINDER:

$$h_H = 0.725 \left[ \frac{8.9 \Delta T k^3 \left[ h_{fg} + \frac{3}{8} C_{PL} \Delta T \right]}{\mu \Delta T} \right]^{1/4}$$

$$= 0.725 \left[ \frac{61.3(32.2)(4.3)(0.0562)(3400)(997)}{0.29 \times 10^{-3} (0.0656)(45)} \right]^{1/4}$$

$$= 397 \frac{\text{Btu}}{\text{hr ft}^2 \text{F}} = 2250 \text{ W/m}^2 \cdot \text{K}$$

$$h_V = h_H \left[ 1.3 \left( \frac{D}{L} \right)^{1/4} \right] = 1000 \text{ W/m}^2 \cdot \text{K}$$

$$21.16 \text{ CONT. } \dot{m}_{\text{cond}} = \frac{h_A \Delta T}{h_{fg}}$$

$$= 2250(\pi)(0.02)(1.5)(25)$$

$$2.32 \times 10^6$$

$$= 2.29 \times 10^{-3} \text{ kg/s}$$

HORIZONTAL

$$= 1.02 \times 10^{-3} \text{ "}$$

VERTICAL

$$21.17 \quad h_{\text{avg}} = \bar{h} \left( \frac{1}{8} \right)^{1/4} = \frac{h}{1.681}$$

HORIZONTAL TUBE CASE { SEE PROB 21.16 }

$$h_{\text{HORIZ}} = 2250 \text{ W/m}^2 \cdot \text{K}$$

$$h_{\text{AVG}} = 2250 / 1.681 = 1341 \text{ W/m}^2 \cdot \text{K}$$

$$q = h_{\text{avg}} A \Delta T$$

$$= 1341(8)(\pi)(0.02)(1.5)(25)$$

$$= 25.3 \text{ kW}$$

21.18 SINGLE HORIZONTAL TUBE:

$$h = 0.725 \left\{ \frac{8.9 \Delta T k^3 \left( h_{fg} + \frac{3}{8} C_{PL} \Delta T \right)}{D \mu \Delta T} \right\}^{1/4}$$

$$= 0.725 \left\{ \frac{(60.1)(32.2)(60.1)(0.392)(1015)(100)}{5/96 (0.206 \times 10^{-3})(100)} \right\}^{1/4}$$

$$= 1600 \text{ Btu/hr ft}^2 \text{F}$$

$$21.19 \quad h_{\text{avg}} = h \left( \frac{1}{8} \right)^{1/4} = h / 1.681$$

$$h_{\text{HORIZ}} = 2250 \text{ W/m}^2 \cdot \text{K} \quad \left\{ \text{FROM PROB 21.16} \right\}$$

$$\text{FOR BANK: } h_{\text{avg}} = \frac{2250}{1.681} = 1341 \text{ W/m}^2 \cdot \text{K}$$

21.19 CONT. For n TUBES -

$$q = h_{avg, n} n A_{TUBE} \Delta T$$

$$= h_{avg, n-1} (n-1) A_{TUBE} \Delta T + h_n A_{TUBE} \Delta T$$

$$\bar{T}_n n A \Delta T = \bar{T}_{n-1} (n-1) A \Delta T + h_n A \Delta T$$

$$n^{th} TUBE: h_n = n \bar{T}_n - (n-1) \bar{T}_{n-1}$$

$$Top Tube: h_1 = 2250 W/m^2 \cdot K$$

$$3^{rd} TUBE: \bar{T}_2 = 1890 W/m^2 \cdot K$$

$$\bar{T}_3 = 1710 "$$

$$h_3 = 3(1710) - 2(1890) = 1350 W/m^2 \cdot K$$

$$8^{th} TUBE: \bar{T}_8 = 1341 W/m^2 \cdot K$$

$$\bar{T}_7 = 1383 "$$

$$h_8 = 8(1341) - 7(1383) = 1047 W/m^2 \cdot K$$

$$21.20 \frac{4A \Gamma_c}{P \mu_f} = Re_c = 2000$$

$$\frac{4A \Gamma_c}{P \mu_f} = \frac{4 \frac{K}{m} \frac{m}{s}}{\frac{m}{s}} \frac{1}{\mu} = \frac{4}{P \mu} h \Gamma_c \Delta T$$

$$L = \frac{\mu h \Gamma_c}{4 h \Delta T} (2000)$$

$$= \frac{(0.0206 \times 10^{-3})(970)(2000)}{4(100)[2250(1.3)(0.02)]^{1/4}} (3600)$$

$$L^{3/4} = 3.27 \quad L = 4.85 FT$$

21.21

$$h_c = 0.943 \left[ \frac{S_{LG} AS (h_{fg} + \frac{3}{8} C_{pL} \Delta T)}{L \mu \Delta T} \right]^{1/4}$$

$$= 0.943 \left[ \frac{37.2(32.2)(37.2)(0.294)^3(505)}{2(14 \times 10^{-5})(25)} \right]^{1/4}$$

$$= 694 Btu/hr ft^2 F$$

$$q = h A \Delta T = 694(2)(25)$$

$$= 34,700 Btu/hr Per foot OF WIDTH$$

$$21.22 \quad q = \frac{k \Delta y}{y} = Sh_{fg} \frac{dy}{dt} \quad \left\{ \text{For Thickness} \right\}$$

$$\int_0^t dt = \frac{Sh_{fg}}{k \Delta T} \int_0^y y dy = \frac{Sh_{fg}}{k \Delta T} \frac{y^2}{2}$$

$$t = 60,2(972)(0.02/0.3048)^2$$

$$0.390(39.6)(2)$$

$$= 16.3 \text{ Hours} - \text{IF PAN IS HORIZONTAL}$$

FOR PAN INCLINED!



$$\text{Per UNIT DEPTH: } Vol = \frac{1}{2} \frac{0.02}{\tan \theta} (0.02)$$

$$Vol = \frac{2 \times 10^{-4} L}{\tan \theta} \quad m^3$$

$$\text{IF } \theta = 10^\circ \quad Vol = 1.34 \times 10^{-3} m^3$$

$$30^\circ \quad " = 3.46 \times 10^{-4} "$$

LENGTH OF SURFACE EXPOSED!

$$\text{IF } \theta = 10^\circ \quad \text{LENGTH} - 40 \text{ TO } 28.7 \text{ cm}$$

$$\theta = 30^\circ \quad " \quad 40 \text{ TO } 36.5 "$$

21.22 CONT.

ASSUMES: ACCUMULATION OF CONDENSATE  
DUE PRINCIPALLY TO CONDENSATION ON  
EXPOSED SURFACE

$$h = 0.943 \left[ \frac{\rho_L g \sin \theta k^3 \Delta T (h_{fg} + \frac{3}{8} C_{PL} \Delta T)}{L \mu \Delta T} \right]^{1/4}$$

$$= \frac{0.943}{(L/\sin \theta)^{1/4}} \left[ \frac{(60)(32.2)(0.392)^3 (60)(995)(3600)}{0.20 \times 10^{-3} (40)} \right]$$

$$= 1252 \left( \frac{L}{\sin \theta} \right)^{-1/4}$$

$$@ \theta = 10^\circ \quad L_{AVG} = \frac{40 + 28.7}{2} = 33.8 \text{ cm} = 1.109 \text{ FT}$$

$$30^\circ \quad " = \frac{40 + 36.5}{2} = 38.3 \text{ " } = 1.257 \text{ "}$$

$$h_{10} = 788 \text{ Btu/hr ft}^2 \text{ F} \quad h_{30} = 994 \text{ Btu/hr ft}^2 \text{ F}$$

$$\dot{m} = \frac{h A \Delta T}{h_{fg}} = \frac{h L_{AVG} (40)}{980}$$

$$\dot{m}_{10} = 35.7 \text{ lbm/hr} = 0.572 \text{ FT}^3/\text{hr}$$

$$\dot{m}_{30} = 51.0 \text{ " } = 0.817 \text{ "}$$

$$t = V/\dot{V}$$

$$t_{10} = \frac{1.34 \times 10^{-3}}{0.572} / (0.3048)^2 = 0.0252 \text{ hr}$$

$$= 1.51 \text{ MIN}$$

$$\approx 91 \text{ S}$$

$$t_{30} = 0.00456 \text{ hr} = 0.274 \text{ MIN}$$

$$\approx 16.4 \text{ S}$$

21.23

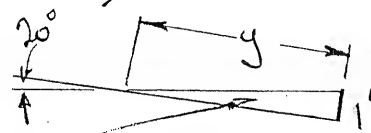
$$h = 0.943 \left[ \frac{\rho_L g \sin \theta k^3 \Delta T (h_{fg} + \frac{3}{8} C_{PL} \Delta T)}{L \mu \Delta T} \right]^{1/4}$$

$$= 0.943 \left[ \frac{(0.601)(32.2)(\sin 20^\circ)(0.392)^3 (60.1)}{10.65/12 (0.206 \times 10^{-3})(30)} \times (981)(3600) \right]^{1/4}$$

$$= 1050 \text{ Btu/hr ft}^2 \text{ F}$$

$$\dot{m} = \frac{h A \Delta T}{h_{fg}}$$

$$= \frac{1050 (1)(30)}{970} = 32.5 \text{ lbm/hr}$$



$$Vol = \frac{1}{2} \left( \frac{1}{12} \right) \left( \frac{1}{\tan 20^\circ} \right) \left( \frac{1}{12} \right)$$

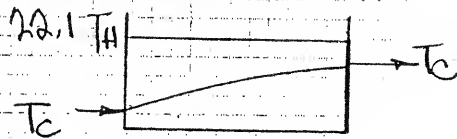
$$= 0.00955 \text{ FT}^3$$

$$t = \frac{V}{\dot{V}} = \frac{0.00955}{32.5/624}$$

$$= 0.183 \text{ Hour}$$

$$= 1.10 \text{ MIN.}$$

## CHAPTER 22



$$q = \dot{m} c_p \Delta T = U A \Delta T_{LM}$$

$$U A = \text{CONST.} = q / \Delta T_{LM}$$

$$U = \frac{1}{\frac{1}{h_i} + \frac{A_i}{2\pi r_i L} \ln \frac{r_o}{r_i} + \frac{A_o}{2\pi r_o L} \ln \frac{r_o}{r_i}} \approx h_i$$

NEGL.      NEGL.

$$\Rightarrow h_i A_i = \text{CONST.}$$

USING DITTUS-BOELTER CORRELATION

$$h_i = \frac{k}{D} (\text{CONST.}) Re^{0.8} Pr^{0.4} = k \left( \frac{D \rho v}{\mu} \right)^{0.8}$$

$$= \frac{k \left( \frac{D \rho v}{\mu} \right)^{0.8}}{D} = (\text{CONST.}) D^{-1.8}$$

$$\Rightarrow h A = \text{CONST.} = A (\text{CONST.}) D^{-1.8}$$

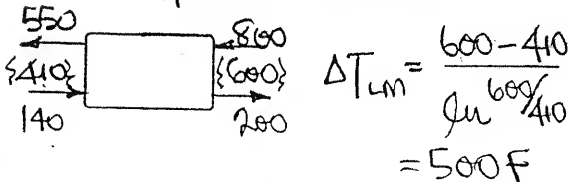
AS DIAMETER INCREASES THE REQUIRED AREA INCREASES AS  $D^{1.8}$

$$22.2 \quad q = \dot{m} c_p \Delta T_w = \dot{m} c_p \Delta T_{pre}$$

$$10^5 (1) (60) = 10^5 (0.24) \Delta T_{pre}$$

$$\Delta T_{pre} = 250 \text{ F} = 800 - T_{pre, out}$$

$$T_{pre, out} = 550 \text{ F}$$



$$\Delta T_{LM} = \frac{600 - 410}{\ln \frac{600}{410}} = 500 \text{ F}$$

$$A = \frac{q}{U \Delta T_{LM}} = \frac{10^5 (1) (60)}{12 (500)} = 1000 \text{ ft}^2$$

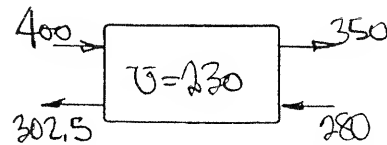
$$22.3 \quad \text{Oil: } T_{in} = 400 \text{ K} \quad T_{out} = 350 \text{ K}$$

$$\dot{m} = 2 \text{ kg/s} \quad q = 1880 \text{ J/kg} \cdot \text{K}$$

$$q = \dot{m} c_p \Delta T = 2 (1880) (50) = 188000 \text{ W}$$

$$\Delta T_w = \frac{q}{\dot{m} c_p} = \frac{188000}{2 (4187)} = 22.5 \text{ K}$$

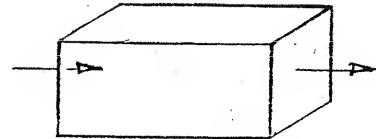
$$T_{w, in} = 280 \text{ K} \quad T_{w, out} = 302.5 \text{ K}$$



$$\Delta T_{LM} = \frac{97.5 - 70}{\ln \frac{97.5}{70}} = 83 \text{ K}$$

$$A = \frac{q}{U \Delta T_{LM}} = \frac{188000}{230 (83)} = 9.85 \text{ m}^2$$

22.4



$$D_{equiv} = \frac{4 (0.1) (0.2)}{2 (0.1 + 0.2)} = 0.0667 \text{ m}$$

$$T_{b, avg} = 295 \text{ K} \quad T_f = 345 \text{ K}$$

$$Re = 0.0667 \frac{800}{\mu} = 0.0667 \frac{\dot{m}}{A \mu}$$

$$q = h A \Delta T_{LM}$$

$$\Delta T_{LM} = \frac{105 - 95}{\ln \frac{105}{95}} = 99.9 \text{ K}$$

## 22.4 CONTINUED

Assuming Turbulent flow:

$$\dot{m} c_p \Delta T = 850 \left[ 0.023 k^{-0.2} \mu^{-0.4} \right] A_s \Delta T_{LM}$$

$$\dot{m}(10) = \frac{\dot{m}}{A} \left[ 0.023 \left( \frac{0.0667}{0.205 \times 10^{-5}} \frac{\dot{m}}{A} \right)^{-0.2} \right. \\ \left. \times (0.698)^{-2/3} \right] (2)(0.3)(2.5) 999$$

Solving for  $\dot{m}$ :  $\dot{m} = 105 \text{ kg/s}$

$$\dot{q} = \dot{m} c_p \Delta T = 105 (1009)(10) = 1060 \text{ kW}$$

## 22.5 CONTINUED

$$C_o = C_{min}$$

$$\frac{UA}{C_{min}} = \frac{50(290)}{4500} = 3.22$$

$$\frac{C_{min}}{C_{max}} = 0.625$$

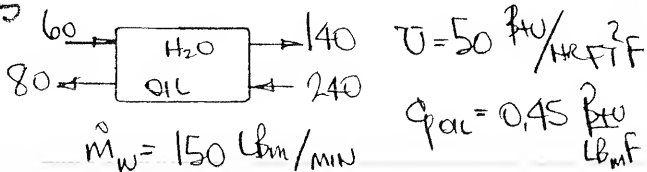
$$\text{Fig 22.12} \sim \epsilon \approx 0.86$$

$$\dot{q} = \epsilon C_{min} (180) = 7200 \Delta T$$

$$\Delta T_w = \frac{0.86(4500)(180)}{7200} \approx 97 \text{ F}$$

$$T_{w \text{ EXT}} = 157 \text{ F}$$

## 22.5



$$a) \dot{q} = 150(1)(80) = 12000 \text{ Btu/min}$$

$$= UA \Delta T_{LM}$$

$$\Delta T_{LM} = \frac{100 - 20}{\ln \frac{100}{20}} = 49.7 \text{ F}$$

$$A = \frac{\dot{q}}{U \Delta T_{LM}} = \frac{12000(60)}{50(49.7)} = 290 \text{ ft}^2$$

b) WATER IN SHELL; OIL ~ 2 PASSES

$$Y = \frac{80 - 240}{60 - 240} = 0.889$$

$$Z = \frac{60 - 140}{80 - 240} = 0.5$$

$A = \infty \sim \text{CAN'T BE DONE}$

$$c) \dot{q} = \dot{m} c_p \Delta T|_w = \dot{m} c_p \Delta T|_o$$

$$C_o = \frac{12000(60)}{160} = 75(60) \\ = 4500 \text{ Btu/hr.F}$$

$$22.6 \Delta T_w = 340 - 255 = 85 \text{ K}$$

$$\Delta T_o = 350 - 305 = 45 \text{ K}$$

$$\dot{q} = \epsilon C_{min} (350 - 255) = C_w (85)$$

$$\epsilon = 85/95 = 0.895$$

$$22.7 \text{ WATER } T_{in} = 50 \text{ F}$$

$$\dot{m} = 400 \text{ lbm/hr}$$

$$c_p = 1 \text{ Btu/lb.m.F}$$

$$\text{OIL: } T_{in} = 250 \text{ F } c_p = 0.45 \text{ Btu/lb.m.F}$$

$$\dot{m} = ?$$

$$U = 60 \text{ Btu/hr.ft}^2 \text{ F } A = 18 \text{ ft}^2$$

$$T_{w \text{ OUT, MAX}} = 212 \text{ F } T_{o \text{ OUT, MAX}} = 160 \text{ F}$$

$$\dot{q} = \dot{m}_w c_{pw} \Delta T_w = \dot{m}_o c_{po} \Delta T_o$$

$$= 400(1)(T_w - 50) = \dot{m}_o(0.45)(250 - T_o)$$

$$= \epsilon C_{min} (200)$$

# 22.7 CONTINUED -

For  $T_{wout} = 212$   $\Delta T_w = 162$

$$q = 400(162) = 64800$$

$$= C_o \Delta T_o = \epsilon C_{min}(200)$$

$\{ q = \dot{m}_o c_{p_o} \Delta T_o, \therefore \text{MAX } \dot{m}_o \text{ will be ASSOCIATED WITH MINIMUM } \Delta T_o \}$

If  $T_{oout \text{ MAX}} = 160 \sim \Delta T_{o \text{ MIN}} = 90$

For  $H_2O$  AS MINIMUM FLUID:

$$C_{min} = 400$$

$$400(162) = \epsilon (400)(200)$$

$$\epsilon = 0.81$$

$$NTU = UA / C_{min} = \frac{60(18)}{400} = 2.7$$

$\{ \text{FIG 22.12 a } C_{min} / C_{max} \approx 0.65 \}$

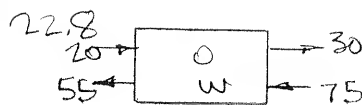
$$C_{max} = \dot{m} c_{p_o} = \frac{400}{0.65} = 615$$

$$\Delta T_o = \frac{q}{C_o} = \frac{64800}{615} = 105.3 \text{ F}$$

$T_{oout} = 144.7 - \text{OK}$

FINALLY:  $\dot{m}_{max} C_p = 615$

$$\dot{m}_{max} = \frac{615}{0.45} = \underline{\underline{1370 \text{ Lbm/hr}}}$$



$$\dot{m}_o = 12 \text{ kg/s}$$

$$c_{p_o} = 2.2 \text{ kJ/kg}\cdot\text{K}$$

$$c_{p_w} = 4.18$$

$$U = 1080 \text{ W/m}^2\cdot\text{K}$$

(PROBLEM STATEMENT)  
USES CH AS  $c_{p_w}$  &  
 $C_C$  AS  $C_{p_o}$

$$q = UAF \Delta T_{LM}$$

$$= \dot{m}_o c_{p_o} \Delta T_o = \dot{m}_w c_{p_w} \Delta T_w$$

$$= (12)(2200)(10) = \dot{m}_w (4180)(20)$$

$$\rightarrow \dot{m}_w = 3.16 \text{ kg/s}$$

$$\Delta T_{LM} = \frac{45-35}{\ln 45/35} = 39.8$$

To FIND F:  $Y = 10/55 = 0.182$

$$Z = 20/10 = 2$$

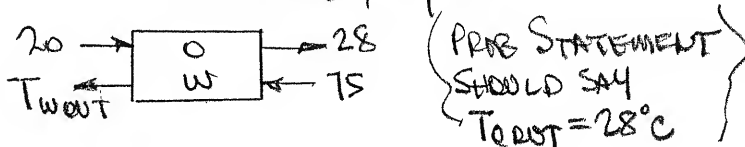
FIG 22.9 a:  $F \approx 1$

$$F \Delta T_{LM} = 39.8 \text{ C}$$

$$A = \frac{q}{U F \Delta T_{LM}} = \frac{(12)(2200)(10)}{1080(39.8)}$$

$$= \underline{\underline{6.14 \text{ m}^2}}$$

22.9 SAME EXCHANGER & ENTRANCE CONDITIONS AS PROB 22.8 -



(PROB STATEMENT)  
SHOULD SAY  
 $T_{oout} = 28^\circ\text{C}$

$$q = \dot{m}_o c_{p_o} \Delta T_o = 12(2200)(8)$$

$$= \dot{m}_w c_{p_w} \Delta T_w = 3.16(4180) \Delta T_w$$

$$\Delta T_w \approx 16^\circ\text{C} \quad T_{wout} \approx 59^\circ\text{C}$$

22.9 CONTINUED -

$$\Delta T_{LM} = \frac{47-39}{\ln 47/39} = 42.9^\circ\text{C}$$

To FIND F:  $Y = 8/55 = 0.145$   
 $Z = 16/8 = 2$

Fig 22.9a -  $F \approx 1$

$$U = \frac{q}{A F \Delta T_{LM}} = \frac{(12)(2200)(8)}{6.14(42.9)} = 802 \text{ W/m}^2 \cdot \text{K}$$

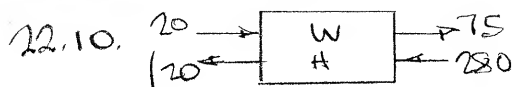
$$UA|_0 = \frac{1}{\sum R_0} \quad UA|_1 = \frac{1}{\sum R_1}$$

$$\sum R_0 = \frac{1}{(1080)(6.14)} = 1.508 \times 10^{-4}$$

$$\sum R_1 = \frac{1}{(802)(6.14)} = 2.031 \times 10^{-4}$$

Fouling Resistance

$$= \sum R_1 - \sum R_0 = 0.523 \times 10^{-4} \text{ K/W}$$



$$\dot{m}_w = 2.7 \text{ kg/s} \quad U = 160 \text{ W/m}^2 \cdot \text{K}$$

$$q = \dot{m}_w c_{pw} \Delta T_w = (2.7)(4200)(55)$$

$$= \dot{m}_A (1200)(160)$$

$$\dot{m}_A = 3.25 \text{ kg/s} \quad \text{a)}$$

$$q = U A F \Delta T_{LM}$$

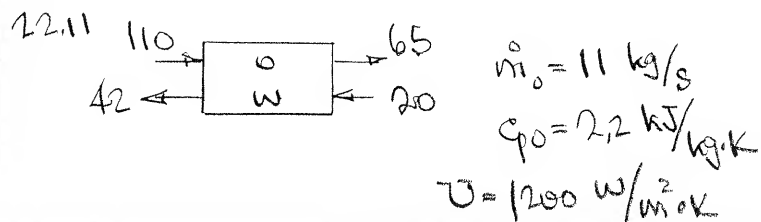
$$\Delta T_{LM} = \frac{205-100}{\ln 205/100} = 146.3^\circ\text{C}$$

22.10 CONTINUED -

To FIND F:  $Y = 55/260 = 0.211$   
 $Z = 160/55 = 2.91$

Fig 22.10a  $F \approx 0.96$

$$A = \frac{q}{U F \Delta T_{LM}} = \frac{(2.7)(4200)(55)}{(160)(0.96)(146.3)} = 27.8 \text{ m}^2 \quad \text{b)}$$



$$\Delta T_{LM} = \frac{68-35}{\ln 68/35} = 49.7^\circ\text{C}$$

$$q = \dot{m}_w c_{pw} \Delta T_w = \dot{m}_w (4200)(22)$$

$$= \dot{m}_o c_{po} \Delta T_o = (11)(2200)(45)$$

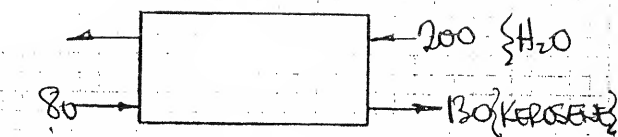
$$\dot{m}_w = 11.79 \text{ kg/s} \quad \text{a)}$$

To FIND F:  $Y = \frac{110-65}{110-20} = 0.5$   
 $Z = \frac{42-20}{110-65} = 0.489$

Fig 22.9a ~  $F \approx 1$

$$A = \frac{q}{U F \Delta T_{LM}} = \frac{11(2200)(45)}{(1200)(1)(49.7)} = 18.26 \text{ m}^2 \quad \text{b)}$$

22.12



$$\dot{m}_k = 2500 \text{ lbm/hr} \quad \dot{m}_w = 900 \text{ lbm/hr}$$

$$U = 260 \text{ Btu/hr-ft}^2\text{-F}$$

$$\Delta T_w = \frac{2500(130-80)(0.5)}{900(1)} = 70.8$$

$$T_{wout} = 129 \text{ F} \quad \Delta T_{LM} = \frac{70-49}{\ln(70/49)} = 58.9 \text{ F}$$

$$AF = \frac{q}{U \Delta T_{LM}} = \frac{2500(50)(0.51)}{58.9(260)} = 4.16$$

FIGURE 22.9 a

$$\left. \begin{aligned} Y &= \frac{130-80}{260-80} = \frac{50}{120} = 0.416 \\ Z &= \frac{260-129}{130-80} = \frac{71}{50} = 1.4 \end{aligned} \right\} F \approx 0.83$$

$$A = 4.16 / 0.83 = \underline{\underline{5.01 \text{ ft}^2}}$$

22.13 INPUT DATA - Set Prob 22.3

$$U = 230 \text{ W/m}^2\text{-K} \quad T_{oil,in} = 400 \text{ K}$$

$$A = 9.85 \text{ m}^2 \quad T_{w,in} = 280 \text{ K}$$

$$(a) \text{ CROSSFLOW: } C_o = \dot{m} c_p = 3760 \text{ J/kg}\cdot\text{s}$$

$$C_w = 8374 \quad "$$

$$\left. \begin{aligned} \frac{UA}{C_{min}} &= \frac{230(9.85)}{3760} = 0.603 \\ C_{min}/C_{max} &= 0.45 \end{aligned} \right\} \epsilon = 0.43$$

$$q = \epsilon C_{min}(400-280) = 194000 \text{ W}$$

$$T_{wout} = 280 + \frac{194000}{8374} = 303 \text{ K}$$

$$T_{oat} = 400 - \frac{194000}{3760} = \underline{\underline{248 \text{ K}}}$$

22.13 CONTINUED -

(b) SHELL AND TUBE

SAME DATA -  $\epsilon = 0.40$ 

$$q = 0.4(3760)(120) = 180500 \text{ W}$$

$$T_{wout} = 301.5 \text{ K} \quad T_{oat} = \underline{\underline{352 \text{ K}}}$$

22.14



$$q = \dot{m} c_p \Delta T_w = C_w \Delta T_w$$

$$C_w = 3A U c_p = \frac{2 \times 10^8}{18} = 1.11 \times 10^7$$

$$UA/C_{min} = \frac{4600 A}{1.11 \times 10^7} = 4.14 \times 10^{-4} A$$

$$A = n \pi D L = n \pi \left( \frac{1.37}{12} \right) L = 0.359 n L$$

$$\frac{UA}{C_{min}} = 1.485 \times 10^{-4} n L$$

NEGLECTING TUBE RESIST

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} \quad \frac{1}{h_i} = \frac{1}{4600} - \frac{1}{10600}$$

$$h_i = 8130 \quad Nu_i = \frac{8130(1.37/12)}{0.015} = 1509$$

USING DITUS-BOECHE EON.

$$Nu = 1509 = 0.023 \left[ \frac{8U(1.37/12)}{8.25 \times 10^{-6}} \right]^{0.8} (5.65)^{0.4}$$

$$8U = \frac{q}{A \phi \Delta T} = \frac{2 \times 10^8}{\frac{\pi D^2}{4} (479)(18)}$$

$$= 3192 \quad n = 81 \text{ TUBES}$$

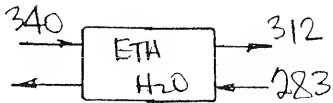
22.14 CONTINUED -

$$q = \epsilon C_{\min}(65) \quad \epsilon = 0,271$$

$$\text{From 22.2 c } \frac{UA}{C_{\min}} \approx 0,38$$

$$L = \frac{0,38}{(1,48 \times 10^{-4})(81)} = \underline{\underline{31,7 \text{ m}}}$$

22.15



$$\dot{m}_{\text{ETH}} = 6,93 \text{ kg/s}$$

$$q = \dot{m} c_p \Delta T = 6,93(3810)(28)$$

$$= \dot{m} c_p \Delta T_w = 6,30(4182)\Delta T$$

$$\Delta T_w = 28,1$$

(a) COUNTERFLOW:  $\Delta T_{\text{LM}} \approx 29 \text{ F}$

$$A = \frac{q}{U \Delta T_{\text{LM}}} = \frac{6,93(3810)(28)}{568(29)} = \underline{\underline{44,9 \text{ m}^2}}$$

(b) PARALLEL FLOW:

$$\Delta T_{\text{LM}} = \frac{57 - 0,9}{\ln 57/0,9} = 13,52$$

$$A = \frac{q}{U \Delta T_{\text{LM}}} = \frac{96,3 \text{ m}^2}{568(13,52)} = \underline{\underline{96,3 \text{ m}^2}}$$

(c) CROSSFLOW:

$$C_{\text{MIXED}} = \dot{m} c_{p,w} = 26350$$

$$C_{\text{UNMIXED}} = \dot{m} c_{p,e} = 26403$$

$$Y = \frac{312 - 240}{283 - 340} = 0,491 \quad Z = \frac{28,1}{28} \approx 1$$

$$F \approx 0,85 \quad A = \frac{44,9}{0,85} = \underline{\underline{52,8 \text{ m}^2}}$$

22.16

$$q = C_A \Delta T_A = C_W \Delta T_W = 95000 \frac{\text{Btu}}{\text{hr}}$$

$$C_W = \frac{95000}{35} = 2720 \frac{\text{Btu}}{\text{hr}}$$

$$C_A = \frac{95000}{80} = 1188 \text{ " } \sim C_{\min}$$

$$\frac{C_{\min}}{C_{\max}} = \frac{1188}{2720} = 0,437 \quad \epsilon = \frac{q}{C_{\min}(110)} \approx 0,73$$

a) COUNTERFLOW:  $\frac{UA}{C_{\min}} \approx 1,65$

$$V = \frac{A}{130} = \frac{1,65(1188)}{30(130)} = \underline{\underline{0,502 \text{ ft}^3}}$$

b) CROSSFLOW - AIR MIXED

$$\frac{UA}{C_{\min}} \approx 2$$

$$V = \frac{A}{100} = \frac{2(1188)}{40(100)} = \underline{\underline{0,593 \text{ ft}^3}}$$

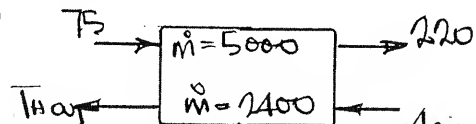
c) CROSSFLOW - BOTH MIXED

$$\frac{UA}{C_{\min}} \approx 1,75$$

$$V = \frac{A}{90} = \frac{1,75(1188)}{50(90)} = \underline{\underline{0,462 \text{ ft}^3}}$$

CONFIGURATION (c) IS MOST COMPACT

22.17



$$\bar{U} = 300 \text{ W/m}^2 \cdot \text{K} = 52,8 \frac{\text{Btu}}{\text{hr ft}^2 \cdot \text{F}}$$

$$q = 5000(1)(45) = 2400(1)\Delta T_H$$

$$\Delta T_H = 302 \quad T_{\text{Hout}} = 98 \text{ F}$$

$$\Delta T_{\text{LM}} = \frac{180 - 23}{\ln 180/23} = 76,3 \text{ F}$$

22.17 CONTINUED -

FOR COOL FLUID IN TUBES:

$$\left. \begin{aligned} Y &= \frac{220-75}{400-75} = 0.446 \\ Z &= \frac{302}{145} = 2.08 \end{aligned} \right\} F \approx ?$$

HOT FLUID IN TUBES:

$$\left. \begin{aligned} Y &= \frac{-302}{-325} = 0.929 \\ Z &= \frac{145}{302} = 0.480 \end{aligned} \right\} F \approx ?$$

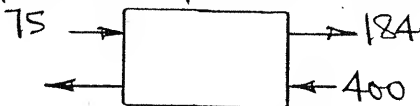
BOTH ARE OFF THE CHARTS

NEITHER IS POSSIBLE ~  
CAN'T USE THIS CONFIGURATION

22.18 IF COUNTERFLOW:

$$AU_{old} = \frac{1}{\left[ \frac{1}{A h_{ci}} + R_f + \frac{1}{A h_{co}} \right]} = 36.7(300)$$

FOR NEW OPERATING CONDITIONS:



$$\begin{aligned} Q &= 5000(1)(184-75) = 2400(1)\Delta T_H \\ &= 545000 \quad \Delta T_H = 227 \quad T_{Hout} = 173 \end{aligned}$$

$$\Delta T_{LM} = \frac{216-98}{\ln \frac{216}{98}} = 149.3$$

$$AU_{new} = \frac{Q}{\Delta T_{LM}} = \frac{545000}{149.3} = 3650$$

$$= \frac{1}{\left[ \frac{1}{A h_{ci}} + R_f + \frac{1}{A h_{co}} \right] + R_f}$$

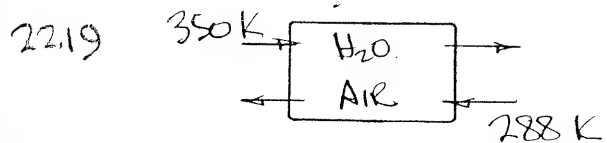
$$AU_{new} = \frac{1}{\frac{1}{36.7(300)} + R_f}$$

22.18 CONTINUED -

$$\frac{1}{36.7(300)} + R_f = \frac{1}{3650}$$

$$R_f = 1.83 \times 10^{-4} \text{ K/W}$$

$$\begin{aligned} \text{OR } AR_f &= 36.7 R \\ &= 6.72 \times 10^{-3} \text{ m}^2 \cdot \text{K/W} \end{aligned}$$



$$h_w = 470 \text{ W/m}^2\text{K} \quad \dot{m}_w = 10 \text{ kg/s}$$

$$h_A = 210 \text{ " } \quad \dot{m}_A = 16$$

$$\begin{aligned} Q &= (10 \text{ kg/s})(4181 \text{ J/kg}\cdot\text{K})(350 - T_{wout}) \\ &= (16 \text{ kg/s})(1007 \text{ J/kg}\cdot\text{K})(T_{Aout} - 288) \end{aligned}$$

$$C_w = 41810 \quad C_A = 16112 = C_{min}$$

$$C_{min}/C_{max} = 0.385$$

$$\begin{aligned} A &= \pi O L (50) = \pi (0.026)(6.7)(50) \\ &= 27.36 \text{ m}^2 \end{aligned}$$

{ ASSUMES TOTAL LENGTH OF EACH  
TUBE IS 67 m }

$$U = \frac{1}{\frac{1}{h_{ci}} + R_{cond} + \frac{1}{h_{co}}} = \frac{1}{\frac{1}{470} + \frac{1}{210}} = 145$$

$$UA/C_{min} = \frac{145(27.36)}{16112} = 0.246$$

$$\underline{\underline{\epsilon \approx 0.20}}$$

22.19 CONTINUED -

$$q = \epsilon C_{\min} (T_{wi} - T_{ci})$$

$$= 0,2 (16112) (62) = \underline{199800 \text{ W}}$$

$$T_{wout} = \underline{345,2 \text{ K}} \quad T_{Aout} = \underline{300,4 \text{ K}}$$

FOR FOULING RESISTANCE = 0.0021

$$U = \frac{1}{\frac{1}{470} + \frac{1}{210} + 0.0021} = 111,2$$

$$UA/C_{\min} = 0,188 \quad \epsilon \approx 0,10$$

$$q = 0,1 (16112) (62) = \underline{99900 \text{ W}}$$

$$T_{wout} = \underline{347,6} \quad T_{Aout} = \underline{294,2}$$

22.20  $311 \text{ K} \rightarrow 3,8 \text{ kg/s} \rightarrow 328 \text{ K}$   
 $333 \text{ K} \leftarrow 1,9 \text{ " } \leftarrow 367 \text{ K}$

$$U = 1420 \text{ W/m}^2 \cdot \text{K}$$

$$\text{Tubes: } ID = 0,01905 \text{ m}$$

$$U = 0,366 \text{ m/s}$$

$$L_{\max} = 2,44 \text{ m}$$

$$q = \dot{m} c_p \Delta T_{shell} = \dot{m} c_p \Delta T_{tubes}$$

$$\Delta T_s = \frac{3,8 (17)}{1,9} = 34$$

$$C_{\min} = 1,9 (4180) = 7942 \text{ W/K}$$

$$\rho = 983 \text{ kg/m}^3$$

$$\dot{m} = \rho A V = 3,8 = n (983) \left( \frac{\pi}{4} \right) (0,01905) (0,366)$$

$$n = \underline{37 \text{ Tubes}}$$

22.20 CONTINUED -

$$q = \epsilon C_{\min} (367 - 311) \quad \frac{C_{\min}}{C_{\max}} = 0,5$$

$$\epsilon = \frac{C_{\min} (34)}{C_{\min} (56)} = 0,607$$

$$UA/C_{\min} \approx 1,3 \quad \left\{ \text{fig. 22.12 c} \right\}$$

$$A = \frac{7942 (1,3)}{1420} = 7,27 \text{ m}^2$$

$$= \frac{\pi}{4} (0,01905) L$$

$$L = 1,64 \text{ m}$$

2 TUBE PASSES WILL WORK  
 37 TUBES PER PASS  
 L = 1,64 m PER PASS

22.21  $NTU = 1,25$

$$C_{\min}/C_{\max} = 0 \quad \epsilon \approx 0,72$$

$$q = \epsilon C_{\min} (T_{w,in} - T_{c,in})$$

$$= 0,72 (0,07) (4,18) (93)$$

$$= 19,59 \text{ kW}$$

$$= C_w \Delta T_w = 4,18 (0,07) \Delta T_w$$

$$\Delta T_w = \frac{0,72 (0,07) (4,18) (93)}{4,18 (0,07)} = 67 \text{ K}$$

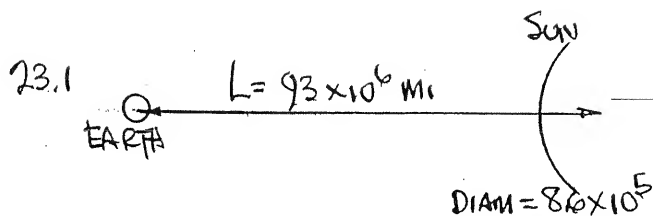
$$T_{wout} = 280 + 67 = \underline{347 \text{ K}}$$

22.21 CONTINUED -

STEAM CONDENSATION RATE

$$\begin{aligned}\dot{m}_{\text{cond}} &= \dot{Q}/h_{\text{fg}} = \frac{19.59}{2256} \\ &= \underline{\underline{8.68 \times 10^{-3} \text{ kg/s}}}\end{aligned}$$

# CHAPTER 23



RADIANT EMISSION FROM SUN =  $A_s E_{bs}$

ALL PASSES THROUGH A SPHERICAL SURFACE OF RADIUS,  $L$ .

AT THE EARTH

$$\frac{q}{A} = \frac{\pi D_s^2 E_{bs}}{4\pi L^2} = \left(\frac{D}{2L}\right)^2 E_{bs}$$

FLUX AT EARTH =  $360 + 90 = 450$  BTU HR FT<sup>2</sup>

$$450 = \left[ \frac{8.6 \times 10^5}{2(93 \times 10^6)} \right]^2 \sigma T_s^4$$

$T_s = 10530$  R

23.2

$0 < \lambda < 0.35 \mu$	$T = 0$
$0.35 < \lambda < 2.7 \mu$	$T = 0.92$
$2.7 < \lambda$	$T = 0$

For  $T = 5800$  K

$\lambda_1 T = 2030$        $F = 0.072$

$\lambda_2 T = 15660$        $F = 0.972$

$\Delta F = 0.90$

PER CENT  $T_x = 0.90(0.92) = 0.828$   
 $\approx 83\%$

For  $T = 300$  K:

$\lambda_1 T = 105$        $F \approx 0$   
 $\lambda_2 T = 810$        $F \approx 0$  }  $\Delta F \approx 0$

PERCENT  $T_x = 0$

23.3

FROM WIEN'S DISPLACEMENT LAW:

$\lambda_{max} T = 5215.6 \mu R$

$\lambda_{max} = \frac{5215.6}{4000} = 1.304 \mu$

FRACTION IN VISIBLE BAND!

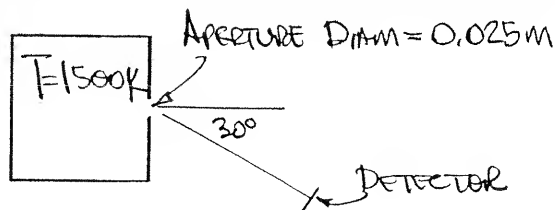
$$= \frac{\int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda} = \frac{1}{\sigma T^4} \left( E_b \Big|_0^{\lambda_2 T} - E_b \Big|_0^{\lambda_1 T} \right)$$

{TABLE 23.1}  $\lambda_1 T = 667 \mu K$   $F \approx 0$

$\lambda_2 T = 1667 \mu K$   $F = 0.0256$

OR 2.56%

23.4



(a)  $I = \frac{q}{A_A \cos \theta_A \omega}$

$I = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi} = 9.137 \times 10^4 \text{ W/m}^2 \text{Sr}$

$\omega = \frac{A_D}{r^2} = \frac{0.001 \text{ m}^2}{1 \text{ m}^2} = 0.001 \text{ Sr}$

$q = I A_A \cos \theta_A \omega$   
 $= (9.137 \times 10^4) \left( \frac{\pi}{4} \right) (0.025)^2 \cos 30^\circ \times (0.001)$

$= 3.88 \times 10^{-2} \text{ W}$

23.4 CONTINUED -

(b) WITH WINDOW

$$q = T I_{A, \omega} \theta_A \omega$$

$$\begin{aligned} T &= \frac{\int_0^\infty T_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} \\ &= \frac{\int_0^\infty T_\lambda E_{\lambda, b} d\lambda}{\int_0^\infty E_{\lambda, b} d\lambda} \\ &= 0.8 \int_0^\infty \frac{E_{\lambda, b}}{E_b} d\lambda + 0 \int_2^\infty \sim \\ &= 0.8 F(0-2\mu\text{m}) \end{aligned}$$

$$\lambda T = 2(1500) = 3000$$

$$F_{0-2\mu} = 0.273$$

$$T = 0.8(0.273) = 0.218$$

$$\begin{aligned} q &= 0.218(3.88 \times 10^{-2}) \\ &= \underline{8.47 \times 10^{-3} \text{ W}} \end{aligned}$$

23.5 CONTINUED -

$$T_s = 0.9(0.966 - 0.033) = \underline{0.84}$$

TINTED GLASS:

$$\lambda_1 T = 0.5(5800) = 2900 \quad F = 0.25$$

$$\lambda_2 T = 1.5(5800) = 8700 \quad F = 0.881$$

$$T = 0.9(0.881 - 0.25) = \underline{0.568}$$

IN THE VISIBLE RANGE:

$$\lambda_1 = 0.38 \mu\text{m} \quad \lambda_2 = 0.76 \mu\text{m}$$

{FOR TINTED GLASS  $\lambda_1 = 0.5 \mu$ }

$$\lambda_1 = 0.38 \quad F_{0-\lambda_1 T} = 0.1017$$

$$\lambda_1 = 0.5 \quad F_{0-\lambda_1 T} = 0.250$$

$$\lambda_2 = 0.76 \quad F_{0-\lambda_2 T} = 0.550$$

$$\begin{aligned} \text{PLAIN GLASS: } T &= 0.9(0.550 - 0.1017) \\ &= \underline{0.404} \end{aligned}$$

$$\begin{aligned} \text{TINTED GLASS: } T &= 0.9(0.550 - 0.250) \\ &= \underline{0.27} \end{aligned}$$

$$\begin{aligned} 23.5 \quad T_{\text{SOLAR}} &= T_\lambda F_{\lambda_1-\lambda_2} \\ &= T_\lambda (F_{0-\lambda_2} - F_{0-\lambda_1}) \end{aligned}$$

FOR SOLAR IRRADIATION:

PLAIN GLASS:

$$\lambda_1 T = 0.3(5800) = 1740 \quad F = 0.033$$

$$\lambda_2 T = 2.5(5800) = 14500 \quad F = 0.966$$

$$\begin{aligned} 23.6 \quad \lambda_1 &= 0.8 \mu\text{m} \\ \lambda_2 &= 5 \mu\text{m} \quad F_{\lambda_1 T - \lambda_2 T} = F_{0-\lambda_2 T} - F_{0-\lambda_1 T} \end{aligned}$$

$T, K$	$F_{0-\lambda_1 T}$	$F_{0-\lambda_2 T}$	$F_{\lambda_1 T - \lambda_2 T}$
500	0	0.1613	0.1613
2000	0.0197	0.9142	0.8945
3000	0.1402	0.9689	0.8287
4500	0.4036	0.9896	0.586



$$23.7 \quad T = 5800 \text{ K}$$

$$\text{for } \lambda_1 = 0.4 \mu\text{m} \quad \lambda_1 T = 2320$$

$$\lambda_2 = 0.7 \mu\text{m} \quad \lambda_2 T = 4060$$

$$F_{0-\lambda_1 T} = 0.1220 \quad F_{0-\lambda_2 T} = 0.4916$$

$$\left. \begin{array}{l} \text{FRACTION IN} \\ \text{VISIBLE RANGE} \end{array} \right\} = \underline{\underline{0.3696}}$$

$$\text{IN UV RANGE: } 0.01 < \lambda < 0.4$$

$$F_{0-\lambda_1 T} = 0 \quad F_{0-\lambda_2 T} \approx 0.12$$

$$\left. \begin{array}{l} \text{FRACTION IN} \\ \text{UV RANGE} \end{array} \right\} \approx \underline{\underline{0.12}}$$

$$\text{IN IR RANGE } 0.4 < \lambda < 10$$

$$F_{0-\lambda_1 T} = 0.12 \quad F_{0-\lambda_2 T} = 1.00$$

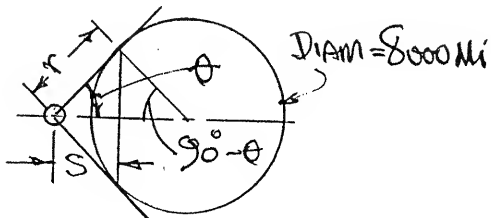
$$\left. \begin{array}{l} \text{FRACTION IN} \\ \text{IR RANGE} \end{array} \right\} \approx \underline{\underline{0.88}}$$

WIEN'S LAW -

$$\lambda_{\text{max}} T = 2897.6 \mu\text{m} \cdot \text{K}$$

$$\lambda_{\text{max}} \approx \underline{\underline{0.500 \mu\text{m}}}$$

23.8



$$\theta = \sin^{-1} \frac{4000}{4500} = 62.7^\circ = 1.096 \text{ RAD}$$

$$S = 4500 - 4000 \cos(90^\circ - \theta) = 945 \text{ mi}$$

$$r = S / \cos \theta = 2060 \text{ mi}$$

23.8 CONTINUED -

AREA SUBTENDED BY EARTH

$$\Delta A = \int_0^\theta 2\pi r^2 \sin \theta d\theta = 2\pi r^2 (1 - \cos \theta)$$

$$= 2\pi r^2 (0.541) \text{ mi}^2$$

$$F_{S-E} = \frac{\Delta A}{4\pi r^2} = 0.271$$

$$F_{S-SPACE} = 0.729$$

$$\text{INCIDENT SOLAR ENERGY} = 450 \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2}$$

{from Prob 23.1}

$$q_{\text{SUN-SAT}} = 450 \left( \frac{\pi}{4} \right) \left( \frac{50}{12} \right)^2 = 6150 \frac{\text{Btu}}{\text{hr}}$$

$$q_{\text{ABSORBED BY SAT}} = 0.3 (6150) = 1845 "$$

$$q_{\text{REFLECTED}} = 4305 \text{ Btu/hr}$$

$$q_{E-SAT} = \epsilon A_s F_{S-E} \sigma T_E^4$$

$$= 0.195 \left[ (\pi) \left( \frac{50}{12} \right)^2 \right] (0.271) (0.1714) (5.1)^4$$

$$= 164 \text{ Btu/hr}$$

$$q_{\text{ABSORBED}} = 0.05 (164) = 8.2 \text{ Btu/hr}$$

$$q_{\text{REFLECTED}} = 155.8 \text{ Btu/hr}$$

$$q_{\text{EMITTED BY SAT.}} = 0.05 (0.1714) \left( \frac{T_s}{100} \right)^4 \pi \left( \frac{50}{12} \right)^2$$

$$= 0.467 \left( \frac{T_s}{100} \right)^4$$

ENERGY BALANCE:

$$6150 + 164 = 4305 + 155.8 + 0.467 \left( \frac{T_s}{100} \right)^4$$

$$\underline{\underline{T_s = 794 \text{ R} = 334 \text{ F}}}$$

23.9

$$\begin{aligned}
 \frac{q}{A}|_{\text{NET}} &= \frac{q}{A}|_{\text{IN}} - \frac{q}{A}|_{\text{OUT}} \\
 &= 1000 - h(T_s - T_\infty) - \epsilon\sigma\left[\left(\frac{T_s}{100}\right)^4 - \left(\frac{T_\infty}{100}\right)^4\right] \\
 &= 1000 - 12(30 - 20) - 5.676(0.3)(106) \\
 &= \underline{\underline{862 \text{ W/m}^2}}
 \end{aligned}$$

23.10

ENERGY BALANCE FOR COLLECTOR:

$$\begin{aligned}
 \dot{q}_{\text{IN}} &= 800 \text{ A W} \\
 \dot{q}_{\text{OUT}} &= \dot{q}_{\text{RAD}} + \dot{q}_{\text{CONV}} \\
 &= \sigma A (T^4 - T_\infty^4) + hA (T - T_\infty)
 \end{aligned}$$

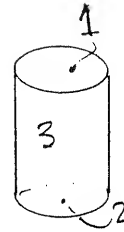
EQUATING:

$$\begin{aligned}
 800 &= \sigma (T^4 - T_\infty^4) + h(T - T_\infty) \\
 &= 5.676 \left(\frac{T}{100}\right)^4 - 5.676(3.03)^4 \\
 &\quad + 35T - 10605 \\
 \left(\frac{T}{100}\right)^4 + 6.17T &= 2094
 \end{aligned}$$

By TRIAL & ERROR:  $T = \underline{\underline{322 \text{ K}}}$ 

$$\begin{aligned}
 \dot{q}_{\text{RAD}} &= \sigma A (T^4 - T_\infty^4) \\
 &= 5.676(60) \left[3.22^4 - 3.03^4\right] \\
 &= \underline{\underline{7910 \text{ W}}}
 \end{aligned}$$

23.11



$$F_{12} = 0.12 \quad \left\{ \begin{array}{l} F_{16} \\ 23.14 \end{array} \right\}$$

$$F_{13} = 0.88$$

$$\begin{aligned}
 F_{31} = F_{32} &= \frac{A_1 F_{13}}{A_3} \\
 &= \frac{\pi D^2}{4} \frac{0.88}{\pi D L} = 0.17
 \end{aligned}$$

$$F_{3-\text{SURF}} = 0.34$$

$$\begin{aligned}
 \dot{q}_{3-\text{SURF}} &= \pi(0.075)(0.1)(0.34)(5.676)(7^4 - 3.1^4) \\
 &= \underline{\underline{105 \text{ W}}}
 \end{aligned}$$

23.12 ENTIRE HOLE INTERIOR IS SURF 2  
OPENING (SURROUNDINGS) IS "1"

$$F_{12} = 1 \quad A F_{12} = \frac{\pi D^2}{4} (1)$$

$$\begin{aligned}
 \dot{q}_{21} &= A_2 F_{21} \epsilon \sigma (T_2^4 - T_1^4) = A F_{12} \epsilon \sigma (T_2^4 - T_1^4) \\
 &= \frac{\pi}{4} (0.075)^2 (1)(5.676)(7^4 - 3.1^4) \\
 &= \underline{\underline{57.9 \text{ W}}}
 \end{aligned}$$

$$23.13 \quad \frac{q}{A} = \frac{1200 \text{ W}}{5(0.49 \text{ m}^2)} = 490 \text{ W/m}^2$$

$$= \epsilon \sigma \left[ \left(\frac{T}{100}\right)^4 - 2.8^4 \right]$$

$$490 = 0.7(5.676) \left[ \left(\frac{T}{100}\right)^4 - 2.8^4 \right]$$

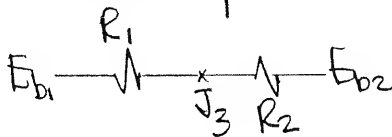
$$T = \underline{\underline{369 \text{ K}}}$$

23.14 WITH NO INTERVENING PLATE:

$$q_{12} = A F_{12} \sigma (T_1^4 - T_2^4)$$

$$\frac{q}{A} = 5.676 \left[ 9^4 - 5.8^4 \right] = \underline{\underline{30.8 \text{ kW/m}^2}}$$

WITH INTERVENING PLATE PRESENT:



PER UNIT AREA:

$$\frac{q}{A} = \frac{E_{b1} - E_{b2}}{\frac{1}{F_{13}} + \frac{1}{F_{23}}} = \frac{E_{b1} - E_{b2}}{2}$$

$$= \underline{\underline{15.4 \text{ kW/m}^2}}$$

$$\frac{q}{A} = (E_{b1} - J_3) F_{13} = (J_3 - E_{b2}) F_{32}$$

$$J_3 = \sigma T_3^4 - \frac{q}{A} = 15.4 \text{ kW/m}^2$$

$$= \sigma T_3^4$$

$$T_3^4 = \frac{15.4 \times 10^3}{\sigma} \quad T_3 = \underline{\underline{722 \text{ K}}}$$

{ EMISSIVITY OF INTERVENING PLATE }  
HAS NO EFFECT

23.15 FILAMENT AT 2910 K

$$q = 100 \text{ W}$$

$$\lambda_{\max} = \frac{2897.6}{2910} = \underline{\underline{0.999 \mu\text{m}}} \quad \text{a)}$$

VISIBLE RANGE:  $0.38 < \lambda < 0.76$

$$\lambda T_1 = 0.38(2910) = 1102 \quad F_{0-\lambda T} = 0.0009$$

$$\lambda T_2 = 0.76(2910) = 2204 \quad F_{0-\lambda T} = 0.1017$$

$$\text{FRACTION IN V.R.} = \underline{\underline{0.1008}} \quad \text{b)}$$

23.16 FOR SURROUNDINGS AT 0 K:

$$E_b = \sigma T^4 = (5.676)(20)^4 = 9.08 \times 10^5 \text{ W/m}^2$$

$$100 \text{ W} = 9.08 \times 10^5 A$$

$$A = 1.109 \times 10^{-4} \text{ m}^2 = \pi D^2/4$$

$$D = 0.01188 \text{ m} = 1.188 \text{ cm} \quad \text{a)}$$

IN VISIBLE RANGE:  $0.4 < \lambda < 0.7 \mu\text{m}$

$$\lambda T|_1 = 2000(0.4) = 800 \quad F \approx 0$$

$$\lambda T|_2 = 2000(0.7) = 1400 \quad F = 0.0078$$

$$\text{FRACTION} = \underline{\underline{0.0078}} \quad \text{b)}$$

IN UV RANGE:  $0 < \lambda < 0.4$

$$\lambda T|_1 = 0$$

$$\lambda T|_2 \approx 0$$

$$\text{FRACTION} = \underline{\underline{0}} \quad \text{c)}$$

IN IR RANGE:  $0.7 < \lambda < 100$

$$\lambda T|_1 = 0.0078$$

$$\lambda T|_2 \approx 1.0$$

$$\text{FRACTION} = \underline{\underline{0.992}} \quad \text{d)}$$

23.17

$q = 8 \text{ W}$  THROUGH HOLE WITH  $D = 0.0025 \text{ m}^2$

$$E_b = \frac{8}{0.0025} = 3200 \text{ W/m}^2 = \sigma T^4$$

$$T = \underline{\underline{487 \text{ K}}}$$

23.18

$$\lambda_{\max} T = 2897.6 \mu\text{m} \cdot \text{K}$$

	T
SUN	5730 K
L. BULB	2910 K
SURFACE	1550 K
SKIN	308 K

$\lambda_{\max}$
1.998 $\mu\text{m}$
1.004 "
0.535 "
0.1063 "

23.19  $T = 1500\text{K}$  Peenblue  $D = 10\text{ cm}$

$\mathcal{J} = 0.78$  for  $0.4\lambda < 3.2\text{ }\mu\text{m}$   
 $0.08$   $3.2 < \lambda < \infty$

$$q_{\text{max}} = 5.676(15)^4 \left(\frac{\pi}{4}\right) (0.01)^2$$

$$= 22.57\text{ W}$$

for  $\lambda T_1 = 0$   $F_{0-\lambda T_1} = 0$   
 $\lambda T_2 = 4800$   $F_{0-\lambda T_2} = 0.6075$   
 $\lambda T_3 = \infty$   $F_{0-\lambda T_3} = 1$

TOTAL HT LOSS

$$= 22.57 \left[ 0.78(0.6075) + 0.08(0.3925) \right]$$

$$= 11.40\text{ W}$$

23.20  $E_{b1} \xrightarrow{R_1} \mathcal{J}_1 \xrightarrow{R_2} \mathcal{J}_2 \xrightarrow{R_3} E_{b2}$

1 IS INNER CYLINDER  
 2 " OUTER "

$$E_{b1} = \sigma(T_1)^4 = 2.10\text{ W/m}^2$$

$$E_{b2} = \sigma(300)^4 = 460 "$$

$$R_1 = \frac{S_1}{A_1 E_1} = \frac{0.8}{\pi(0.02)(1)(0.2)} = 63.7\text{ m}^2$$

$$R_2 = \frac{1}{A_1 F_{12}} = \frac{1}{\pi(0.02)(1)} = 15.9 "$$

$$R_3 = \frac{S_2}{A_2 E_2} = \frac{0.95}{\pi(0.05)(1)(0.05)} = 121 "$$

$$\Sigma R = 201\text{ m}^2$$

$$q = \frac{E_{b2} - E_{b1}}{\Sigma R} = \frac{460 - 2}{201} = 2.28\text{ W/m}$$

23.20 CONT. - WITH RADIATION SHIELD

$$E_{b1} \xrightarrow{R_1} R_2 \xrightarrow{R_4} R_3 \xrightarrow{R_3} E_{b2}$$

$$R_1 = 63.7 \quad R_3 = 121$$

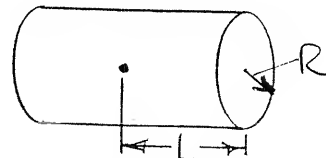
$$R_2 = 15.9$$

$$R_4 = \frac{1}{A_2 F_{21}} = \frac{1}{\pi(0.05)(1)} = 9.09$$

$$\Sigma R = 209.7$$

$$q = \frac{460 - 2}{209.7} = 2.18\text{ W/m}$$

23.21 ASSUMING THERMOCOUPLE AT GEOMETRIC CENTER OF DUCT



SOLID ANGLE OF DUCT OPENING

$$\Omega \approx \frac{\pi R^2}{\pi R_0^2} = \frac{\text{DUCT AREA}}{\text{HEMISPHERE SURFACE}}$$

$$= \frac{(15/12)^2}{1^2} = 0.0156$$

{ THERMOCOUPLE SEES DUCT, PERMANENT }

FOR THERMOCOUPLE:  $q_{\text{rad}} = q_{\text{conv}}$

$$A F_{\text{ew}} (E_{\text{bc}} - E_{\text{bw}}) = h A (T_6 - T_c)$$

$$E_{\text{bc}} \xrightarrow{R_1} R_2 \xrightarrow{R_3} E_{\text{bw}}$$

23,21 CONT.

$$A_c F_{cw} = \frac{1}{\frac{S_c}{A_c \epsilon_c} + \frac{1}{A_c F_{cw}} + \frac{S_w}{A_w \epsilon_w}}$$

$$F_{cw} = \frac{1}{\frac{S_c}{\epsilon_c} + \frac{1}{F_{cw}} + \frac{S_w A_c}{A_w \epsilon_w}}$$

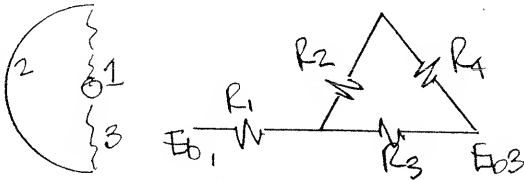
$$F_{cw} \approx 1 \quad A_c/A_w \approx 0$$

$$\therefore F_{cw} = \frac{1}{\frac{1-\epsilon_c}{\epsilon_c} + 1} \approx \epsilon_c = 0.6$$

$$20(T_g - T_c) = \epsilon_c (0.1714) \left[ \left( \frac{T_c}{100} \right)^4 - \left( \frac{T_w}{100} \right)^4 \right]$$

$$\underline{T_c = 316 F}$$

23,22



$$R_1 = \frac{S_1}{A_1 \epsilon_1} = \frac{0.2}{\pi/6 (0.8)} = \frac{1.5}{\pi}$$

$$R_2 = \frac{1}{A_1 F_{12}} = \frac{1}{\pi/6 (0.5)} = \frac{12}{\pi}$$

$$R_3 = \frac{1}{A_1 F_{13}} = \frac{1}{\pi/6 (0.5)} = \frac{12}{\pi}$$

$$R_4 = \frac{1}{A_2 F_{23}} = \frac{1}{A_3 F_{32}} = \frac{1}{1.5 - 0.167} = 0.75$$

$$R_{equiv} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_2 + R_4}} = 2.08$$

23,22 CONT.

$$\Sigma R = 1.5/\pi + 2.08 = 2.557$$

$$q = \frac{0.1714 (24.6^4 - 5.3^4)}{2.557} = 24,500 \frac{\text{Btu}}{\text{hr ft}} \quad (a)$$

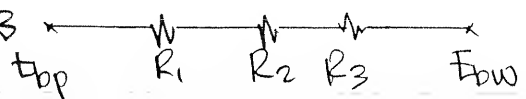
WITH NO REFLECTOR:

$$q = \epsilon A (E_{b1} - E_{b3})$$

$$= 0.8 \left( \frac{\pi}{6} \right) (0.1714) (24.6^4 - 5.3^4)$$

$$= \underline{25,500 \text{ Btu/hr ft}} \quad (b)$$

23,23



$$R_1 = \frac{S_1}{A_1 \epsilon_1} = \frac{0.3}{\pi (1/4) (0.7)} = 0.546$$

$$R_2 = \frac{1}{A_1 F_{12}} = \frac{1}{\pi (1/4) (1)} = 1.273$$

$$R_3 = \frac{S_w}{A_w \epsilon_w} = \frac{0.2}{(A_w) \epsilon_w} \approx \text{VERY SMALL}$$

$$q = \frac{0.1714 (6.65^4 - 5.3^4)}{1.819}$$

$$= \underline{110 \text{ Btu/hr per foot}} \left\{ \begin{array}{l} \text{RADIANT} \\ \text{LOSS} \end{array} \right\}$$

CONVECTION:

$$q = h A \Delta T$$

$$h = \frac{k}{D} \left\{ 0.60 + \frac{0.387 Ra^{1/4}}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{1/4} \right]^{4/3}} \right\}^2$$

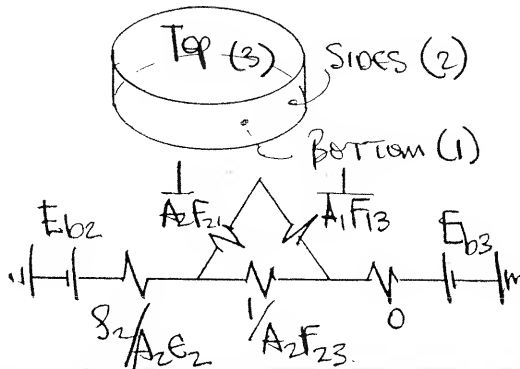
23.23 CONTINUED -

$$F_{R T_f} = 137 F$$

$$h = 1.21 \text{ Btu/hr ft}^2 F$$

$$q_{\text{conv}} = 1.21 (\pi \times 1/4) (135) = \underline{\underline{128 \frac{\text{Btu}}{\text{hr}}}}$$

23.24



{Fig 23.14}  $F_{13} = 0.38$   $F_{12} = 0.62$

$$A F_{12} = A_2 F_{21} \quad F_{21} = F_{23} = A F_{12} / A_2$$

$$\frac{1}{A_2 F_{21}} = \frac{1}{A F_{12}} = \frac{1}{\pi (6)^2 (0.62)}$$

$$\frac{1}{A F_{13}} = \frac{1}{\pi (6)^2 (0.38)}$$

$$\frac{1}{A_2 F_{13}} = \frac{1}{A_3 F_{32}} = \frac{1}{A F_{12}} = \frac{1}{A_2 F_{21}} = \frac{1}{\pi (6)^2 (0.62)}$$

$$\begin{aligned} \frac{1}{R_{\text{conv}}} &= A_2 F_{23} + \frac{1}{\frac{1}{A_2 F_{21}} + \frac{1}{A_1 F_{13}}} \\ &= \pi (6)^2 (0.62) + \frac{1}{\frac{1}{\pi (6)^2 (0.62)} + \frac{1}{\pi (6)^2 (0.38)}} \\ &= \pi (6)^2 (0.856) \end{aligned}$$

23.24 CONTINUED -

$$\frac{R_2}{A_2 E_2} = \frac{0.2}{\pi (6) (12) (0.8)} = \frac{1}{\pi (6)^2 (8)}$$

$$\Sigma R = \frac{1}{\pi (6)^2 (0.856)} + \frac{1}{\pi (6)^2 (8)}$$

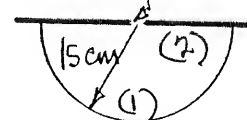
$$= \frac{1.29}{\pi (6)^2}$$

$$q = \frac{\sigma (T_2^4 - T_1^4)}{\Sigma R}$$

$$= \frac{0.1714 (\pi) (6)^2 (10^4 - 5^4)}{1.29}$$

$$= \underline{\underline{140,900 \text{ Btu/hr}}}$$

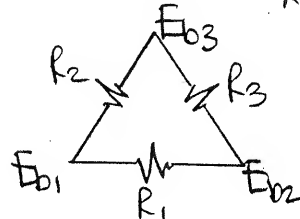
23.25 (3) Diam of Hole = 5 cm



$$F_{11} + F_{12} + F_{13} = 1 \quad F_{21} = F_{31} = 1$$

$$F_{12} = F_{21} \frac{A_2}{A_1} = \frac{\pi (0.15^2 - 0.025^2)}{2\pi (0.15)^2} = 0.486$$

$$F_{13} = F_{31} \frac{A_3}{A_1} = \frac{\pi (0.025^2)}{2\pi (0.15)^2} = 0.0139$$



BLACK CASE

$$R_1 = \frac{1}{A_1 F_{12}} = \frac{1}{2\pi (0.15)^2 (0.486)} = 14.55$$

$$R_2 = \frac{1}{A_1 F_{13}} = \frac{1}{2\pi (0.15)^2 (0.0139)} = 509$$

$$R_3 = \infty$$

23.25 CONTINUED -

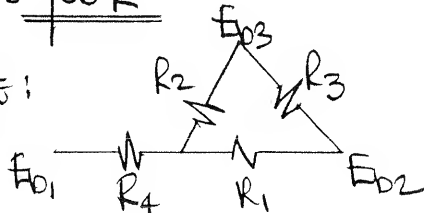
$$q_{13} = A_1 F_{13} E_{b1} = A_3 F_{31} E_{b3}$$

$$= \frac{\pi (0.05)^2}{4} (1) (5.676) (7^4)$$

$$= \underline{16.7 \text{ W}}$$

$$T_2 = T_1 = \underline{700 \text{ K}}$$

GRAY CASE:

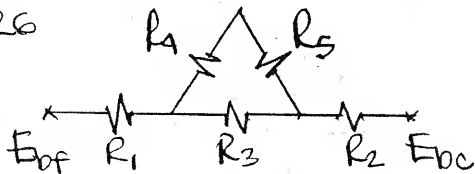


$$R_4 = \frac{S_1}{A_1 \epsilon_1} = \frac{0.3}{2\pi (0.15^2) (0.7)} = 3.03$$

$$q = \frac{E_{b1} - 0}{\sum R} = \frac{\sigma T_1^4}{512.03} = \underline{26.7 \text{ W}}$$

$$T_2 = T_1 = \underline{700 \text{ F}}$$

23.26



{ WALLS ASSUMED TO BE AT A UNIFORM TEMPERATURE }

$$R_1 = \frac{0.2}{12(20)(0.8)} = 0.00104$$

$$R_2 = \text{same} = 0.00104$$

$$R_3 = \frac{1}{A F_{F-C}} = \frac{1}{(12)(20)(0.145)} = 0.00903$$

$$R_4 = \frac{1}{A F_{F-W}} = \frac{1}{(12)(20)(0.55)} = 0.0076$$

$$R_5 = R_4 = 0.0076$$

23.26 CONTINUED

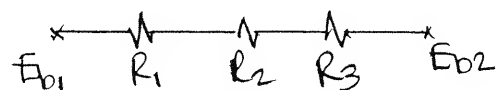
$$R_{\text{equiv}} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_4 + R_5}} = 0.0058$$

$$\sum R = R_1 + R_2 + R_{\text{equiv}} = 0.00785$$

$$q = \frac{\sigma (T_F^4 - T_C^4)}{\sum R} = \frac{0.1714 (5.45^4 - 5.25^4)}{0.00785}$$

$$= \underline{2680 \text{ Btu/hr}}$$

23.27



{ EQUIVALENT CIRCUIT }

$$R_1 = \frac{S_1}{A_1 \epsilon_1} \quad R_2 = \frac{1}{A F_{12}} = \frac{1}{A_2 F_{21}} \quad R_3 = \frac{S_2}{A_2 \epsilon_2}$$

$$T_1 = 300 \text{ K} \quad T_2 = 78 \text{ K}$$

$$A_1 = \pi D_1^2 = \pi (1.3)^2 = 1.69 \pi \text{ m}^2$$

$$A_2 = \pi D_2^2 = \pi (1)^2 = \pi \text{ m}^2$$

$$R_1 = \frac{0.8}{(1.69 \pi)(0.2)} = \frac{2.37}{\pi} \text{ m}^{-1}$$

$$R_2 = \frac{1}{\pi (1)^2 (1)} = \frac{1}{\pi} \text{ m}^{-1}$$

$$R_3 = \frac{0.8}{\pi (0.2)} = \frac{4}{\pi} \text{ m}^{-1}$$

$$\sum R = \frac{7.37}{\pi} = 2.35 \text{ m}^{-1}$$

23.27 CONTINUED -

$$q = \frac{E_{b1} - E_{b2}}{\sum R} = \frac{\sigma(T_1^4 - T_2^4)}{\sum R}$$

$$= \frac{5.676(3^4 - 0.78^4)}{2.35} = 194.8 \text{ W}$$

Boil-off Rate:  $\dot{m} = q / h_{fg}$

$$\dot{m} = \frac{1948}{2 \times 10^5} = 9.74 \times 10^{-4} \text{ kg/s}$$

$$= \underline{\underline{3.51 \text{ kg/hr}}}$$

23.28 OPENING DIAM = 5 mm

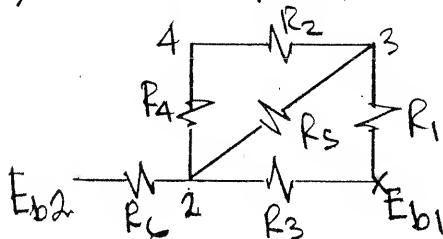
(a) EQUIV. SURFACE ① SEES INTERIOR AS A SINGLE SURFACE

$$q = \frac{E_{b2} - E_{b1}}{R} = \frac{\sigma T_2^4 - 0}{1/A_1}$$

$$= \frac{\pi(5)^2(5.676)(6)^4(10^{-6} \frac{\text{m}^2}{\text{mm}^2})}{1}$$

$$= \underline{\underline{0.144 \text{ W}}}$$

b) ANALOGY CIRCUIT:



$$R_1 = \frac{1}{A_1 F_{12}} = 0.060 \text{ mm}^{-2} \quad F_{12} \approx 0.15$$

$$R_2 = \frac{1}{A_3 F_{34}} = 1.61 \times 10^{-3} \quad F_{13} = 0.85$$

$$R_3 = \frac{1}{A_1 F_{12}} = 0.339 \quad F_{2-(1+4)} \approx 0.1$$

$$R_4 = \frac{1}{A_2 F_{24}} = 0.0148 \quad F_{23} = 0.9$$

$$F_{24} = 4.167 \times 10^{-3}$$

23.28 CONTINUED -

$$R_5 = \frac{1}{A_2 F_{23}} = 1.57 \times 10^{-3} \quad F_{24} = 0.0958$$

$$R_6 = \frac{8}{A_2 \epsilon_2} = 9.43 \times 10^{-4} \quad F_{42} = 0.0985$$

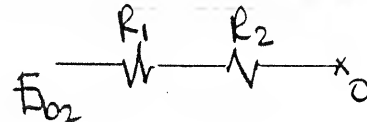
$$F_{43} = 0.9015$$

$$\frac{1}{R_{23 \text{ EQUIV}}} = \frac{1}{R_5} + \frac{1}{R_4 + R_2} \quad R_{23 \text{ EQUIV}} = 1.433 \times 10^{-3}$$

$$\frac{1}{R_{21 \text{ EQUIV}}} = \frac{1}{R_3} + \frac{1}{R_5 + R_1} \quad R_{21 \text{ EQUIV}} = 0.052$$

$$q = \frac{\sigma T^4}{R_6 + R_{21 \text{ EQUIV}}} = \underline{\underline{0.139 \text{ W}}}$$

(c) ALL INTERIOR MAY BE CONSIDERED A SINGLE SURFACE -



$$R_2 = \frac{1}{A_1 F_{12}} = \frac{1}{A_1} = 0.0509$$

$$R_1 = \frac{8}{A \epsilon} = \frac{0.4}{A_1 (0.6)} = 1.291 \times 10^{-4}$$

$$q = \frac{\sigma T^4}{\sum R} = \underline{\underline{0.144 \text{ W}}}$$

(SLIGHTLY LESS THAN IN PART (a))

23.29 PROBLEM STATEMENT ASKS FOR RADIANT ENERGY REACHING TANK BOTTOM - I.E. THE IRRADIATION

a)  $q_{\text{TOTAL}} = q_{\text{FROM HTR}} + q_{\text{FROM SPACE}}$

23.29 CONTINUED

$$q_{HTR} = A_1 F_{12} E_{b1}$$

$$\{f_{12} \approx 0.39 - f_{11} \approx 0.15\}$$

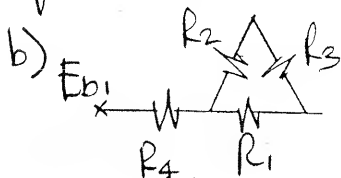
$$= \frac{\pi}{4} (0.2)^2 (0.39) \sigma T_1^4$$

$$= 1826 \text{ W}$$

$$q_{\text{space}} = \frac{\pi}{4} (0.2)^2 (0.61) \sigma T_2^4$$

$$= 8.8 \text{ W}$$

$$q_{\text{TOTAL}} \approx \underline{1835 \text{ W}}$$



$$R_1 = \frac{1}{A_1 F_{12}} = \frac{1}{\frac{\pi}{4} (0.2)^2 (0.39)} = 81.62$$

$$R_2 = \frac{1}{A_1 F_{13}} = \frac{1}{\frac{\pi}{4} (0.2)^2 (0.61)} = 52.18$$

$$R_3 = \frac{1}{A_2 F_{23}} = R_2 = 52.18$$

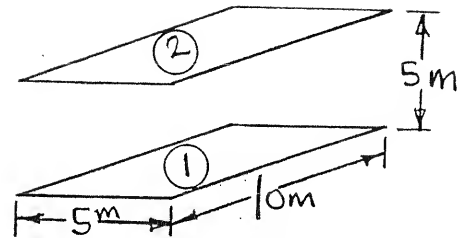
$$R_4 = \frac{S_1}{A_1 \epsilon_1} = \frac{0.4}{\frac{\pi}{4} (0.2)^2 (0.6)} = 21.22$$

$$\frac{1}{R_{12 \text{ EQUIV}}} = \frac{1}{R_1} + \frac{1}{R_2 + R_3} \quad R_{12 \text{ EQUIV}} = 45.8$$

$$\Sigma R = R_4 + R_{12 \text{ EQUIV}} \approx 67$$

$$q_{\text{surroundings self}} = \frac{\sigma T^4}{67} = \underline{2224 \text{ W}}$$

23.30

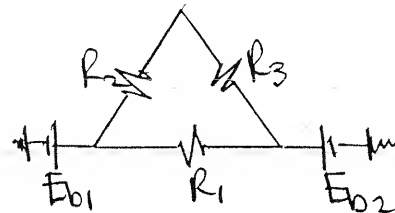


SURROUNDINGS ARE CONSIDERED AN EQUIVALENT SURFACE (3) AT 0K

$$T_1 = 100 \text{ K} \quad F_{12} \approx 0.28 \quad \left\{ \begin{array}{l} F_{14} \\ 23, 14 \end{array} \right\}$$

$$T_2 = 200 \text{ K} \quad F_{13} = 0.72$$

$$T_3 = 0 \text{ K} \quad F_{23} = 0.72$$



$$R_1 = \frac{1}{50 (0.28)} = 0.0714$$

$$R_2 = R_3 = \frac{1}{50 (0.72)} = 0.0278$$

(a)

$$q_{12} = \frac{E_{b1} - E_{b2}}{R_1} = \frac{\sigma (T_1^4 - T_2^4)}{R_1} = \underline{-1192 \text{ W}}$$

(b)

$$q_1 = q_{12} + q_{13}$$

$$q_{12} = -1192$$

$$q_{13} = \frac{E_{b1} - 0}{R_2} = 204 \text{ W}$$

$$q_1 = \underline{-988 \text{ W}}$$

23.30 CONTINUED-

$$q_2 = q_{21} + q_{23}$$

$$q_{21} = 1192$$

$$q_{23} = \frac{E_{02} - 0}{R_3} = 3270 \text{ W}$$

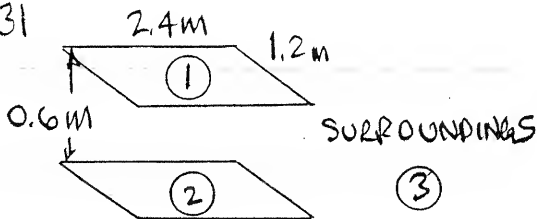
$$q_2 = \underline{4462 \text{ W}}$$

c) TO SURROUNDINGS

$$q_{13} = \underline{204 \text{ W}}$$

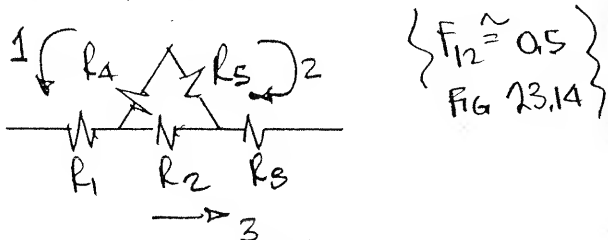
$$q_{23} = \underline{3270 \text{ W}}$$

23.31



$$\epsilon_1 = 0.6 \quad T_1 = 1000 \text{ K} \quad A_1 = 2.88 \text{ m}^2$$

$$\epsilon_2 = 0.9 \quad T_2 = 400 \text{ K} \quad A_2 = "$$



{NOTE 3 "Loops"}

$$R_1 = \frac{S_1}{A_1 \epsilon_1} = 0.231 \quad R_2 = \frac{1}{A_1 F_{12}} = 0.694$$

$$R_3 = \frac{S_2}{A_2 \epsilon_2} = 0.039 \quad R_4 = R_5 = \frac{1}{A_1 F_{13}} = 0.694$$

23.31 CONTINUED-

WRITING EQNS FOR LOOPS AS SHOWN:

$$E_{01} - 0 = (I_1 + I_3) R_1 + I_1 R_4$$

$$E_{02} - 0 = (I_2 - I_3) R_3 + I_2 R_5$$

$$E_{01} - E_{02} = (I_1 + I_3) R_1 + I_3 R_2 + (I_3 - I_2) R_3$$

SUBSTITUTING VALUES & SOLVING SIMULTANEOUS EQNS.

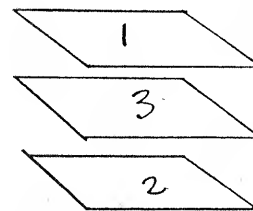
$$I_1 = 59550 \quad I_2 = 4695 \quad I_3 = 42970$$

$$q_{\text{NET}} = I_1 + I_3 = \underline{102.5 \text{ kW}}$$

$$q_{12} = I_3 = \underline{42.97 \text{ kW}}$$

{ THESE RESULTS PRESUME NO HT TX FROM OTHER SIDES OF PLATES }

23.32



$$T_1 = 1000 \text{ K} \quad A_1 = 2.88 \text{ m}^2$$

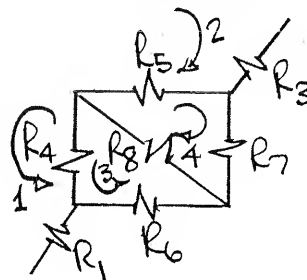
$$\epsilon_1 = 0.6$$

$$T_3 = ? \quad A_3 = 2.88$$

$$\epsilon = 0.8$$

$$T_2 = 400 \text{ K}$$

$$A = 2.88 \quad \epsilon = 0.9$$



$$R_1 = 0.231$$

$$R_3 = 0.694$$

$$R_4 = 1.157$$

$$R_7 = 0.496$$

$$R_5 = 1.157$$

$$R_8 = 0.579$$

$$R_6 = 0.496$$

23.32 CONTINUED -

EQUATIONS FOR LOOPS SHOWN:

$$E_{b1} - 0 = I_1 R_1 + (I_1 - I_3) R_4$$

$$E_{b2} - 0 = I_2 R_3 + (I_2 - I_4) R_5$$

$$0 = (I_3 - I_1) R_4 + I_3 R_6 + (I_3 + I_4) R_8$$

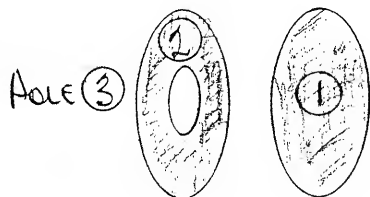
$$0 = (I_3 + I_4) R_8 + I_4 R_7 + (I_4 - I_2) R_5$$

SOLVING:  $I_1 = 62100$   $I_2 = 25500$

$I_3 = 16940$   $I_4 = 25600$

$q_1 = \underline{62.1 \text{ kW}}$

23.33



FROM FIGS 23.14 & 23.15

$$F(2 \rightarrow 3) - 1 = F_{1-(2+3)} \approx 0.04$$

$$F_{31} \approx 0.04$$

$$F_{13} = 0.04 \frac{(2.5)^2}{(4)^2} = 0.0156$$

$$F_{12} = 0.04 - 0.0156 = 0.0244$$

FOR BLACK DISKS:

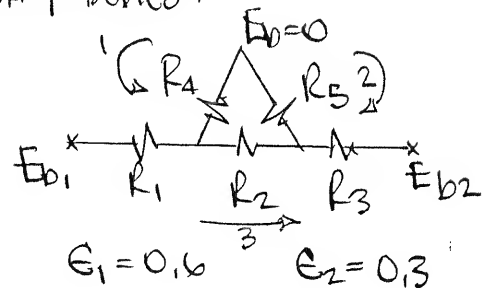
$$q_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

$$= \frac{\pi}{4} \left( \frac{4}{12} \right)^2 (0.0244) (0.1714) \times (9.6^4 - 6.7^4)$$

$$= \underline{2.36 \text{ Btu/hr}} \quad (a)$$

23.33 CONTINUED -

FOR GRAY BODIES:



$$R_1 = \frac{S_1}{A_1 \epsilon_1} = \frac{0.4}{\frac{\pi}{4} \left( \frac{4}{12} \right)^2 (0.6)} = 76.4$$

$$R_2 = \frac{1}{A_1 F_{12}} = 470$$

$$R_3 = \frac{S_2}{A_2 \epsilon_2} = 65$$

$$R_4 = \frac{1}{A_1 F_{1-5}} = 11.74$$

$$R_5 = \frac{1}{A_2 F_{2-5}} = 19.6$$

FOR LOOPS SHOWN:

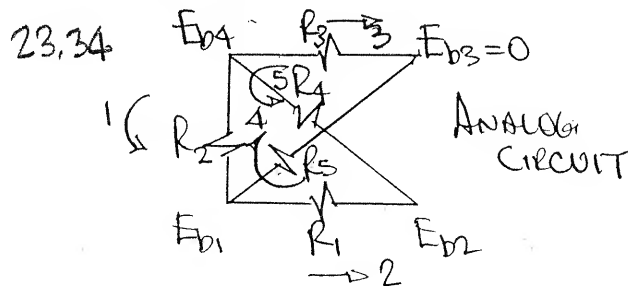
$$E_{b1} - 0 = I_1 R_1 + I_1 R_4 + I_3 R_1$$

$$E_{b2} - 0 = I_2 R_3 + I_2 R_5 - I_3 R_3$$

$$E_{b1} - E_{b2} = (I_1 + I_3) R_1 + I_3 R_2 + (I_3 - I_2) R_3$$

SOLVING SIMULTANEOUSLY:

$$q_{1-2} = I_3 = \underline{1.67 \text{ Btu/hr}} \quad (b)$$



23.34 CONTINUED-

$$\begin{aligned} R_1 &= 470 & E_{b1} &= 1460 \\ R_2 &= 11.94 & E_{b2} &= 345 \\ R_3 &= 30.6 & E_{b3} &= 0 \\ R_4 &= 735 & & \text{4 IS ADIABATIC} \\ R_5 &= 19.6 & & \end{aligned}$$

FOR BLACK SURFACES:

$$q_{12} = \frac{E_{b1} - E_{b2}}{R_{\text{equiv},12}}$$

$$R_{\text{equiv},12} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_4}} = 288$$

$$q_{12} = \frac{1460 - 345}{288} = 3.87 \frac{\text{Btu}}{\text{hr}}$$

$$q_{\text{lost through HOLE}} = q_{13} = \frac{E_{b1} - 0}{R_{\text{equiv}}}$$

$$R_{\text{equiv}} = \frac{1}{\frac{1}{R_5} + \frac{1}{R_1 + R_2}} = 18.83$$

$$q_{\text{lost}} = \frac{1460}{18.83} = 77.5 \frac{\text{Btu}}{\text{hr}}$$

FOR GRAY SURFACES:  $\epsilon_1 = 0.6$   $\epsilon_2 = 0.3$

ADDITIONAL RESISTANCES  $R_A, R_B$

$$R_A = \frac{S_1}{A_1 \epsilon_1} = \frac{0.4}{\frac{\pi}{4} \left(\frac{4}{12}\right)^2 0.6} = 7.64$$

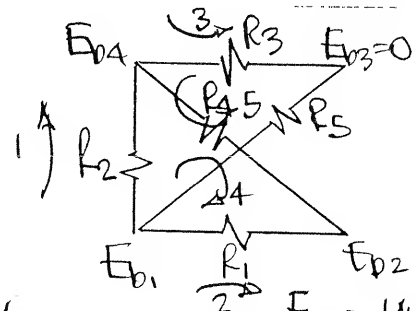
$$R_B = \frac{0.3}{\frac{\pi}{4} \left[\left(\frac{4}{12}\right)^2 - \left(\frac{0.5}{12}\right)^2\right] 0.7} = 8.06$$

23.34 CONTINUED-

$$q_{12} = \frac{1460 - 345}{288 + 7.64 + 8.06} = 3.67 \frac{\text{Btu}}{\text{hr}}$$

$$q_{\text{lost}} = \frac{1460}{18.83 + 7.64} = 55.2 \frac{\text{Btu}}{\text{hr}}$$

23.35



$$\begin{aligned} R_1 &= 470 & R_4 &= 735 & E_{b1} &= 1460 \\ R_2 &= 11.94 & R_5 &= 19.6 & E_{b2} &= 345 \\ R_3 &= 30.6 & & & E_{b3} &= 0 \\ & & & & E_{b4} &= 738 \end{aligned}$$

WRITING LOOP EQNS:

- 1:  $E_{b1} - E_{b4} = R_2(I_1 + I_4 + I_5)$
- 2:  $E_{b4} - 0 = R_3(I_3 - I_5)$
- 3:  $E_{b1} - E_{b2} = R_1(I_2 - I_4)$
- 4:  $0 = R_1(I_4 - I_2) + R_2(I_4 + I_1 - I_5) + I_4 R_4$
- 5:  $0 = R_3(I_5 - I_3) + R_2(I_5 - I_1 - I_4) + I_5 R_5$

SOLVING:

$$I_1 = 134.8 \quad I_3 = 98.7 \quad I_5 = 74.6$$

$$I_2 = 2.84 \quad I_4 = 0.47$$

$$q_{12} = I_2 = 2.84 \frac{\text{Btu}}{\text{hr}}$$

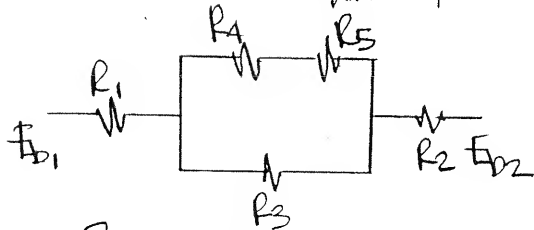
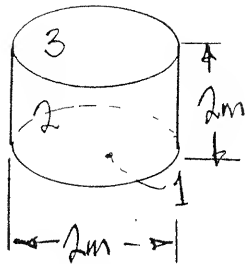
$$q_{\text{lost}} = I_3 + I_5 = 173.3 \frac{\text{Btu}}{\text{hr}}$$

23.36

{F<sub>16</sub> 23.14}

F<sub>13</sub> = 0.18

F<sub>12</sub> = 0.82



$$R_1 = \frac{S_1}{A_1 \epsilon_1} = \frac{0.69}{\frac{\pi}{4}(2)^2(0.31)} = 0.708 \text{ m}^2$$

$$R_2 = \frac{S_2}{A_2 \epsilon_2} = 0.996 \text{ m}^2$$

$$R_3 = \frac{1}{A F_{12}} = 0.388 \text{ m}^2$$

$$R_4 = \frac{1}{A F_{13}} = 1.77 \text{ m}^2$$

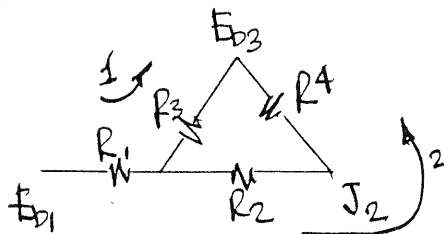
$$R_5 = 0.388 \text{ m}^2$$

$$R_{\text{equiv}} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_4 + R_5}} = \frac{1}{\frac{1}{0.388} + \frac{1}{1.77 + 0.388}} = 0.329$$

$$\Sigma R = 0.708 + 0.329 + 0.996 = 2.03 \text{ m}^2$$

$$q = \frac{E_{01} - E_{02}}{\Sigma R} = \frac{5.676(7.55^4 - 3.95^4)}{2.03} = 8400 \text{ W} = 8.4 \text{ kW}$$

23.37



{SURFACE 3 IS SURROUNDING}

23.37 CONTINUED.

$$A_1 = A_2 = \frac{\pi}{4}(0.15)^2 = 0.0177 \text{ m}^2$$

$$R_1 = \frac{S_1}{A_1 \epsilon_1} = \frac{0.2}{A(0.8)} = 14.12$$

$$\{F_{16} 23.14 \quad F_{12} \approx 0.37\} \Rightarrow F_{13} = 0.63$$

$$R_2 = \frac{1}{A F_{12}} = 153$$

$$R_3 = \frac{1}{A F_{13}} = 89.7$$

$$R_4 = \frac{1}{A_2 F_{23}} = 89.7$$

$$E_{03} = \sigma(3.5)^4 = 852 \text{ W/m}^2$$

Loop Eqs:

$$E_{01} - E_{03} = (I_1 + I_2)R_1 + I_1 R_3$$

$$E_{01} - E_{03} = (I_1 + I_2)R_1 + I_2(R_2 + R_4)$$

Solving:  $I_1 = 2.706 I_2$

$$I_1 + I_2 = 300$$

$$\therefore I_2 \approx 81 \quad I_1 = 219$$

$$J_2 = E_{02} + I_2 R_4$$

$$= 57.9 + 81(89.7) = 7319$$

$$J_1 = J_2 + I_2 R_2$$

$$J_1 = 7319 + 81(153) = 19710$$

$$E_{01} = J_1 + 300(R_1)$$

$$= 19710 + 300(14.12)$$

$$= 23950$$

23.37 CONTINUED -

$$\text{FINALLY: } T_1 = \left( \frac{E_{b1}}{\sigma} \right)^{1/4} = \underline{806 \text{ K}} \quad (a)$$

$$T_2 = \left( \frac{J_2}{\sigma} \right)^{1/4} = \underline{599 \text{ K}} \quad (b)$$

$$q_{\text{TO SOLR}} = I_1 + I_2 = \underline{300 \text{ W}} \quad (c)$$

$$q_{1-2} = I_2 = \underline{81 \text{ W}} \quad (d)$$

23.37 ALTERNATE SOLUTION USING  
EQNS 23.37 & 23.38

APPLYING THEM TO EACH NODE:

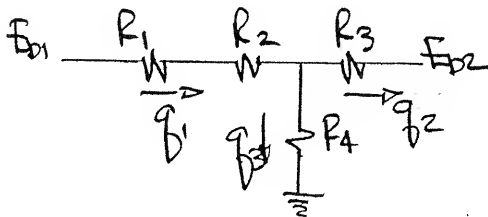
$$\frac{300}{A_1} = J_1 - F_{12} J_2 - F_{13} J_3$$

$$0 = J_2 - F_{21} J_1 - F_{23} J_3$$

$$E_{b3} = J_3$$

SOLVING THESE EQNS SIMULTANEOUSLY  
GIVES SAME RESULTS AS ABOVE

23.38 TEST SPECIMEN IS 1  
TUBE IS 2  
VIEWING PORT W



$$R_1 = \frac{R_1}{A_1 \epsilon_1} = \frac{0.2}{0.833(0.8)} = 0.30$$

$$R_2 = \frac{1}{A_1 F_{12}} = \frac{1}{A_1} = 1.133 \quad \{F_{12} \approx 1\}$$

23.38 CONTINUED -

$$R_3 = \frac{R_3}{A_2 \epsilon_2} = \frac{0.77}{340(0.23)} = 9.85 \times 10^{-4}$$

$$R_4 = \frac{1}{A_w F_w}$$

$$A_1 = 0.883 \text{ in}^2 \quad A_w = 0.049 \text{ in}^2$$

$$A_2 = \frac{\pi}{4}(16) + 8\pi + 4\pi(24) = 340 \text{ in}^2$$

$$q_1 = q_2 + q_3$$

$$q_2 = \frac{A_2 \epsilon_2 (J_2 - E_{b2})}{S_2}$$

$$q_3 = A_w J_3$$

$$q_1 = \frac{E_{b1} - J_1}{R_1 + R_2} = A_1 \epsilon_1 (E_{b1} - J_1)$$

$J_2$  BECOMES:

$$J_2 = \frac{E_{b1} + \frac{A_1 \epsilon_2}{A_2 S_2 \epsilon_1} E_{b2}}{1 + \frac{A_2 \epsilon_2}{A_1 S_2 \epsilon_1} + \frac{A_w}{A_1 \epsilon_1}}$$

$$E_{b1} = 131,500 \text{ Btu/hr ft}^2 \quad E_{b2} = 16$$

$$\Rightarrow J_2 = 344 \text{ Btu/hr ft}^2$$

$$q_1 = \frac{0.883}{144} (0.2)(131,500 - 344) = \underline{161 \frac{\text{Btu}}{\text{hr}}} \quad (b)$$

~ FROM SPECIMEN

$$q_3 = \frac{0.049}{144} (344) = \underline{0.117 \frac{\text{Btu}}{\text{hr}}} \quad (c)$$

~ LOSS THROUGH WINDOW

$$F_{1w} \approx \frac{A_w F_{w1}}{A_1} = \frac{\pi/4 (1/16)}{2\pi (144)} \approx \underline{5 \times 10^{-5}} \quad (a)$$

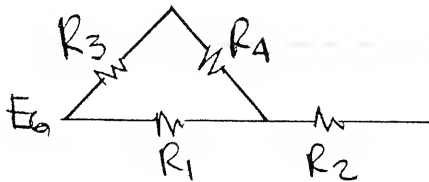
$$23.39 \quad \dot{q}_{\text{GAS-WALL DIRECT}} = A_1 F_{1G} \alpha_G \sigma (T_G^4 - T_1^4)$$

$$\dot{q}_{\text{GAS TO RADIATING WALLS}} = A_2 F_{2G} \alpha_G \sigma (T_G - T_2^4)$$

$$\dot{q}_{\text{RADIATING WALLS TO (1)}} = A_1 F_{12} \mathcal{T}_G \sigma (T_2^4 - T_1^4)$$

$$\dot{q}_{G2} = \dot{q}_{21} = \dot{q}_R = \frac{\sigma (T_G^4 - T_1^4)}{\frac{1}{A_1 F_{12} \mathcal{T}_G} + \frac{1}{A_2 F_{2G} \alpha_G}}$$

$$\dot{q}_{\text{TOTAL TO 1}} = \dot{q}_{G1} + \dot{q}_R$$



$$L = \frac{3A(0.2)(0.2)(1)}{4(0.2)(1)} = 0.17 \text{ m}$$

$$p = 1 \text{ ATM} \quad \alpha_G = 0.22$$

$$p_L = 0.558 \text{ ATM-FT} \quad \mathcal{T}_G = 0.78$$

$$R_1 = \frac{1}{0.2(1)(0.22)} = 22.7$$

$$R_2 = \frac{0.2}{0.2(1)(0.8)} = 1.25$$

$$R_3 = \frac{1}{3(0.2)(1)(1)(0.22)} = 7.58$$

$$R_4 = \frac{1}{0.2(1)(1)(0.78)} = 6.41$$

$$R_{\text{EQUIV}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_3 + R_4}} = 8.66$$

23.39 CONTINUED-

$$\Sigma R = 8.66 + 1.25 = 9.91$$

$$\dot{q} = \frac{5.676 (6^4 - 4.2^4)}{9.91} = \frac{564 \text{ W}}{\text{PER M}}$$

$$23.40 \quad \dot{q}_{\text{NET}} = \sigma A (\epsilon_G T_G^4 - \alpha_G T_W^4)$$

$$A = 4\pi r^2 = \pi (3\text{m})^2 = 28.27$$

$$T_G = 1000 \text{ K} \quad T_W = 600 \text{ K}$$

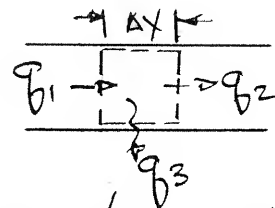
$$L = \frac{2}{3} D = 2 \text{ m}$$

$$pL = 0.15(5)(6.56) = 4.92 \text{ ATM-FT}$$

$$\alpha_G = 0.18 \quad \epsilon_G = 0.22$$

$$\dot{q}_{\text{NET}} = 5.676 (9\pi) [0.22(10^4) - 0.18(6^4)] = \underline{\underline{316 \text{ kW}}}$$

23.41



$$\dot{q}_2 - \dot{q}_1 = A \delta V c_p (T_{x+\Delta x} - T_x)$$

$$\dot{q}_3 = h p \Delta x (T - T_w) + A_w F_{wG} \sigma \epsilon_w [\epsilon_G T_G^4 - \alpha_G T_W^4]$$

$$\dot{q}_1 = \dot{q}_2 + \dot{q}_3$$

$$A \delta V c_p \frac{T_{x+\Delta x} - T_x}{\Delta x} + h p (T - T_w)$$

$$+ p F_{wG} \sigma \epsilon_w (\epsilon_G T_G^4 - \alpha_G T_W^4) = 0$$

23.41 CONTINUED -

IN LIMIT AS  $\Delta x \rightarrow 0$ :

$$A \rho V c_p \frac{dT}{dx} + hP(T - T_w) + P \epsilon_w \sigma (\epsilon_g T_g^4 - \alpha_g T_w^4) = 0$$

$$P_c = 0.20 \quad L = \frac{3.4 \text{ WD}^2}{4 \text{ WD}} = 0.425$$

$$P_c L = 0.085 \sim @ 2000 \text{ F} \quad \epsilon_g \approx 0.035$$

$$@ 1000 \quad \epsilon_g \approx 0.065$$

$$T_w = 1260 \text{ R} \quad \alpha_g = 0.071$$

$$\begin{aligned} \frac{dT}{dx} &= \left[ -\frac{hP}{\rho V c_p} (T - T_w) - \frac{P \epsilon_w \sigma}{\rho V c_p} (\epsilon_g T_g^4 - \alpha_g T_w^4) \right] \frac{1}{A} \\ &= \left\{ -\frac{1.5(2)(T - 1260)}{0.4(0.28)} - \frac{2(0.9)(0.0114)}{(0.4)(0.28)} \right. \\ &\quad \left. \times \left[ \epsilon_g \left( \frac{T_g}{100} \right)^4 - 0.071(12.6)^4 \right] \right\} \frac{4}{3600} \end{aligned}$$

$$\frac{dT}{dx} = \frac{1}{900} \left\{ -26.8(T - 1260) - 2.75 \left[ \epsilon_g \left( \frac{T_g}{100} \right)^4 - 1772 \right] \right\}$$

$$dx = - \left[ \frac{1}{-26.8(T - 1260) - 2.75 \epsilon_g \left( \frac{T_g}{100} \right)^4 + 4870} \right] \times 900 dT$$

BY GRAPHICAL INTEGRATION

$$x = \int_{1000}^{2000} \left[ \right] 900 dT = \underline{\underline{35.2 \text{ FT}}} \quad (a)$$

$$q_{\text{TOTAL}} = SA V c_p \Delta T$$

$$= 0.4 \left( \frac{1}{4} \right) (0.28) (1000)$$

$$= 128 \text{ BTU/s}$$

23.41 CONTINUED -

$$\begin{aligned} q_{\text{conv}} &= hP(x)(T - 800) \\ &= \frac{1.5(2)(x)(T - 800)}{3600} = x \frac{T - 800}{1200} \end{aligned}$$

$T_{\text{avg}}$	$x$	$q_{\text{conv}, i}$
1900	3.2	2.93
1700	4.0	3.0
1500	5.4	3.15
1300	8.0	3.33
1100	14.6	3.65

$$q_{\text{conv}} = \sum q_i = 16.06 \text{ BTU/s}$$

$$q_{\text{RAD}} = 28 - 16.06 = 11.94 \text{ BTU/s}$$

$$\text{RADIANT FRACTION} = \frac{11.94}{28} = 0.43 \quad (b)$$

FOR  $V$  DOUBLED:

$$dx = \left[ \frac{900}{-13.4(T - 1260) - 1.375 \left[ \epsilon_g \left( \frac{T_g}{100} \right)^4 - 1772 \right]} \right] dT$$

INTEGRATE GRAPHICALLY UNTIL  $x = 35.2$

AT THIS LOCATION  $T = \underline{\underline{1265 \text{ F}}}$

## CHAPTER 24

24,1 BASIS 1g mole LNG

	g/mol	MW	g	WT FRACTION
CH <sub>4</sub>	0,935	16	14,96	0,871
C <sub>2</sub> H <sub>6</sub>	0,046	30	1,38	0,080
C <sub>3</sub> H <sub>8</sub>	0,012	44	0,528	0,031
CO <sub>2</sub>	0,007	44	0,308	0,018
			17.176	1.00

WT FRACTION ETHANE - 0,080 a)

Aug. M.WT - 17.176 g/mol b)

DENSITY:

$$\rho = \frac{PM}{RT} = \frac{1,4 \times 10^5 (17,176)}{8,314 (201)} = 1397 \frac{\text{g}}{\text{m}^3} = 1,397 \frac{\text{kg}}{\text{m}^3} \quad \text{C)}$$

$$P_{CH_4} = y_{CH_4} P = (0.935)(1.4 \times 10^5) = \underline{131 \text{ kPa}} \quad d)$$

MASS FRACTION  $\text{CO}_2$

$$= \frac{0,308}{17,176} = \underline{\underline{0,0179}} \quad \text{e)}$$

24,2 BASIS - 1 kg MOLE

	kg/mole	M.W.	kg	WT FRACTION
$\text{SiCl}_4$	0.40	32.12	12.85	0.914
$\text{H}_2$	0.60	2.02	1.21	0.086
			14.06	1.0

↑  
a

24,2 CONTINUED -

$$\underline{M.Wt. = 14.06 \text{ kg/kg mole}} \quad (b)$$

$$C_A, SiCl_2 = y_{AC}$$

$$P = \frac{60}{760} (1.013 \times 10^5) = 7.99 \times 10^3 \text{ Pa}$$

$$C = \frac{P}{RT} = \frac{7.99 \times 10^3}{8.314(900)} = 1.08 \text{ mole/m}^3$$

$$C_A = (0.40)(1.068) = \underline{0.427 \text{ MOLE}/\text{m}^3} \quad \text{c)}$$

24.3 Basis 1g mole

	g	MOLE	M.W.T.	g
O <sub>2</sub>	0.21	0.21	32	6.72
N <sub>2</sub>	0.79	0.79	28	22.12
		1.0		28.84

MOLE FRACTION OF  $O_2 = \underline{\underline{0,21}}$  a)

VOLUME " " " = 0,21 b)

WT of MIXTURE = 28.84 g c)

$$\text{Vol/mole} = \frac{RT}{P} = \frac{8,314(400)}{1,013 \times 10^5} = 0.0328 \text{ m}^3/\text{mole}$$

$$\rho_{O_2} = 6,72 / 0,0328 = \underline{\underline{204,9 \text{ g/m}^3}} \quad d)$$

$$S_{N_2} = 22.12 / 0.0328 = \underline{\underline{674.4 \text{ } \mu \text{E}}}$$

$$S_{m14} = \underline{\underline{879 \text{ " f)}}$$

M, wt of mixture = 28.84 g)

$$24.4 \quad N_{A2} = -CD_{AB} \frac{dy_A}{dz} + y_A (N_{A2} + N_{B2})$$

$$N_{B2} = -CD_{BA} \frac{dy_B}{dz} + y_B (N_{A2} + N_{B2})$$

ADDING:

$$N_{A2} + N_{B2} = -CD_{AB} \frac{dy_A}{dz} - CD_{BA} \frac{dy_B}{dz} + (y_A + y_B)(N_{A2} + N_{B2})$$

$$CD_{AB} \frac{dy_A}{dz} + CD_{BA} \frac{dy_B}{dz} = 0$$

$$y_A + y_B = 1$$

$$\therefore \frac{dy_A}{dz} + \frac{dy_B}{dz} = 0 \quad \therefore \frac{dy_A}{dz} = -\frac{dy_B}{dz}$$

GIVING  $D_{AB} = D_{BA}$

IN HIRSHFELDER EQN:

$$\sigma_{AB}^2, \sqrt{2} \left[ \frac{1}{m_A} + \frac{1}{m_B} \right]^{1/2}$$

WILL BE THE SAME FOR  $D_{AB}, D_{BA}$   
 $\therefore$  AGREEMENT — Q.E.D.

$$24.5 \quad \vec{N}_A = -CD_{AB} \nabla y_A + y_A (\vec{N}_A + \vec{N}_B)$$

$C = \text{CONST.}$ ; MULTIPLY BY  $M_A$

$$\vec{N}_A M_A = -D_{AB} M_A \nabla C_A + y_A M_A (\vec{N}_A + \vec{N}_B)$$

$$W_A = \frac{y_A M_A}{x_A M_A + x_B M_B} = \frac{y_A M_A}{M_{\text{Ave}}}$$

$$\vec{N}_A = -D_{AB} \nabla P_A + W_A (\vec{N}_A + \vec{N}_B)$$

$$\therefore \vec{N}_A = -D_{AB} P \nabla W_A + W_A (\vec{N}_A + \vec{N}_B)$$

(a) 219

24.5 CONTINUED

$$\vec{N}_A = -D_{AB} \nabla C_A + C_A \left[ \frac{C_A \vec{U}_A + C_B \vec{U}_B}{C} \right]$$

$$C_A \vec{U}_A = -D_{AB} \nabla C_A + C_A \vec{U}$$

$$C_A (\vec{U} - \vec{U}) = -D_{AB} \nabla C_A$$

$$\therefore \vec{J}_A = -D_{AB} \nabla C_A \quad b)$$

$$24.6 \quad \vec{N}_A + \vec{N}_B = \left[ C_A \vec{U}_A + C_B \vec{U}_B \right] \frac{C}{C}$$

$$= C \vec{U} \quad a)$$

$$n_A + n_B = \left( P_A \vec{U}_A + P_B \vec{U}_B \right) P$$

$$= P \vec{U} \quad b)$$

$$\vec{J}_A + \vec{J}_B = -D_{AB} P \nabla W_A - D_{AB} P \nabla W_B$$

$$\text{AS } W_A + W_B = 1; \quad \nabla W_A + \nabla W_B = 0$$

$$\therefore \vec{J}_A + \vec{J}_B = 0 \quad c)$$

$$24.7 \quad -\frac{dC_A}{dz} = P_{AB} \frac{P_A}{M_A M_B} (U_{A2} - U_{B2})$$

$$+ P_{AC} \frac{P_A P_C}{M_A M_C} (U_{A2} - U_{C2}) + \dots$$

$$\text{AS } C = \frac{P_C}{M_C} = \frac{P_i}{RT}$$

$$-\frac{1}{RT} \frac{dP_A}{dz} = P_{AB} \left[ \frac{P_B}{RT} C_A U_{A2} - \frac{P_A}{RT} C_B U_{B2} \right]$$

$$+ P_{AC} \left[ \frac{P_C}{RT} C_A U_{A2} - \frac{P_A}{RT} C_C U_{C2} \right] + \dots$$

24.7 CONTINUED -

$$\begin{aligned}
 -\frac{1}{RT} \frac{dP_A}{dz} &= \frac{P_{AB}}{RT} [P_B N_{Az} - P_A N_{Bz}] \\
 &\quad + \frac{P_{AC}}{RT} [P_C N_{Az} - P_A N_{Cz}] + \dots \\
 -\frac{dP_A}{dz} &= \left[ P_{AB} P_B + P_{AC} P_C + \dots \right] N_{Az} \\
 &\quad - P_A [P_{AB} N_{Bz} + P_{AC} N_{Cz} + \dots]
 \end{aligned}$$

$$\text{Let } D_{Ai} = \frac{RT}{P_{Ai} P} \quad \text{or } P_{Ai} = \frac{RT}{D_{Ai} P}$$

$$\begin{aligned}
 -\frac{dP_A}{dz} &= \left[ \frac{RT}{P} \left( \frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \dots \right) \right] N_{Az} \\
 &\quad - \frac{P_A RT}{P} \left( \frac{N_{Bz}}{D_{AB}} + \frac{N_{Cz}}{D_{AC}} + \dots \right)
 \end{aligned}$$

FOR A DIFFUSING THROUGH NON-DIFFUSING B, C, D, ...

$$N_{Bz} = N_{Cz} = N_{Dz} = \dots = 0$$

GIVING

$$\begin{aligned}
 -\frac{dP_A}{dz} &= \frac{RT}{P} \left( \frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \dots \right) N_{Az} \\
 \therefore \frac{P}{RT N_{Az}} \left( -\frac{dP_A}{dz} \right) &= \frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \dots \quad (1)
 \end{aligned}$$

NOW - CONSIDER A BINARY CASE  
WITH  $N_{Bz} = 0$

$$N_{Az} = -C_{AB} \frac{dy_A}{dz} + y_A N_{Az}$$

24.7 CONTINUED -

$$N_{Az} = -\frac{P}{RT} \frac{D_{AB}}{1-y_A} \frac{dy_A}{dz}$$

$$= -\frac{P}{RT} \frac{D_{AB}}{P-y_A} \frac{dP_A}{dz}$$

$$\text{OR } \frac{P-y_A}{D_{AB}} = \frac{P}{RT} \left[ \frac{-dP_A/dz}{N_{Az}} \right] \quad (2)$$

COMBINING (1) & (2)

$$\frac{P-y_A}{D_{A-MIX}} = \frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \frac{P_D}{D_{AD}} + \dots$$

$$\therefore D_{A-MIX} = \frac{P-y_A}{\frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \frac{P_D}{D_{AD}} + \dots}$$

DIVIDING NUMERATOR & DENOMINATOR  
BY P WE GET

$$D_{A-MIX} = \frac{1-y_A}{y_B/D_{AB} + y_C/D_{AC} + y_D/D_{AD} + \dots}$$

DIVIDING NUMERATOR & DENOMINATOR  
BY  $1-y_A$  & DESIGNATING  $y'_i = y_i / (1-y_A)$

WE HAVE, FINALLY,

$$D_{A-MIX} = \frac{1}{y'_B/D_{AB} + y'_C/D_{AC} + y'_D/D_{AD} + \dots}$$

24.8 CO<sub>2</sub> IN AIR @ 310 K,  $1.5 \times 10^5$  Pa

APPENDIX J:  $D_{AB}P = 1.378 \text{ m}^2/\text{s Pa}$

$$D_{AB}@T_2 P_2 = D_{AB}|_{T_1 P_1} \left( \frac{P_1}{P_2} \right) \left( \frac{T_2}{T_1} \right)^{3/2} \frac{\Omega_{D,T_1}}{\Omega_{D,T_2}}$$

$$\text{CO}_2: \epsilon_L/k = 190$$

$$\text{AIR } \epsilon_L/k = 97$$

$$\epsilon_{AB}/k = \sqrt{190(97)} = 135.76$$

$$T_1: \frac{TK}{\epsilon_{AB}} = \frac{273}{135.76} = 2.01 \quad \Omega_D = 1.673$$

$$T_2: \frac{TK}{\epsilon_{AB}} = \frac{310}{135.76} = 2.283 \quad \Omega_D = 1.028$$

$$D_{AB} = \frac{1.378}{1.5 \times 10^5} \left( \frac{310}{273} \right)^{3/2} \frac{1.673}{1.028}$$

$$= 1.16 \times 10^{-5} \text{ m}^2/\text{s} \quad (a)$$

ETHANOL IN AIR @ 325 K  $2 \times 10^5$  Pa

SAME PROCEDURE AS ABOVE-

$$D_{AB}P|_{298} = 1.337 \text{ m}^2/\text{s Pa}$$

$$\epsilon_{AB}/k = 194.7 \quad \Omega_{D,T_1} = 1.188 \quad \Omega_{D,T_2} = 1.148$$

$$D_{AB} = \frac{1.337}{2 \times 10^5} \left( \frac{325}{298} \right)^{3/2} \left( \frac{1.188}{1.148} \right)$$

$$= 7.88 \times 10^{-6} \text{ m}^2/\text{s} \quad (b)$$

24.8 CONTINUED --

CO IN AIR @ 310 K,  $1.5 \times 10^5$  Pa

MUST USE HIRSHFELDER EQN

$$D_{AB} = \frac{0.001858 T^{3/2} \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2}}{P \sigma_{AB}^2 \Omega_D}$$

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.265$$

$$P = 1.4807 \text{ ATM}$$

$$\sigma_{AB}^2 = 12.985 \quad \epsilon_{AB}/k = 103.29$$

$$TK/\epsilon_{AB} = 3.0 \quad \Omega_D = 0.949$$

SUBSTITUTING & SOLVING:

$$D_{AB} = 1.47 \times 10^{-5} \text{ m}^2/\text{s} \quad (c)$$

CCl<sub>4</sub> IN AIR @ 298 K,  $1.913 \times 10^5$  Pa

AGAIN - HIRSHFELDER EQN - SEE PART (c)

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.202$$

$$P = 1.888 \text{ ATM}$$

$$\sigma_{AB}^2 = 22.553 \quad \epsilon_{AB}/k = 178.1$$

$$TK/\epsilon_{AB} = 1.67 \quad \Omega_D = 1.148$$

SUBSTITUTING & SOLVING:

$$D_{AB} = 3.95 \times 10^{-6} \text{ m}^2/\text{s} \quad (d)$$

24.9 n BUTANE - i BUTANE @ 673 K  
2.0 ATM

USE HIRSHFELDER EQN - SEE PROB 24.8

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.1857$$

$$\sigma_{AB}^2 = 26.718 \quad \epsilon_{AB}/K = 358.2$$

$$TK/\epsilon_{AB} = 1.88 \quad \Omega_D = 1.098$$

SUBSTITUTING & SOLVING:

$$\underline{D_{AB} = 1.03 \times 10^{-5} \text{ m}^2/\text{s}}$$

FULLER-SCHETTLER-GIDDINGS

$$D_{AB} = \frac{10^{-3} T^{1.75} \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2}}{P \left[ (\sum V_A)^{1/3} + (\sum V_B)^{1/3} \right]^2}$$

$$\sum V_A = \sum V_B = [4(4.8) + 10(3.7)]$$

$$= 96.2$$

SUBSTITUTING VALUES & SOLVING

$$\underline{D_{AB} = 9.9 \times 10^{-6} \text{ m}^2/\text{s}}$$

24.10 CH<sub>4</sub> IN AIR, 373 K,  $1.5 \times 10^5 \text{ Pa}$

HIRSHFELDER EQN - SEE PROB 24.8

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.311$$

$$\sigma_{AB}^2 = 13.834 \quad \epsilon_{AB}/K = 115.07$$

$$TK/\epsilon_{AB} = 3.24 \quad \Omega_D = 0.930$$

SUBSTITUTING & SOLVING:

$$\underline{D_{AB} = 2.19 \times 10^{-5} \text{ m}^2/\text{s} \text{ (a)}}$$

24.10 CONTINUED -

$$\text{WILKE EQN: } D_{A-\text{MIX}} = \frac{1}{\frac{0.21}{D_{A-O_2}} + \frac{0.79}{D_{A-N_2}}}$$

$$A = \text{CH}_4$$

MUST USE HIRSHFELDER EQN FOR  $D_{A,i}$   
- SEE PROB 24.8 FOR EQN.

FOR  $D_{A-O_2}$ :

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.306$$

$$\sigma_{AB}^2 = 13.159 \quad \epsilon_{AB}/K = 124.19$$

$$TK/\epsilon_{AB} = 3.0 \quad \Omega_D = 0.949$$

$$\text{SUBSTITUTING } D_{A-O_2} = 2.22 \times 10^{-5} \text{ m}^2/\text{s}$$

FOR  $D_{A-N_2}$

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.313$$

$$\sigma_{AB}^2 = 14.074 \quad \epsilon_{AB}/K = 111.76$$

$$\frac{TK}{\epsilon_{AB}} = 3.337 \quad \Omega_D = 0.923$$

$$\text{SUBSTITUTING: } D_{A-N_2} = 2.19 \times 10^{-5} \text{ m}^2/\text{s}$$

FOR MIXTURE: (WILKE EQN)

$$D_{A-AIR} = \frac{1 \cdot (10^{-5})}{0.21/2.22 + 0.79/2.19}$$

$$= \underline{2.19 \times 10^{-5} \text{ m}^2/\text{s} \text{ (b)}}$$

24.11  $\text{NH}_3$ -AIR 373 K  $1.013 \times 10^5 \text{ Pa}$

BROOKAW METHOD:  $\mu_{\text{NH}_3} = 1.46 \text{ DEBYE}$

$$\delta_{\text{NH}_3} = \frac{1.94 \times 10^3 (1.46)^2}{25 (239.7)} = 0.690$$

$$\delta_{\text{AIR}} = 0 \quad \therefore \delta_{\text{AB}} = 0$$

VALUES:  $\epsilon_{\text{AB}}/K = 210.75$

$$T^* = \frac{KT}{\epsilon_{\text{AB}}} = 1.77$$

EQU 14-46

$$\Omega_D = \frac{1.06036}{(T^*)^B} + \frac{0.19300}{\exp(DT^*)} + \frac{1.03587}{\exp(FT^*)} + \frac{1.03587}{\exp(UT^*)} = 1.1238$$

$$\delta_{\text{NH}_3} = 2.900 \quad \sigma_{\text{AB}}^2 = 10.49$$

SUBSTITUTING INTO H.E. & SOLVING:

$$D_{\text{AB}} = 3.47 \times 10^{-5} \text{ m}^2/\text{s}$$

FROM APPENDIX: @ 273 K

$$D_{\text{AB}} = 2.006 \text{ m}^2/\text{s Pa}$$

$$T^* = Tk/\epsilon_{\text{AB}} = 1.295$$

$$\Omega_D = \frac{1.06036}{1.295} + \dots \left\{ \begin{array}{l} \text{SEE PART 1} \\ \text{FOR FORM.} \end{array} \right\}$$

$$= 1.272$$

$$D_{\text{AB}} = \frac{(2.006)}{1.015 \times 10^5} \left( \frac{373}{273} \right)^{3/2} \left( \frac{1.272}{1.1238} \right)$$

$$= 3.52 \times 10^{-5} \text{ m}^2/\text{s}$$

24.12  $\text{SiCl}_4$  in  $\text{H}_2$  1073 K,  $1.5 \times 10^5 \text{ Pa}$

HIRSHFELDER EQU - SEE PROB 24.8

VALUES:  $\left[ 1/M_A + 1/M_B \right]^{1/2} = 0.7084$

$$P = \frac{1.5 \times 10^5}{1.015 \times 10^5} = 1.478 \text{ atm}$$

$$\sigma_{\text{AB}}^2 = 16.193 \quad \epsilon_{\text{AB}}/K = 109.19$$

$$Tk/\epsilon_{\text{AB}} = 9.83 \quad \Omega_D = 0.7446$$

SUBSTITUTING INTO H.E. & SOLVING:

$$D_{\text{AB}} = 2.596 \times 10^{-4} \text{ m}^2/\text{s} \quad (a)$$

FOR  $\text{SiCl}_4$  in  $\text{HCl}$  - SAME  $P \& T$

VALUES:  $\left[ 1/M_A + 1/M_B \right]^{1/2} = 0.182$

$$\sigma_{\text{AB}}^2 = 17.58 \quad \epsilon_{\text{AB}}/K = 359$$

$$Tk/\epsilon_{\text{AB}} = 2.99 \quad \Omega_D = 0.9586$$

SUBSTITUTING INTO H.E. & SOLVING:

$$D_{\text{AC}} = 4.77 \times 10^{-5} \text{ m}^2/\text{s}$$

FOR MIXTURES -  $y_{\text{SiCl}_4} = 0.40 \quad y_{\text{H}_2} = 0.40$

$$y_{\text{HCl}} = 0.20$$

$$y'_{\text{H}_2} = \frac{0.4}{0.6} = 0.667, \quad y'_{\text{HCl}} = \frac{0.2}{0.6} = 0.333$$

$$D_{\text{SiCl}_4-\text{MIX}} = \frac{1 \times 10^{-5}}{\frac{0.667}{2.596} + \frac{0.333}{4.77}}$$

$$= 1.0482 \times 10^{-5} \text{ m}^2/\text{s} \quad (b)$$

24.13  $H_2S$  in MIXTURE 350 K, 1 ATM

A =  $H_2S$  B =  $N_2$  C =  $SO_2$

FOR A INTO B: USE H.E. (PROB 24.8)

VALUES:  $\left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.255$

$\sigma_{AB}^2 = 14.27$   $\epsilon_{AB}/k = 162.2$

$KT/\epsilon_{AB} = 2.158$   $Z_D = 1.048$

SUBSTITUTING & SOLVING

$D_{AB} = 2.07 \times 10^{-5} \text{ m}^2/\text{s}$

FOR A INTO C: -- SAME PROCEDURE

VALUES:  $\left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.212$

$\sigma_{AB}^2 = 16.66$   $\epsilon_{AB}/k = 269.1$

$KT/\epsilon_{AB} = 1.30$   $Z_D = 1.273$

$D_{AC} = 1.20 \times 10^{-5} \text{ m}^2/\text{s}$

MIXTURE:  $y_A = 0.03$   $y_B = 0.92$   $y_C = 0.05$

$y'_B = 0.948$   $y'_C = 0.0515$

$$D_{H_2S-MIX} = \frac{1 \times 10^{-5}}{\frac{0.948}{2.07} + \frac{0.0515}{1.20}} = 2.00 \times 10^{-5} \text{ m}^2/\text{s}$$

24.14  $D_{AB} = \frac{kT}{6\pi r \mu_B} \sim r = \frac{kT}{6\pi D_{AB} \mu_B}$

GIVEN  $D_{AB} = 5.94 \times 10^{-11} \text{ m}^2/\text{s}$

$T = 293 \text{ K}$   $\mu_B = 998 \times 10^{-6} \text{ Pa}\cdot\text{s}$

SUBSTITUTING:  $r = 3.637 \text{ nm}$

24.15  $O_2$  in  $C_2H_5OH$  293 K

FOR  $C_2H_5OH$  -  $\mu = 1.25 \text{ cP}$   $M_B = 46$   $\phi_B = 1.5$   
 $V_{O_2} = 25.6$

$D_{AB} = \frac{T}{\mu_B} \frac{(7.4 \times 10^{-8}) (\phi_B M_B)^{1/2}}{V_A^{0.6}}$  (a)

SUBSTITUTING VALUES -  $D_{AB} = 2.06 \times 10^{-9} \text{ m}^2/\text{s}$

$CH_3OH$  in  $H_2O$ , 298 K

$\mu_B = 1.14 \text{ cP}$   $M_B = 18$   $\phi_B = 2.26$

$V_{CH_3OH} = 14.8 + 4(3.7) + 7.4 = 37$

SUBSTITUTION INTO EQ (24-52) { SEE PART A }

$D_{AB} = 1.326 \times 10^{-9} \text{ m}^2/\text{s}$  (b)

$H_2O$  in  $CH_3OH$  298 K

$\mu_B = 0.62 \text{ cP}$   $M_B = 32$   $\phi_B = 1.9$

$V_A = 18.9$

~ SUBSTITUTING INTO EQ (24-52)

$D_{AB} = 4.59 \times 10^{-9} \text{ m}^2/\text{s}$  (c)

$C_2H_5OH$  in  $H_2O$  298 K

$\mu_B = 1.14 \text{ cP}$   $M_B = 18$   $\phi_B = 2.26$

~ SUBSTITUTION INTO EQ (24-52)

$D_{AB} = 7.37 \times 10^{-10} \text{ m}^2/\text{s}$  (d)

FROM TEXT - APPENDIX J

$D_{AB} = 7.7 \times 10^{-10} \text{ m}^2/\text{s}$

24.16  $\text{Cl}_2$  in  $\text{H}_2\text{O}$  289 K

$\mu_B = 1.13 \text{ eV}$   $M_B = 18$   $\phi_B = 2.26$   
 $V_A = 48.4$

SUBSTITUTION INTO EQN (24-52)

$D_{AB} = 1.17 \times 10^{-9} \text{ m}^2/\text{s}$

USING EQN (24-53)

$D_{AB} = (13.26 \times 10^{-5}) \mu_B^{-1.14} V_A^{-0.589}$   
 $= 1.14 \times 10^{-9} \text{ m}^2/\text{s}$

APPENDIX J:  $D_{AB} = 1.26 \times 10^{-9} \text{ m}^2/\text{s}$

24.17  $\text{C}_6\text{H}_6$  in  $\text{C}_2\text{H}_5\text{OH}$  288 K

$\mu_B = 1.3 \text{ eV}$   $M_B = 46$   $\phi_B = 1.5$   
 $V_A = 96$

SUBSTITUTION INTO EQN (24-52)

$D_{AB} = 8.81 \times 10^{-10} \text{ m}^2/\text{s}$

$\text{C}_2\text{H}_5\text{OH}$  INTO  $\text{C}_6\text{H}_6$

$\mu_B = 0.75 \text{ eV}$   $M_B = 78$   $\phi_B = 1.0$   
 $V_A = 59.2$

SUBSTITUTION INTO EQ. (24-52)

$D_{AB} = 2.17 \times 10^{-9} \text{ m}^2/\text{s}$

24.18  $\text{O}_2$  in  $\text{H}_2\text{O}(\ell)$  288 K

~~EQN (24-52)~~  $\mu_B = 1.14 \text{ eV}$

$M_B = 18$   $\phi_B = 2.26$   $V_A = 25.6$

SUBSTITUTION:  $D_{AB} = 1.70 \times 10^{-9} \text{ m}^2/\text{s}$

~~EQN. (24-53)~~  $D_{AB} = 1.69 \times 10^{-9} \text{ m}^2/\text{s}$

24.19  $\text{P}$  in  $\text{Si}(\text{s})$

@ 1316 K  $D_{AB} = 1 \times 10^{-17} \text{ m}^2/\text{s}$

1408 K  $D_{AB} = 1 \times 10^{-16} \text{ m}^2/\text{s}$

$D_i = D_0 e^{-Q/RT}$

$\ln D_i = \ln D_0 - Q/RT$

SUBSTITUTION:  $Q/R = 4.645 \times 10^4$

$D_0 = 213.31$

@ 1373 K  $\ln D_i = -28.47$

$D_{AB} = 4.33 \times 10^{-17} \text{ m}^2/\text{s}$

24.20  $\text{C}$  in FCC  $\text{Fe}$  1000 K

$D_0 = 2.5 \times 10^{-6} \text{ m}^2/\text{s}$   $Q = 144.2 \text{ kJ/mol}$

$D_i = D_0 e^{-Q/RT} = 7.34 \times 10^{-10} \text{ m}^2/\text{s}$

$\text{C}$  in BCC  $\text{Fe}$

$D_0 = 2.0 \times 10^{-6} \text{ m}^2/\text{s}$   $Q = 84.1 \text{ kJ/mol}$

$D_i = D_0 e^{-Q/RT} = 8.09 \times 10^{-9} \text{ m}^2/\text{s}$

24.21 EFFECTIVE DIFFUSION OF  
H<sub>2</sub> IN N<sub>2</sub> 373 K, 1 ATM

STRAIGHT PORES (D=100 Å) IN PARALLEL

$$\Delta p = 1 \times 10^{-8} \text{ m}$$

$$\begin{aligned} \text{EQN (24-58)} \quad D_{KA} &= 4850 \Delta p \sqrt{T/M_A} \\ &= 4850 (10^{-8}) \left[ \frac{373}{2.015} \right]^{1/2} \\ &= \underline{6.6 \times 10^{-8} \text{ m}^2/\text{s}} \end{aligned}$$

$$\text{AT 288 K} \quad D_{AB} = 0.743 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{AT 373 K} \quad D_{AB} = 1.095 \times 10^{-6} \text{ m}^2/\text{s}$$

ASSUMING DILUTE N<sub>2</sub>

$$\begin{aligned} \text{EFFECTIVE} &= \frac{1 \times 10^{-6}}{1/1.095 + 1/0.066} \\ &= \underline{0.062 \times 10^{-6} \text{ m}^2/\text{s}} \quad (a) \end{aligned}$$

RANDOM PORES - VOID FRACTION = 0.4

$$\begin{aligned} D_{eff} &= \epsilon^2 D_e = (0.4)^2 (0.062 \times 10^{-6}) \\ &= \underline{9.92 \times 10^{-7} \text{ m}^2/\text{s}} \quad (b) \end{aligned}$$

RANDOM PORES 1000 Å  $\epsilon = 0.4$

$$\text{EQN. (24-58)} \quad D_{KA} = 0.16 \times 10^{-6} \text{ m}^2/\text{s}$$

$$D_{AE} = 0.383 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\begin{aligned} D_{AE}' &= (0.4)^2 (0.383 \times 10^{-6}) \\ &= \underline{0.0614 \times 10^{-6} \text{ m}^2/\text{s}} \quad (c) \end{aligned}$$

24.21 CONTINUED.  $\Delta p = 20,000 \text{ Å}$  PARALLEL

$$\text{EQN (24-58)} \quad D_{KA} = 1.3197 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\begin{aligned} D_{AE} &= \frac{1 \times 10^{-6}}{1/1.095 + 1/1.3197} \\ &= \underline{1.011 \times 10^{-6} \text{ m}^2/\text{s}} \quad (d) \end{aligned}$$

24.22 A = CH<sub>4</sub> ~ 20 mol %

B = H<sub>2</sub>O ~ 80 " "

USE H.E. - EQN. (24-33)

$$\text{VALUES } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.3436$$

$$\sigma_{AB}^2 = 10.468 \quad \epsilon_{AB}/K = 220.4$$

$$KT/\epsilon_{AB} = 2.60 \quad \Omega_0 = 0.9878$$

$$\text{SUBSTITUTING: } D_{AB} = 1.694 \times 10^{-6} \text{ m}^2/\text{s}$$

$$D_{AE} = \frac{1}{1/D_{AB} + 1/D_{AK}}$$

$$\begin{aligned} D_{AK} &= 4850 (2 \times 10^{-7} \text{ m}) \sqrt{\frac{573}{16}} \\ &= 0.580 \times 10^{-6} \text{ m}^2/\text{s} \end{aligned}$$

$$\text{SUBSTITUTING: } D_{AE} = \underline{0.432 \times 10^{-6} \text{ m}^2/\text{s}}$$

KUNSEN DIFFUSION IS ~ 75% OF TOTAL

24.23  $H_2O$  INTO  $CO$  353 K 2 ATM

$A = H_2O$   $B = CO$

$$D_{AB} @ 273 K, 1 \text{ ATM} = 0.651 \times 10^{-4} \text{ m}^2/\text{s}$$

$$AT \left\{ \begin{matrix} 353 K \\ 2 \text{ ATM} \end{matrix} \right\} D_{AB} = 0.651 \left( \frac{353}{273} \right)^{3/2} \left( \frac{1}{2} \right)$$

$$= 0.479 \times 10^{-4} \text{ m}^2/\text{s}$$

$$D'_{AE} = 0.036 \times 10^{-4} \text{ m}^2/\text{s}$$

$$= (0.3)^2 D_{AE}$$

$$D_{AE} = 0.4 \text{ m}^2/\text{s}$$

$$0.4 = \frac{1 \times 10^{-4}}{1/0.479 + 1/D_{AK}}$$

$$D_{AK} = 2.425 \times 10^{-4} \text{ m}^2/\text{s}$$

FROM EQN (24-58)

$$2.425 \times 10^{-4} = 4850 d_p \left[ \frac{353}{2.0158} \right]^{1/2}$$

$$\underline{d_p = 3.78 \times 10^{-7} \text{ m}}$$

24.24  $O_2$  INTO  $He$  ~ A INTO B

$$d_p = 5 \times 10^{-6} \text{ m} \quad P = 300 \text{ Pa}$$

$$T = 373 K \quad M_A = 32 \quad M_B = 4$$

$$C = \frac{P}{RT} = \frac{300}{8.314(373)} = 0.0967 \text{ mol/m}^3$$

$$C_{O_2} = 0.01(0.0967) = \underline{9.67 \times 10^{-4} \text{ mol/m}^3}$$

(a)

24.24 CONTINUED -

$$\text{IN PORES} - D_{eff} = \frac{1}{1/D_{AB} + 1/D_{AK}}$$

USE EQN (24-33) TO FIND  $D_{AB}$ :

$$\text{VALUES: } \left[ 1/M_A + 1/M_B \right]^{1/2} = 0.530$$

$$\sigma_{AB}^2 = 9.027 \quad \epsilon_{AB}/K = 33.98$$

$$KT/\epsilon_{AB} = 10.98 \quad \Omega_0 = 0.8161$$

$$\text{SUBSTITUTING: } D_{AB} = 0.0325 \text{ m}^2/\text{s}$$

USE EQN (24-58) TO FIND  $D_{AK}$

$$D_{AK} = 8.28 \times 10^{-4} \text{ m}^2/\text{s}$$

$$D_{AE} = \frac{1 \times 10^{-4}}{1/0.325 + 1/8.28} = \underline{8.08 \times 10^{-4} \text{ m}^2/\text{s}}$$

24.25  $C_6H_6$  IN  $H_2O(l)$

$$d_p = 1.50 \times 10^{-7} \text{ m} \quad \mu_B = 0.95 \text{ cP}$$

$$\epsilon = 0.4 \quad \phi_B = 2.26 \quad M_B = 18$$

$$V_A = 96.38$$

SUBSTITUTING INTO EQN (24-52)

$$D_{AB} = 0.955 \times 10^{-9} \text{ m}^2/\text{s}$$

USE EQN (24-58) TO GET  $D_{AK}$

$$D_{AK} = 0.142 \times 10^{-4} \text{ m}^2/\text{s}$$

$$D_{AE} = \frac{1 \times 10^{-9}}{1/14200 + 1/0.955} \approx 0.955 \times 10^{-9} \text{ m}^2/\text{s}$$

$$D_{AE} = \epsilon^2 D_{AE} = \underline{1.528 \times 10^{-10} \text{ m}^2/\text{s}}$$

24.26 CO IN H<sub>2</sub>

CO ~ A H<sub>2</sub> ~ B

$$d_p = 1.5 \times 10^{-8} \text{ m} \quad \epsilon = 0.10$$

$$T = 673 \text{ K} \quad P = 5.0 \text{ atm}$$

APPENDIX J:  $D_{AB} = 0.651 \times 10^{-4} \text{ @ 1 atm}$

$$\sim D_{AB} = 0.130 \times 10^{-4} \text{ m}^2/\text{s} \text{ @ 5 atm}$$

$$\epsilon_{AB}/k = 60.52$$

@ 273 K -  $kT/\epsilon_{AB} = 4.51 \quad \Omega_D = 0.8606$

@ 673 K -  $kT/\epsilon_{AB} = 11.12 \quad \Omega_D = 0.7345$

$$D_{AB} \Big|_{673} = 0.130 \times 10^{-4} \left( \frac{673}{273} \right)^{3/2} \left( \frac{0.8606}{0.7345} \right)$$

$$= 0.5891 \times 10^{-4} \text{ m}^2/\text{s}$$

OBTAIN  $D_{AK}$  FROM EQN (24-58)

$$D_{AK} = 4850 (1.5 \times 10^{-8}) \sqrt{\frac{673}{28}}$$

$$= 0.0357 \times 10^{-4} \text{ m}^2/\text{s}$$

$$D_{AE} = \frac{1 \times 10^{-4}}{1/0.5891 + 1/0.0357}$$

$$= 0.337 \times 10^{-4} \text{ m}^2/\text{s}$$

$$D'_{AE} = (0.1)^2 (0.337 \times 10^{-4})$$

$$= \underline{\underline{0.337 \times 10^{-6} \text{ m}^2/\text{s}}}$$

$$K.D. = \frac{0.0357}{0.589 + 0.0357} \approx \underline{\underline{5.7\%}}$$

24.27 GLUCOSE (A) IN H<sub>2</sub>O

$$T = 303 \text{ K} \quad d_p = 3 \times 10^{-9} \text{ m}$$

$$d_A = 0.86 \times 10^{-9} \text{ m}$$

$$\mu_B = 825 \text{ g/cm.s}$$

USE STOKES-EINSTEIN EQN: (24-50)

$$D_{AB} = \frac{kT}{6\pi\mu_B r_A} = \frac{(1.38 \times 10^{-16})(303)}{6\pi(825)(0.86 \times 10^{-7})}$$

$$= 6.25 \times 10^{-15} \text{ m}^2/\text{s}$$

USE EQN (24-62) TO OBTAIN  $D_{AE}$

$$\phi = \frac{8.6 \times 10^{-10} \text{ m}}{30 \times 10^{-10} \text{ m}} = 0.2867$$

$$F_1 = (1 - \phi)^2 = 0.508$$

$$F_2 = 1 - 2.104(0.2867) + 2.09(0.2867)^3$$

$$- 0.95(0.2867)^5 = 0.444$$

$$D_{AE} = D_{AB} F_1 F_2 = (6.25 \times 10^{-15})(0.508)(0.444)$$

$$= \underline{\underline{1.41 \times 10^{-15} \text{ m}^2/\text{s}}}$$

24.28 UREASE (A) INTO SUPPORT (B)

$$D_{AB} = 3.46 \times 10^{-11} \text{ m}^2/\text{s}$$

$$\lambda_{\text{molecule}} = 1238 \text{ nm} \quad d_p = 100 \text{ nm}$$

$$\phi_A = \frac{1238}{100} = 0.1238$$

$$F_1(\phi) = (1 - 0.1238)^2 = 0.7677$$

$$F_2(\phi) = 1 - 2.104(0.1238) + 2.09(0.1238)^3$$

$$- 0.95(0.1238)^5$$

$$= 0.743$$

24.28 CONTINUED -

$$D_{AE} = (3.46 \times 10^{-7}) (0.767) (0.743) \\ = \underline{\underline{1.97 \times 10^{-11} \text{ m}^2/\text{s}}}$$

24.29 RIBONUCLEASE (A)  
INTO SUPPORT (B)

$$D_{AE} = 5.0 \times 10^{-11} \text{ m}^2/\text{s}$$

$$D_{AB} = 1.19 \times 10^{-10} \text{ m}^2/\text{s}$$

$$d_m = 3.6 \text{ nm}$$

$$D_{AE} = D_{AB} f_1(\phi) f_2(\phi)$$

$$f_1(\phi) f_2(\phi) = \frac{5.0 \times 10^{-11}}{1.19 \times 10^{-10}} \\ = 0.4202$$

TRIAL  $\frac{1}{2}$  ERROR -

$$\phi \approx 0.183$$

$$f_1(\phi) = (1 - 0.183)^2 = 0.6675$$

$$f_2(\phi) = 1 - 2.104(0.183) \\ + 2.09(0.183)^3 - 0.95(0.183)^5 \\ = 0.6276$$

$$f_1 f_2 \approx 0.4190$$

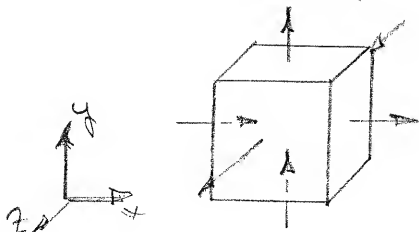
CLOSE ENOUGH -

$$d_p = \frac{3.6 \text{ nm}}{0.183} = \underline{\underline{19.67 \text{ nm}}}$$

# CHAPTER 25

## 25.1 CONSERVATION OF MASS:

$$\iint_{CS} \rho(\vec{U} \cdot \vec{n}) dA + \frac{\partial}{\partial t} \iiint_{CV} \rho dV = 0$$



MASS FLOW:

$$\begin{aligned} & N_{Ax} \Delta y \Delta z \Big|_{x+\Delta x} - N_{Ax} \Delta y \Delta z \Big|_x \\ & + N_{Ay} \Delta x \Delta z \Big|_{y+\Delta y} - N_{Ay} \Delta x \Delta z \Big|_y \\ & + N_{Az} \Delta x \Delta y \Big|_{z+\Delta z} - N_{Az} \Delta x \Delta y \Big|_z \end{aligned}$$

ACCUMULATION:  $\frac{\partial \rho}{\partial t} \Delta x \Delta y \Delta z$

PRODUCTION:  $R_A \Delta x \Delta y \Delta z$

PROCEDURE:

1. RELATE ACCORDING TO BASIC EQN.
2. DIVIDE THROUGH BY  $\Delta x \Delta y \Delta z$
3. CANCEL  $\Delta$  TERMS WHERE APPLICABLE
4. TAKE LIMIT AS  $\Delta x, \Delta y, \Delta z \rightarrow 0$

RESULT: 
$$\nabla \cdot \vec{N}_A + \frac{\partial \rho_A}{\partial t} - R_A = 0$$

$$25.2 \quad \nabla \cdot \vec{N}_A + \frac{\partial \rho_A}{\partial t} - \rho_A v_A = 0$$

FOR  $\rho \propto D_{AB}$  CONSTANT

$$\vec{N}_A = -D_{AB} \nabla \rho_A + \rho_A \vec{U}$$

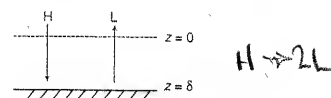
$$\nabla \cdot \vec{N}_A = -D_{AB} \nabla^2 \rho_A + \nabla \cdot \rho_A \vec{U}$$

SUBSTITUTION YIELDS:

$$\frac{\partial \rho_A}{\partial t} - D_{AB} \nabla^2 \rho_A + \nabla \cdot \rho_A \vec{U} = \rho_A v_A$$

## 25.3

$$\nabla \cdot \vec{N}_H + \frac{\partial \rho_H}{\partial t} = \rho_H v_H$$



ONE-DIRECTIONAL, STEADY STATE, NO HOMOGENEOUS REACTION -

$$\frac{d}{dz} N_{Az} = 0 \quad (a)$$

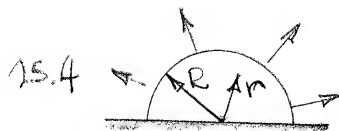
$$N_{Hz} = -CD_{HL} \frac{dy_H}{dz} + y_H (N_{Hz} + N_{Lz})$$

$$\text{AS } N_{Lz} = -2N_{Hz}$$

$$N_{Hz} = -CD_{HL} \frac{dy_H}{dz} + y_H (N_{Hz} - 2N_{Hz})$$

$$N_{Hz} (1 + y_H) = -CD_{HL} \frac{dy_H}{dz}$$

$$N_{Hz} = - \frac{CD_{HL}}{1 + y_H} \frac{dy_H}{dz}$$



1.  $T, P$  CONSTANT;  $C = \text{CONSTANT}$
2. STEADY STATE
3. NO HOMOGENEOUS REACTION,  $R_A = 0$
4. ONE DIRECTIONAL DIFFUSION
5. CONCENTRATION CONSTANT @  $r = R$
6.  $N_{AIR} = 0$

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

$$\frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) = 0 \quad r^2 N_{Ar} = \text{CONST.}$$

$$N_{Ar} = -C D_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

$$\underline{N_{Ar} = -\frac{C D_{AB}}{1 - y_A} \frac{dy_A}{dr}}$$

15.5  $O_2 \sim A$   $H_2O \sim B$

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

FOR Z-DIRECTION

$$\frac{d}{dz} N_{Az} = 0 \quad N_{Az} = \text{CONST}$$

$$N_{Az} = -C D_{AB} \frac{dy_A}{dz} + y_A (N_{Az} + N_{Bz})$$

SINCE DILUTE:  $y_A \approx 0, C \approx \text{CONST}$

$$\underline{\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2}}$$

FOR  $R_A = -k C_A$

$$\underline{\frac{\partial C_A}{\partial t} - D_{AB} \frac{\partial^2 C_A}{\partial z^2} + k C_A = 0}$$

15.6



(a)

1. DIFFUSION IN  $r$ -DIRECTION ONLY
2. NO HOMOGENEOUS REACTION,  $R_A = 0$
3.  $C_A @ r = R + 10$  IS KNOWN & CONSTANT
4.  $C_A @ r = R$  IS CONSTANT,  $y_A = P_A / P$
5. MOLECULAR DIFFUSION ONLY
6. STEADY STATE

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} = R_A$$

$$\sim \frac{1}{r} \frac{d}{dr} (r N_{Ar}) = 0 \quad (b)$$

$$\frac{d}{dr} (r N_{Ar}) = 0$$

$$\sim \underline{r N_{Ar} = \text{CONSTANT}} \quad (c)$$

$$N_{Ar} = -C D_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

$$= -C D_{AB} \frac{dy_A}{dr} + y_A N_{Ar}$$

$$\underline{N_{Ar} = -\frac{C D_{AB}}{1 - y_A} \frac{dy_A}{dr}}$$

FOR DILUTE CONCENTRATION:  $y_A \approx 0$

$$\underline{N_{Ar} = -C D_{AB} \frac{dy_A}{dr}}$$

15.7  $\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$

ASSUMPTIONS / CONDITIONS:

1. STEADY STATE
2. NO HOMOGENEOUS REACTION
3. DIFFUSION IN  $x$  &  $y$  DIRECTIONS
4.  $U_y = 0$
5. CONSTANT  $C, D_{AB}$
6.  $U_x = U_y$

# 25.7 CONTINUED-

$$\frac{\partial N_{Ax}}{\partial y} + \frac{\partial N_{Ay}}{\partial y} = 0$$

$$N_{Ax} = -D_{AB} \frac{\partial C_A}{\partial y} + u_y C_A$$

$$N_{Ay} = -D_{AB} \frac{\partial C_A}{\partial y}$$

SUBSTITUTING:

$$-D_{AB} \frac{\partial^2 C_A}{\partial y^2} + u_y \frac{\partial C_A}{\partial y} - D_{AB} \frac{\partial^2 C_A}{\partial y^2} = 0$$

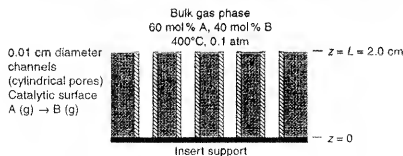
$$D_{AB} \left[ \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial y^2} \right] = u_y \frac{\partial C_A}{\partial y}$$

$$B.C. \quad C_A(0, y) = 0$$

$$C_A(x, 0) = C_{As}$$

$$C_A(x, y) = 0$$

25.8



1. DIFFUSION IN  $r$  &  $z$  DIRECTIONS

2. STEADY STATE

3. NO HOMOGENEOUS REACTION

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r N_{Ar}) + \frac{\partial}{\partial z} N_{Az} = 0$$

IN BOTH DIRECTIONS:

$$N_{Ar} = -N_{Br} \quad N_{Az} = -N_{Bz}$$

~ EQUIMOLAR COUNTERDIFFUSION

# 25.8 CONTINUED-

$$\therefore N_{Ar} = -C D_{AB} \frac{\partial y_A}{\partial r} = -D_{AB} \frac{\partial C_A}{\partial r}$$

$$N_{Az} = -C D_{AB} \frac{\partial y_A}{\partial z} = -D_{AB} \frac{\partial C_A}{\partial z}$$

INTO MASS CONSERVATION EQN:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( -D_{AB} r \frac{\partial C_A}{\partial r} \right) + \frac{\partial}{\partial z} \left( -D_{AB} \frac{\partial C_A}{\partial z} \right) = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C_A}{\partial r} \right) + \frac{\partial^2 C_A}{\partial z^2} = 0$$

$$B.C. \quad \frac{\partial C_A}{\partial r} (0, z) = 0$$

$$C_A(0.005 \text{ cm}, z) = 0$$

$$C_A(r, 2.0 \text{ cm}) = 0.6 C$$

$$25.9 \quad \nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0 \quad \{A \text{ is } O_2\}$$

$$\text{STEADY STATE, } R_A = -m$$

$$\nabla^2 N_A + m = 0$$

DIFFUSION IN  $r$ -DIRECTION ONLY

$$\frac{1}{r} \frac{\partial}{\partial r} (r N_{Ar}) + m = 0 \quad \text{EQUIMOLAR COUNTERD.}$$

$$N_{Ar} = -C D_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

$$\therefore N_{Ar} = -C D_{AB} \frac{dy_A}{dr} = -D_{AB} \frac{dC_A}{dr}$$

$$\text{OR - } N_{Ar} = - \frac{D_{AB}}{RT} \frac{dp_A}{dr}$$

$$25.10 \quad \nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0 \quad \{A \text{ is } O_2\}$$

STEADY STATE

NO HOMOGENEOUS REACTION

ONE-D (SPHERICAL) DIFFUSION

$$\frac{d}{dr}(r^2 N_{Ar}) = 0$$



$$2N_{Ar} = N_{Br}$$

$$y_A(N_{Ar} + N_{Br}) = -y_A N_{Ar}$$

$$N_{Ar} = -C D_{AB} \frac{dy_A}{dr} - y_A N_{Ar}$$

$$= -\frac{C D_{AB}}{1 + y_A} \frac{dy_A}{dr}$$



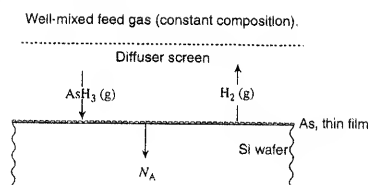
$$y_A(N_{Ar} + N_{Br}) = 0$$

$$N_{Ar} = -C D_{AB} \frac{dy_A}{dr}$$

B.C.  $y_A(r) = 0$

$$y_A(\infty) = 0.21$$

25.11



ASSUMPTIONS:

1. Temp = const,  $D_{AB} \approx P_S$  CONSTANT
2. NO HOMOGENEOUS REACTION

25.11 CONTINUED.

3. SILICON TREATED AS SEMI-INFINITE

4.  $C_A(z, 0) = 0$

5. MOLECULAR DIFFUSION IN SOLID

6. ONE DIRECTIONAL (z) DIFFUSION

GENERAL MASS CONSERVATION EQN IS

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} = 0$$

$$\therefore N_A = -D_{AB} \frac{\partial C_A}{\partial z}$$

COMBINING:  $\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2}$

B.C.  $C_A(z, 0) = 0$

$$C_A(0, t) = C_{As}$$

$$C_A(\infty, t) = 0$$

25.12 A IS  $O_2$

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

ST. ST. NO Rx

$$\frac{\partial N_{Ax}}{\partial x} + \frac{\partial N_{Az}}{\partial z} = 0$$

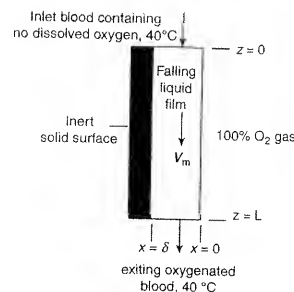
$$N_{Ax} = -D_{AB} \frac{\partial C_A}{\partial x} + y_A(N_{Ax} + N_{Bx})$$

20 - DILUTE CONCENTR.

$$N_{Ax} = -D_{AB} \frac{\partial C_A}{\partial x}$$

$$N_{Az} = -D_{AB} \frac{\partial C_A}{\partial z} + C_A v_m$$

BULK FLOW  $\approx$  MOLECULAR FLOW  
IN z-DIRECTION



25.12 CONTINUED -

SUBSTITUTING INTO MASS CONC. EQ.

$$\frac{\partial}{\partial x} \left( -D_{AB} \frac{\partial C_A}{\partial x} \right) + \frac{\partial}{\partial z} (C_A v_m) = 0$$

$$-D_{AB} \frac{\partial^2 C_A}{\partial x^2} + v_m \frac{\partial C_A}{\partial z} = 0$$

B.C.  $C_A(x, 0) = 0$   
 $C_A(0, z) = C_A^*$   
 $\frac{dC_A}{dx}(8, z) = 0$

25.13  $\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$

ONE-DIMENSIONAL (r) DIFFUSION  
 IN SPHERICAL GEOMETRY  
 NO HOMOGENEOUS REACTION

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 N_{Ar}) + \frac{\partial C_A}{\partial t} = 0$$

FICK'S LAW:

$$N_{Ar} = -D_{AB} \frac{\partial C_A}{\partial r} + C_A v_r$$

COMBINING:

$$-\frac{D_{AB}}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial C_A}{\partial r}) + \frac{\partial C_A}{\partial t} = 0$$

B.C.  $C_A(R, t) = 0$   
 $C_A(r, 0) = C_{A0}$   
 $\frac{\partial C_A}{\partial r}(0, t) = 0$

25.14 SPHERICAL GEOMETRY -

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

No Homog. Rx

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 N_{Ar}) + \frac{\partial C_A}{\partial t} = 0$$

FICK'S LAW:

$$N_{Ar} = -D_{A \text{ eff}} \frac{\partial C_A}{\partial r} + \text{CONTRIB.}$$

0 - NO BULK CONTRIB.

COMBINING:

$$-\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_{A \text{ eff}} \frac{\partial C_A}{\partial r}) + \frac{\partial C_A}{\partial t} = 0$$

$$\frac{\partial C_A}{\partial t} = \frac{D_{A \text{ eff}}}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial C_A}{\partial r})$$

$$C_A(r \leq R, 0) = C_{A0}$$

$$C_A(R, t) = C_A^*$$

$$\frac{\partial C_A}{\partial r}(0, t) = 0$$

25.15 INTO AIR: {A - HERBICIDE}

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

0 - ST. ST. NO HOMOGEN. RX

ONE DIMENSIONAL (z) DIFFUSION

$$\frac{dN_A}{dz} = 0$$

FICK'S LAW

$$N_{Az} = -CD_{AB} \frac{dy_A}{dz} + y_A (N_{Az} + N_{Bz})$$

$$N_{Az} = -\frac{CD_{AB}}{1 - y_A} \frac{dy_A}{dz} \quad (a)$$

25.15 CONTINUED -

INTO SOIL -

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

NO Rx  
NO BULK CONTR.

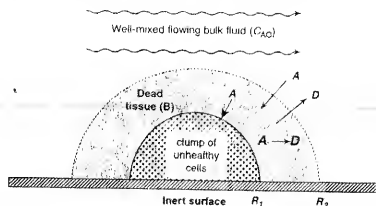
$$N_{Az} = -D_{AB} \frac{dC_A}{dz} + \cancel{C_A v_z}$$

$$\frac{\partial N_{Az}}{\partial z} + \frac{\partial C_A}{\partial t} = 0$$

COMBINING:

$$\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2}$$

25.16



$$R_A = -k C_A$$

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

SPHERICAL GEOMETRY - STEADY STATE

$$\frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) + k C_A = 0$$

$$N_{Ar} = -D_{A-mix} \frac{dC_A}{dr} + \cancel{C_A v_r} \quad \text{DILUTE}$$

COMBINING:

$$\frac{-D_{A-mix}}{r^2} \frac{d}{dr} \left( r^2 \frac{dC_A}{dr} \right) + k C_A = 0$$

## CHAPTER 26

### 26.1 ARNOLD CELL

$$d = 1 \text{ cm}$$

$$T = 308 \text{ K}$$

$$P_A^0 = 0.195 \text{ ATM}$$

$$\rho_L = 0.85 \text{ g/cm}^3$$

$$M_A = 78$$

$$y_{A1} = 0.195$$

$$y_{A2} = 0$$

EQU (26-19) APPLIES

$$D_{AB} = \frac{P_A y_{BLM} / M_A}{C(y_{A1} - y_{A2})L} \left( \frac{z_1^2 - z_2^2}{2} \right)$$

$$y_{BLM} = \frac{1.0 - 0.805}{\ln 1/0.805} = 0.899$$

$$C = \frac{P}{RT} = \frac{1}{(82.06)(308)} = 3.956 \times 10^{-5} \text{ mol/cm}^3$$

$$t = 72 \text{ h} = 2.592 \times 10^5 \text{ s}$$

SUBSTITUTING -

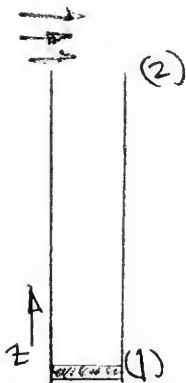
$$D_{AB} = 9.6 \times 10^{-6} \text{ m}^2/\text{s}$$

FROM APPENDIX J.1.

$$\text{AT } 298 \text{ K: } D_{AB} = 9.62 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{AT } 308 \text{ K: } D_{AB} = (9.62 \times 10^{-6}) \left( \frac{308}{298} \right)^{3/2} = 1.01 \times 10^{-5} \text{ m}^2/\text{s}$$

~ IN EXPERIMENT - EDDIES AT TOP OF CELL WOULD ALTER DIFFUSION MECHANISM



### 26.2 CYLINDRICAL GEOMETRY:

STEADY STATE, NO HOMOGENEOUS REACTION -

$$\frac{1}{r} \frac{d}{dr} (r N_{Ar}) = 0$$

$$N_{Ar} = -C D_{AB} \frac{dy_A}{dr} + y_A \overset{\text{O-DILUTE}}{N_A}$$

$$= -C D_{AB} \frac{dy_A}{dr}$$

$$r N_{Ar} \int_{r_i}^{r_o} \frac{dr}{r} = -\frac{D_{AB}}{RT} \int_{P_{Ai}}^{P_{Ao}} dy_A$$

$$r N_{Ar} \ln \frac{r_o}{r_i} = \frac{D_{AB}}{RT} (P_{Ai} - P_{Ao})$$

TABLE J.3. AT 293 K  $D_{AB} = 4.49 \times 10^{-5} \text{ m}^2/\text{s}$

SUBSTITUTING VALUES & SOLVING:

$$N_{Ar} = 3.92 \times 10^{-11} \text{ mol/m}^2 \cdot \text{s}$$

TO GET CONCENTRATION PROFILE:

$$\frac{d}{dr} (r N_{Ar}) = \frac{d}{dr} (-r D_{AB} \frac{dc_A}{dr}) = 0$$

$$\frac{d}{dr} (r \frac{dc_A}{dr}) = 0$$

SOLVING:

$$r \frac{dc_A}{dr} = C_1$$

$$c_A = C_1 \ln r + C_2$$

$$\text{AT } r_i = 5 \text{ mm}$$

$$C_{Ai} = \frac{P_{Ai}}{RT} = \frac{1.5 \times 10^5}{8.314(293)} = 61.58 \text{ mol/m}^3$$

$$\text{AT } r_o = 8 \text{ mm}$$

$$C_{Ao} = \frac{1.0 \times 10^5}{(8.314)(293)} = 41.05 \text{ mol/m}^3$$

26.2 CONTINUED -

UNITS:  $C_A, \text{mol/m}^3$   $r, \text{mm}$

$$@r_1 \quad 61.58 = C_1 \ln 5 + C_2$$

$$@r_0 \quad 41.05 = C_1 \ln 8 + C_2$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \begin{array}{l} C_1 = -43.64 \\ C_2 = 131.8 \end{array}$$

$$\underline{C_A = -43.64 \ln r + 131.8}$$

26.3

ONE DIRECTIONAL

STEADY STATE

B INSOLUBLE IN A & STAGNANT

$$\begin{aligned} N_{A2} &= -CD_{AB} \frac{dy_A}{dz} + y_A(N_{A2} + N_{B2}) \\ &= \frac{CD_{AB}}{z_2 - z_1} \ln \left( \frac{1 - y_{A2}}{1 - y_{A1}} \right) \quad \left\{ \begin{array}{l} \text{Eq. (24-5)} \\ (26-5) \end{array} \right\} \end{aligned}$$

$$y_A(3.0) = 1.0 \quad y_A(0.5) = \frac{1.63}{760} = 0.214$$

$$C = \frac{P}{RT} = \frac{1.0}{(82.06)(303)} = 4.02 \times 10^{-5} \frac{\text{mol}}{\text{cm}^3}$$

$$\text{APP. J: @ 298 K } -D_{AB} = 1.62 \times 10^{-4} \text{ m}^2/\text{s}$$

$$\begin{aligned} \text{AT 303 K } D_{AB} &= (1.62 \times 10^{-4}) \left( \frac{303}{298} \right)^{3/2} \\ &= 1.66 \times 10^{-4} \text{ m}^2/\text{s} \end{aligned}$$

SUBSTITUTING VALUES & SOLVING:

$$N_{A2} = 6.42 \times 10^{-5} \text{ mol/m}^2 \cdot \text{s}$$

$$W_A = (6.42 \times 10^{-5})(32)(3600)(24) A$$

$$A = \pi/4 (1)^2 = 0.785 \text{ m}^2$$

$$\underline{W = 139 \text{ g/day}} \quad (a)$$

26.3 CONTINUED

IF TEMPERATURE IS 313 K:

$$D_{AB} = (1.62 \times 10^{-4}) \left( \frac{313}{298} \right)^{3/2} = 1.74 \times 10^{-4} \text{ m}^2/\text{s}$$

$$y_{A1} = \frac{265}{760} = 0.349$$

ALL OTHER VALUES REMAIN THE SAME - SOLVING:

$$\underline{W_A = 260.6 \text{ g/day}}$$

26.4  $\text{C}_2\text{H}_5\text{OH (A)}$  THROUGH STAGNANT  $\text{H}_2\text{O (B)}$

ONE DIMENSIONAL, STEADY DIFFUSION

DILUTE CONCENTRATION:  $y_A \sim \text{SMALL}$

$$N_{A2} = -D_{AB} \frac{dc_A}{dz} = \frac{D_{AB}}{\delta} (C_{A1} - C_{A2})$$

TO EVALUATE  $D_{AB}$  - USE EQN. (24-53)

$$\left\{ \begin{array}{l} V_A = 2(14.8) + 6(3.7) + 7.4 = 59.2 \text{ cm}^3/\text{mol} \\ \mu_B = 1.45 \text{ cp} \end{array} \right\}$$

$$\begin{aligned} D_{AB} &= (13.26 \times 10^{-9}) (1.45)^{-1.14} (59.2)^{-0.589} \\ &= 7.82 \times 10^{-10} \text{ m}^2/\text{s} \end{aligned}$$

$$C_{A1} = 0.1 \text{ mol/m}^3 \quad C_{A2} = 0.02 \text{ mol/m}^3$$

SUBSTITUTING & SOLVING:

$$\underline{N_{A2} = 1.56 \times 10^{-12} \text{ mol/m}^2 \cdot \text{s}}$$

TO DETERMINE  $C_A(z)$ :

$$\nabla \cdot \vec{N}_A = 0 \sim \frac{dN_A}{dz} = 0$$

$$\text{GIVEN } \frac{d^2 N_{A2}}{dz^2} = 0$$

26.4 CONTINUED

$$C_A = C_1 + C_2$$

B.C.  $C_A(0) = 0.1 \text{ mol/m}^3$

$$C_A(0.004) = 0.02$$

$$C_2 = 0.1 \quad C_1 = \frac{(0.02 - 0.1)}{0.004} = -20$$

$$\underline{C_A = 0.1 - 20z} \quad \left\{ \begin{array}{l} C_A, \text{ mol/m}^3 \\ z, \text{ m} \end{array} \right.$$

FOR  $\text{C}_2\text{H}_5\text{OH (A) IN AIR (B) 283 K}$

$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{8.314(283)} = 43.15 \text{ mol/m}^3$$

$$y_A = \frac{C_A}{C} = \frac{0.1}{43.15} = 2.32 \times 10^{-3}$$

$$y_{A2} = \frac{0.02}{43.15} = 4.64 \times 10^{-4}$$

$$D_{AB} = (1.32 \times 10^{-5}) \left( \frac{283}{298} \right)^{3/2} = 1.22 \times 10^{-5} \text{ m}^2/\text{s}$$

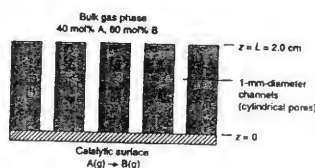
SAME EQN FOR  $N_{A2}$  AS IN PART (a)

$$N_{A2} = \frac{(1.22 \times 10^{-5})(0.1 - 0.02)}{4 \times 10^{-3}} = 2.44 \times 10^{-4} \text{ mol/m}^2 \cdot \text{s}$$

26.5

$$\begin{aligned} P &= 2 \text{ atm} \\ T &= 373 \text{ K} \\ M_A &= 58 \end{aligned}$$

STEADY STATE, 1D DIFFUSION



26.5 CONTINUED -

$$\frac{dN_{A2}}{dz} = 0 \quad N_{A2} = -N_{B2} = \frac{C D_{AB}}{S} (y_{A1} - y_{A2})$$

$$y_{A1}(0) = 0.4$$

$$y_{A2}(0.02 \text{ m}) = 0$$

$$C = \frac{P}{RT} = \frac{2}{(82.06)(373)} = 6.53 \times 10^{-5} \text{ mol/cm}^3$$

$$D_{AB} = 0.1 \left( \frac{373}{298} \right)^{3/2} \left( \frac{1}{2} \right) = 0.07 \text{ cm}^2/\text{s}$$

SUBSTITUTING INTO  $N_{A2}$  EXPRESSION

$$N_{B2} = -N_{A2} = 1.829 \times 10^{-6} \text{ g-mol/cm}^2 \cdot \text{s}$$

$$W_A = N_{A2} \cdot S$$

$$0.01 \text{ mol/min} = (1.829 \times 10^{-6})(60) S$$

$$S = \text{SURFACE AREA} = 91.12 \text{ cm}^2$$

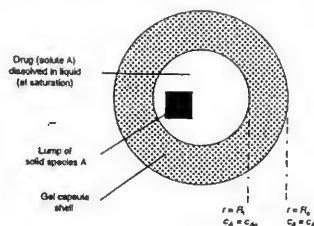
$$\text{PER CHANNEL} - S' = \pi/4 (0.1)^2 = 0.00785$$

$$\text{NO. CHANNELS} = \frac{91.12}{0.00785} = 11608$$

26.6  $D_{AB} = 1.5 \times 10^{-9} \text{ m}^2/\text{s}$

$$C_{AS} = C_A(R_1) = 0.01 \text{ mol/cm}^3$$

$$C_{AO} = C_A(R_0)$$



STEADY STATE, NO HOMOGENEOUS RX

$$\nabla \cdot \vec{N}_A = 0 \quad N_{Ar} = -D_{AB} \frac{dC_A}{dr}$$

$$\frac{d}{dr} (r^2 N_{Ar}) = 0 \quad \sim r^2 N_{Ar} \text{ IS CONST.}$$

$$\frac{d}{dr} \left( r^2 \frac{dC_A}{dr} \right) = 0$$

26.6 CONTINUED -

$$r^2 \frac{dC_A}{dr} = C_1 \quad \frac{dC_A}{dr} = \frac{C_1}{r^2}$$

$$C_A = -\frac{C_1}{r} + C_2$$

USING B.C.

$$C_{A1} = 0.01 = -\frac{C_1}{0.2} + C_2$$

$$C_{A0} = -\frac{C_1}{0.35} + C_2$$

SUBTRACTING -  $C_1 = \frac{C_{A0} - 0.01}{0.466}$

$$W = 4\pi r^2 N_{A2} = 4\pi (-D_{AB}) C_1$$

$$= -\frac{4\pi D_{AB}}{0.466} (C_{A0} - 0.01)$$

$$= -\frac{4\pi (1.5 \times 10^{-5})}{0.466} (C_{A0} - 0.01)$$

$$= -4.045 \times 10^{-4} (C_{A0} - 0.01) \text{ mol/s}$$

$W_A$  IS MAX FOR  $C_{A0} = 0$

$$= 4.045 \times 10^{-4} \text{ mol/s}$$

$$= \underline{\underline{1.456 \text{ mol/h}}}$$

26.7 SPHERICAL GEOMETRY  
STEADY STATE, NO HOMOGENEOUS RX  
A INTO STAGNANT B

$$\nabla \cdot \vec{N}_A = 0$$

$$\frac{d}{dr}(r^2 N_{Ar}) = 0$$

$$N_{Ar} = -C D_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

26.7 - CONTINUED -

$$\nabla \cdot \vec{N}_A = 0 \Rightarrow \frac{d}{dr}(r^2 N_{Ar}) = 0$$

$$N_{Ar} = -C D_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

$$N_{Ar} = -\frac{C D_{AB}}{1-y_A} \frac{dy_A}{dr}$$

$$N_{Ar} (4\pi r^2) \int_R^{\infty} \frac{dr}{r^2} = -4\pi C D_{AB} \int_{y_{A0}}^{y_A=0} \frac{dy_A}{1-y_A}$$

$$W_A \left( -\frac{1}{r} \right) \Big|_R^{\infty} = -4\pi C D_{AB} \left( \ln \frac{1}{1-y_A} \right) \Big|_{y_{A0}}^0$$

$$W_A = 4\pi C D_{AB} R \ln \left( \frac{1}{1-y_{A0}} \right)$$

MASS BALANCE FOR A:

$$4\pi C D_{AB} R \ln \frac{1}{1-y_{A0}} = \frac{S}{M_A} \frac{dV}{dt}$$

$$4\pi C D_{AB} R \ln \frac{1}{1-y_{A0}} = \frac{4\pi S}{M_A} R^2 \frac{dr}{dt}$$

SEPARATING VARIABLES & INTEGRATING:

$$C D_{AB} \ln \left( \frac{1}{1-y_{A0}} \right) = \frac{S}{M_A} \left( \frac{R_1^2 - R_2^2}{2} \right)$$

VALUES:  $D_{AB} = 8.19 \times 10^{-6} \text{ m}^2/\text{s}$

$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{8.314(347)} = 35.11 \text{ g mol/m}^3$$

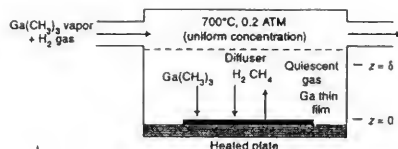
$$P_S = \frac{1.145}{128} (100)^3 = 8945 \text{ g mol/m}^3$$

SOLVING FOR t:

$$t = \frac{(8945) \left( \frac{10^{-4} - 0.0625 \times 10^{-4}}{2} \right)}{(35.11)(8.19 \times 10^{-6}) \ln \left( \frac{1}{0.993} \right)}$$

$$= \underline{\underline{2.076 \times 10^5 \text{ s} = 57 \text{ h}}}$$

26.9



PSEUDO STEADY STATE -  
 No homogeneous rx  
 ONE (Z) DIRECTIONAL DIFFUSION

$$\vec{\nabla} \cdot \vec{N}_A = 0 \quad \frac{dN_{Az}}{dz} = 0$$

$$N_{Az} = -CD_{AB} \frac{dy_A}{dz} + y_A(N_{Az} + N_{Bz} + N_{Cz})$$

$$\left\{ \begin{array}{l} \text{H}_2: N_{Bz} = \frac{3}{2} N_{Az} \\ \text{CH}_4: N_{Cz} = -3 N_{Az} \end{array} \right\}$$

$$N_{Az} = -CD_{AB} \frac{dy_A}{dz} + y_A N_{Az} \left(1 + \frac{3}{2} - 3\right)$$

$$= -\frac{CD_{AB}}{(1 + y_A/2)} \frac{dy_A}{dz}$$

$$N_{Az} \int_{\delta}^0 dz = -CD_{AB} \int_{y_{A0}}^0 \frac{dy_A}{1 + y_A/2} \quad (a)$$

FOR DILUTE A  $\sim y_A$  SMALL

$$N_{Az} = -CD_{AB} \frac{dy_A}{dz}$$

$$N_{Az} \int_{\delta}^0 dz = -CD_{AB} \int_{0.0002}^0 dy_A$$

$$C = \frac{P}{RT} = \frac{0.20}{82.06(973)} = 2.5 \times 10^{-6} \text{ g mol/cm}^3$$

$$D_{AB}|_{T_2, P_2} = 2.0 \text{ cm}^2/\text{s} \left( \frac{1}{0.2} \right)^{3/2} \left( \frac{973}{1023} \right)$$

$$= 9.276 \text{ cm}^2/\text{s} \quad (c)$$

IN TERMS OF  $\delta$ :

$$N_{Az} = \frac{CD_{AB}}{\delta} (0.0002) \quad (b)$$

$$26.9 \quad P = 303.9 \text{ Pa} \quad T = 873 \text{ K}$$

$$y_A = y_{As} = 0 \text{ @ } z = 0$$

$$y_A = 0.2 \text{ @ } z = \delta = 6 \text{ cm}$$

$$MA = 78$$

FOR  $D_{AB}$  - HIRSHFELDER EQN.

$$\text{VALUES: } \left[ \frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2} = 0.716$$

$$\sigma_{AB}^2 = 12.418 \quad \epsilon_{AB}/k = 6913$$

$$\epsilon_{AB}/kT = 7.92 \quad \Omega_0 = 0.8556$$

$$\sim \text{SUBSTITUTING } D_{AB} = 0.0221 \text{ m}^2/\text{s} \quad (a)$$

PHYSICAL SITUATION IS EQUIVALENT TO  
 CASE EXAMINED IN EXAMPLE 2, CH 25

$$N_{Az} = \frac{CD_{AB}}{\delta} \ln \left( \frac{1 + y_{A0}}{1 + y_{As}} \right)$$

$$C = \frac{P}{RT} = \frac{3 \times 10^{-3}}{82.06(873)} = 4.188 \times 10^{-8} \text{ mol/cm}^3$$

$$N_{Az} = \frac{(4.188 \times 10^{-8} \text{ mol/cm}^3)(0.0221 \text{ m}^2/\text{s}) \ln \left( \frac{1.2}{1} \right)}{6 \text{ cm}}$$

$$= 2.814 \times 10^{-7} \text{ mol/cm}^2 \cdot \text{s}$$

$$W_A = N_{Az} A = N_{Az} (\pi/4) D^2$$

$$= (2.814 \times 10^{-7}) \left( \frac{\pi}{4} \right) (15)^2 (60) (78)$$

$$= 0.2327 \text{ g/m}$$

# 26.10 Hemispherical Droplet on a Plane Surface

Steady State, No Homog. Rx. and D

$$\nabla^2 N_A = \frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) = 0$$

$$r^2 N_{Ar} \sim \text{CONSTANT}$$

$$N_{Ar} = -C_{DAB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Er})$$

$$N_{Ar} = -\frac{C_{DAB}}{1-y_A} \frac{dy_A}{dr}$$

$$\underbrace{2\pi r^2 N_{Ar}}_{W_A} \int_R^{\infty} \frac{dr}{r^2} = 2\pi C_{DAB} \int_0^{y_A} \frac{dy_A}{1-y_A}$$

$$@ t=0 \quad r = 0.005 \text{ m}$$

$$y_{A1} = \frac{31.824}{760} = 0.0419$$

$$W_A \left[ -\frac{1}{r} \right]_R^{\infty} = 2\pi C_{DAB} R \ln \left( \frac{1}{0.958} \right)$$

$$W_A = 2\pi C_{DAB} R \ln(1.0437)$$

For Droplet ~ Pseudo S.S. (1)

$$W_A = -\frac{\rho_A}{M_A} \frac{dV}{dt}$$

$$= -\frac{1}{18} \left( 2\pi R^2 \frac{dR}{dt} \right) \quad (2)$$

Equating (1) & (2) & Integrating:

$$C_{DAB} \ln(1.0437) t = 0.0556 \left( \frac{R_i^2 - R_f^2}{2} \right)$$

$$C = \frac{P}{RT} = \frac{1.03 \times 10^5}{8314(303)} = 4.021 \times 10^{-7} \text{ gmol/cm}^3$$

# 26.10 CONTINUED -

$$D_{AB} = 0.260 \left( \frac{303}{298} \right)^{3/2} = 0.246 \text{ cm}^2/\text{s}$$

$$R_i = 0.5 \text{ cm} \quad R_o = 0$$

Substitute & Solve -

$$t = 1.517 \times 10^6 \text{ s} = 421.4 \text{ h}$$

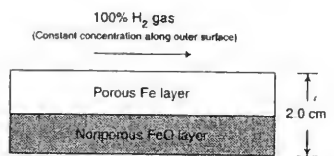
# 26.11

1 atm, 400 K

$$D_{AB} = 1.7 \text{ cm}^2/\text{s}$$

$$\rho_{FeO} = 2.5 \text{ g/cm}^3$$

$$M_{FeO} = 71.85$$



Steady State  
No Homog. Rx

$$\nabla^2 N_A = \frac{dN_{Az}}{dz} = 0 \sim N_{Az} \sim \text{CONST.}$$

$$N_{Az} = -C_{DAB} \frac{dy_A}{dz} + y_A (N_{Az} + N_{Bz})$$

$$\text{As } N_{Bz} = -N_{Az} \quad y_A (N_{Az} + N_{Bz}) = 0$$

$$N_{Az} \int_0^{\delta} dz = -C_{DAB} \int_0^{\delta} \frac{dy_A}{1-y_A}$$

$$N_{Az} = \frac{C_{DAB}}{\delta} \quad (a)$$

$$C = \frac{P}{RT} = \frac{1}{82.06(400)} = 3.047 \times 10^{-5} \text{ gmol/cm}^3$$

$$N_{Az} = 5.18 \times 10^{-5} \text{ gmol/cm}^2 \cdot \text{s} \quad (b)$$

For  $0.1 < \delta < 0.2$

$$W = N_{Az}(1) = \rho \frac{d\delta}{dt}$$

$$\frac{M_B}{\rho_B} D_{AB} C \int_0^t dt = \int_{\delta_1}^{\delta_2} \delta d\delta$$

26.11 CONTINUED -

$$\frac{M_B D_{AB} C}{S_B} t = \frac{\delta_2^2 - \delta_1^2}{2}$$

SUBSTITUTING NUMERICAL VALUES:

$$t = 1007 \text{ s} = 16.78 \text{ min.}$$

26.12

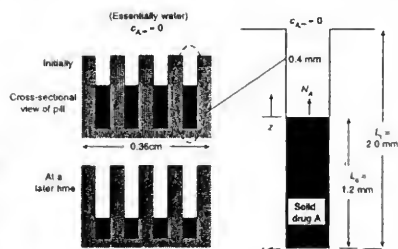
$$T = 310 \text{ K}$$

$$D_{AB} = 2 \times 10^{-5} \text{ cm}^2/\text{s}$$

$$\rho_A = 1.10 \text{ g/cm}^3$$

$$M_A = 120$$

$$C_A^* = 2.0 \times 10^{-4} \text{ g mol/cm}^3$$



PSEUDO STATIONARY, NO HOMOGENEOUS Rx  
ONE-DIMENSIONAL - DILUTE SOLN

$$\nabla \cdot \vec{N}_A = \frac{dN_A}{dz} = 0 \sim N_{A2} \text{ CONST.}$$

$$N_{A2} = -D_{AB} \frac{dc_A}{dz}$$

$$N_{A2} \int_{z_1}^{z_2} dz = -D_{AB} \int_{z_1}^{z_2} dc_A$$

$$N_{A2} = \frac{D_{AB} C_A^*}{z_2 - z_1} \quad (a)$$

FOR A PORE:

$$W_{A2} = N_{A2} A = \frac{D_{AB} C_A^* A}{z_2 - z_1}$$

$$W_A = \frac{(2 \times 10^{-5}) (2.0 \times 10^{-4}) (\pi/4) (0.04)^2}{0.2 - 0.12}$$

$$= 6.3 \times 10^{-11} \text{ g mol/s Per Pore}$$

26.12 CONTINUED -

FOR 1 PORE ~ 16 PORES ~

$$W_A = 6.3 \times 10^{-11} (16) = 1.008 \times 10^{-9} \text{ g mol/s} \quad (b)$$

TIME TO DISSOLVE -

$$\frac{M_B}{S_B} \frac{d\delta}{dt} = \frac{D_{AB} C_A^*}{\delta}$$

$$\int_{0.08}^{0.12} \delta d\delta = \frac{D_{AB} C_A^* M_B}{S_B} \int_0^t dt$$

$$\frac{\delta^2}{2} \Big|_{0.08}^{0.12} = \frac{D_{AB} C_A^* M_B}{S_B} t$$

$$t = 3.65 \times 10^4 \text{ s} = 10.14 \text{ h}$$

26.13 FOR CONDITIONS DESCRIBED -

$$\nabla \cdot \vec{N}_A = \frac{dN_A}{dz} = 0 \quad N_{A2} \sim \text{CONSTANT}$$

$$N_{A2} = -C D_{AB} \frac{dy_A}{dz} + y_A (N_{A2} + N_{B2})$$

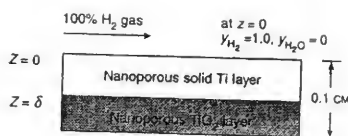
IN EACH REACTION -  $N_{B2} = -N_{A2}$

$$\therefore N_{A2} = -C D_{AB} \frac{dy_A}{dz}$$

$$N_{A2} = \frac{C D_{AB} (y_{A0} - 0)}{\delta}$$

ALL REACTIONS INVOLVE EQUIMOLAR DIFFUSION -

26.14



$$T = 900\text{K}$$

$$P = 1\text{ATM}$$

$$A - \text{H}_2$$

$$B - \text{H}_2\text{O}$$

FOR CONDITIONS STATED:

$$\nabla \cdot \vec{N}_A = \frac{dN_{Az}}{dz} = 0 \quad N_{Az} \text{ CONST.}$$

$$N_{Az} = -CD_{AB} \frac{dy_A}{dz} + y_A(N_{Az} - N_{Bz})$$

$$\text{SINCE } N_{Bz} - N_{Az} = 0$$

$$N_{Az} = -CD_{AB} \frac{dy_A}{dz}$$

INTEGRATING:

$$N_{Az} = \frac{CD_{AB} y_{A0}}{\delta}$$

$$C = \frac{P}{RT} = \frac{1}{(82.06)(900)} = 1.354 \times 10^{-5} \text{ mol/cm}^3$$

FOR  $\delta = 0.05 \text{ cm}$ 

$$N_{Az} = \frac{(0.031)(1.354 \times 10^{-5})}{0.05}$$

$$= 8.39 \times 10^{-6} \text{ mol/cm}^2 \cdot \text{s} \quad (a)$$

BY STOICHIOMETRY:

$$\left\{ \begin{array}{l} \text{RATE OF} \\ \text{Ti DEPOSITED} \end{array} \right\} = \frac{1}{2} \left\{ \begin{array}{l} \text{RATE OF} \\ \text{A}_2 \text{ DIFFUSED} \end{array} \right\}$$

$$\frac{S_{Ti}}{M_{Ti}} \frac{d\delta}{dt} = \frac{1}{2} \frac{D_{AB} C_{A0}}{\delta}$$

$$\frac{S_{Ti}}{M_{Ti}} \int_0^{\delta} d\delta = \frac{D_{AB} C_{A0}}{2} \int_0^t dt$$

$$\delta = \left[ \frac{M_{Ti} D_{AB} C_{A0}}{S_{Ti}} \right]^{1/2} t^{1/2}$$

26.14 CONTINUED -

INSERTING VALUES - FOR  $\delta = 0.1 \text{ cm}$ 

$$t = 1293 \text{ s} = 0.359 \text{ h} \quad (b)$$

$$@ \delta = 0.05 \text{ cm}; N_A = 8.39 \times 10^6 \text{ mol/cm}^2 \cdot \text{s}$$

$$= A \quad (A \text{ CONSTANT})$$

$$A \int_0^z dz = -D_{AB} \int_{C_{A0}}^{C_A} dC_A$$

$$C_A - C_{A0} = -\left[ A / D_{AB} \right] z$$

$$C_A = C_{A0} - \frac{A}{D_{AB}} z \quad (c)$$

26.15 ACETONE (A) DIFFUSING IN AIR (B)

$$D_{AB}|_{298\text{K}} = 0.093 \text{ cm}^2/\text{s}$$

$$D_{AB}|_{323\text{K}} = 0.093 \left( \frac{323}{298} \right)^{3/2} = 0.105 \text{ cm}^2/\text{s}$$

STEADY STATE - NO HOMOGENEOUS Rx

$$\nabla \cdot \vec{N}_A = \frac{dN_{Az}}{dz} = 0 \quad N_{Az} \text{ CONST.}$$

FOR  $T \& P$  CONSTANT  $N_{Az} = -N_{Bz}$ 

$$N_{Az} = -CD_{AB} \frac{dy_A}{dz} = \frac{CD_{AB} (y_{A1} - y_{A2})}{z_2 - z_1}$$

$$C = \frac{P}{RT} = \frac{1\text{ATM}}{82.06(323)} = 3.77 \times 10^{-5} \text{ mol/cm}^3$$

$$z_2 - z_1 = 500 \text{ cm} \quad y_{A1} - y_{A2} = 0.5$$

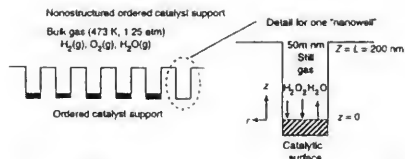
$$\text{SUBSTITUTING: } N_{Az} = 3.96 \times 10^{-9} \text{ mol/cm}^2 \cdot \text{s}$$

$$W_A = N_{Az}(A)$$

$$= (3.96 \times 10^{-9}) \left( \frac{\pi}{4} \right) (10 \text{ cm})^2$$

$$= 3.11 \times 10^{-7} \text{ mol/s}$$

26.16



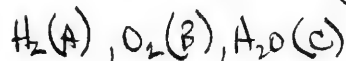
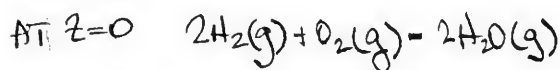
$$d = 5 \times 10^{-6} \text{ cm} \quad \Delta z = 2 \times 10^{-5} \text{ cm}$$

$$T = 473 \text{ K} \quad P = 1.25 \text{ atm}$$

ASSUMPTIONS — STEADY  
NO HOMOGENEOUS RX  
ONE DIMENSIONAL

$$\nabla \cdot \vec{N}_A = \frac{d}{dz} N_{Az} = 0 \quad N_{Az} \text{ CONST.}$$

$$N_{Az} = -C D_{A-MIX} \frac{dy_A}{dz} + y_A (N_{Az} + N_{Bz} + N_{Cz})$$



$$N_{Bz} = \frac{1}{2} N_{Az} \quad N_{Cz} = -N_{Az}$$

$$N_{Az} = -C D_{AB} \frac{dy_A}{dz} + \frac{1}{2} y_A N_{Az}$$

$$N_{Az} \int_0^L dz = C D_{AB} \int_0^1 \frac{dy_A}{1 - y_A/2}$$

$$N_{Az} L = 2 C D_{AB} \ln \frac{1 - 0.1/2}{1 - 0}$$

$$N_{Az} = \frac{2 C D_{AB}}{L} (-0.050)$$

$$C = \frac{P}{RT} = \frac{1.25}{8206(473)} = 3.22 \times 10^{-5} \text{ mol/m}^3$$

$$D_{AB} = \frac{0.697}{1.25} \left( \frac{473}{273} \right)^{3/2} = 1.272 \text{ cm}^2/\text{s}$$

$$D_{AC} = \frac{0.850}{1.25} \left( \frac{473}{273} \right)^{3/2} = 1.551 \text{ cm}^2/\text{s}$$

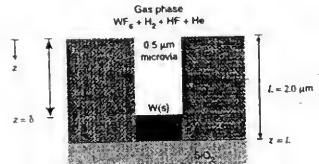
26.16 CONTINUED —

$$D_{H_2-MIX} = \frac{1}{\frac{y_B}{D_{AB}} + \frac{y_C}{D_{AC}}} = 1.274 \text{ cm}^2/\text{s}$$

SUBSTITUTING NUMERICAL VALUES!

$$N_{Az} = -0.0205 \text{ mol/cm}^2 \cdot \text{s}$$

26.17  $W_F(\text{A})$   
KNUDSEN DIFFUSION

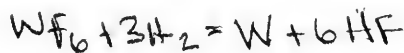


VERY DILUTE

$$\nabla \cdot \vec{N}_A = \frac{d}{dz} N_{Az} = 0 \quad N_{Az} \text{ - CONSTANT}$$

$$N_{Az} = -D_{AB} \frac{dc_A}{dz}$$

$$= \frac{D_{AB} C_{A0}}{\delta}$$



RATE OF FORMATION OF W

$$= N_{Dz}(\text{A}) = \frac{D_{AB} C_{A0}(\text{A})}{\delta} = \frac{P_W}{M_W} \frac{d(A\delta)}{dt}$$

$$\frac{P_W}{M_W} \frac{d\delta}{dt} = \frac{D_{AB} C_{A0}}{\delta}$$

$$\delta^2 = 2 D_{AB} \frac{M_W C_{A0} t}{P_W}$$

$$\delta = \left[ 2 D_{AB} \frac{M_W C_{A0}}{P_W} t \right]^{1/2}$$

FOR KNUDSEN DIFFUSION — EQN(24-58)

$$D_{KA} = 4850 d_p \sqrt{\frac{T}{M_{WF6}}}$$

$$d_p = 2.5 \times 10^{-5} \text{ cm}, T = 700 \text{ K}, M = 298$$

$$D_{KA} = 0.1858 \text{ cm}^2/\text{s}$$

26.17 CONTINUED

$$C = \frac{P}{RT} = \frac{75 \text{ Pa}}{8.314(100)} = 0.0129 \text{ mol/m}^3$$

$$C_{A0} = y_{A0} C = 1.29 \times 10^{-3} \text{ mol/cm}^3$$

SUBSTITUTING INTO EQN FOR  $\delta(t)$

$$t = 8.80 \times 10^4 \text{ s} = 24.44 \text{ h}$$

26.18  $C_6H_6(A)$  IN  $C_7H_8$

$$h_{fg}(A) = 30 \text{ kJ/mol}$$

$$h_{fg}(B) = 33 "$$

$$\nabla \cdot \vec{N}_A = \frac{dN_{Az}}{dz} = 0 \quad N_{Az} \text{ CONST.}$$

$$N_{Az} = -CD_{AB} \frac{dy_A}{dz} + y_A(N_{Az} + N_{Bz})$$

$$N_{Az}(30) = N_{Bz}(33)$$

$$N_{Bz} = -0.909 N_{Az}$$

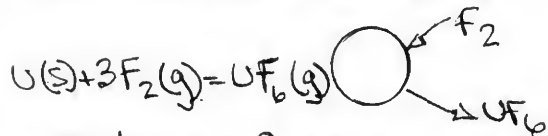
$$N_{Az} = -CD_{AB} \frac{dy_A}{dz} + y_A N_{Az} (1 - 0.909)$$

$$N_{Az} \int_0^{\delta} dz = -CD_{AB} \int_{y_{A0}}^{y_{As}} \frac{dy_A}{1 - 0.091 y_A}$$

$$N_{Az} \delta = \frac{CD_{AB}}{0.091} \ln \left[ \frac{1 - 0.091 y_{As}}{1 - 0.091 y_{A0}} \right]$$

$$N_{Az} = \frac{CD_{AB}}{0.091 \delta} \ln \left[ \frac{1 - 0.091 y_{As}}{1 - 0.091 y_{A0}} \right]$$

26.19 SPHERICAL GEOMETRY -



$$T = 1000 \text{ K} \quad P = 1 \text{ atm}$$

$$D_{AB} = 0.273 \text{ cm}^2/\text{s} \quad d = 0.4 \text{ cm}$$

STEADY STATE, NO HOMOGENEOUS Rx

$$\nabla \cdot \vec{N}_A = \frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) = 0 \quad r^2 N_{Ar} \sim \text{CONST.}$$

$$N_{Ar} = -CD_{AB} \frac{dy_A}{dr} + y_A(N_{Ar} + N_{Br})$$

$$N_{Br} = -3N_{Ar} \Rightarrow N_{Ar} + N_{Br} = -2N_{Ar}$$

$$N_{Ar} = -\frac{CD_{AB}}{1 + 2y_A} \frac{dy_A}{dr}$$

$$\underbrace{4\pi r^2 N_{Ar}}_{W_A} \int_R^{\infty} \frac{dr}{r^2} = 4\pi CD_{AB} \int_{1.0}^0 \frac{dy_A}{1 + 2y_A}$$

$$W_A \left( \frac{1}{R} \right) = \frac{4\pi CD_{AB}}{2} \ln \frac{3.0}{1.0}$$

$$W_A = 2\pi R CD_{AB} \ln 3$$

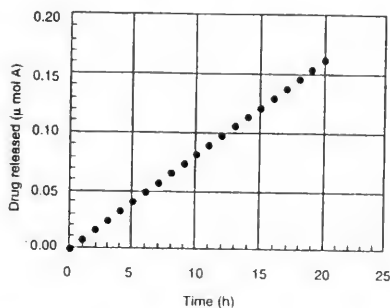
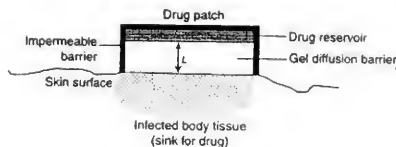
$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{8.314(1000)} = 12.18 \text{ mol/m}^3$$

$$= 1.218 \times 10^{-5} \text{ mol/cm}^3$$

SUBSTITUTING VALUES:

$$W_A = 4.59 \times 10^{-6} \text{ mol/s}$$

26,20

SLOPE OF PLOT IS  $W_A$ :

$$W_A \approx \frac{0.15 \mu\text{mol}}{18.5 \text{ h}} \left( \frac{1}{3600} \right)$$

$$= 2.25 \times 10^{-12} \text{ mol/s}$$

$$A_s = 9 \text{ cm}^2$$

$$N_{A2} = 2.503 \times 10^{-13} \text{ mol/s} \cdot \text{cm}^2$$

SINCE PLOT IS LINEAR --

ALL TRANSPORT IS DIFFUSION

$$N_{A2} = D_{AB} \frac{C_{A1} - C_{A2}}{z_2 - z_1}$$

$$D_{AB} = \frac{2.503 \times 10^{-13} (0.2)}{0.5 \times 10^{-6}}$$

$$= 1.00 \times 10^{-7} \text{ cm}^2/\text{s}$$

MODIFIED WILKE-CRANK - EQN (24-54)

$$D_{AB} \frac{\mu}{T} = \text{CONST}$$

$$D_{AB|35} = D_{AB|20} \left( \frac{293}{308} \right) \left( \frac{\mu_{H_2O|35}}{\mu_{H_2O|20}} \right)$$

$$= 1.0 \times 10^{-7} \left( \frac{293}{308} \right) \left( \frac{0.00993}{0.00742} \right)$$

$$= 1.273 \times 10^{-7} \text{ cm}^2/\text{s}$$

26,20 CONTINUED -

ALL OTHER TERMS REMAIN THE SAME

$$W_A|_{35} = W_A|_{20} \frac{D_{AB|35}}{D_{AB|20}}$$

$$= (2.25 \times 10^{-12}) \frac{1.273 \times 10^{-7}}{1 \times 10^{-7}}$$

$$= 2.864 \times 10^{-12} \text{ mol/s}$$

$$= 2.475 \times 10^{-7} \text{ mol/DAY}$$

$$26.21 \quad J_{A2} = -CD_{AB} \frac{\partial C_A}{\partial z} = \frac{D_{AB}}{\Delta z} (C_{A1} - C_{A2})$$

$$C_{A1} - C_{A2} = k \left( p_{A1}^{1/2} - p_{A2}^{1/2} \right)$$

$$\Rightarrow J_{A2} = D_{AB} k \left( p_{A1}^{1/2} - p_{A2}^{1/2} \right)$$

$$\text{AT 1 ATM } C_{A1} = k p_A^{1/2}$$

$$= \frac{7 \text{ cm}^3}{100 \text{ g}} \left( \frac{9 \text{ g}}{\text{cm}^3} \right) = 0.63$$

$$k = 0.63 \text{ ATM}^{-1/2}$$

$$D_{AB} = 6 \times 10^{-5} \text{ cm}^2/\text{s} \quad p_{A1} = 8 \text{ atm} \quad p_{A2} = 0$$

$$\Delta z = 0.2 \text{ cm}$$

$$\text{SUBSTITUTING: } J_{A2} = 5.346 \times 10^{-4} \text{ cm/s}$$

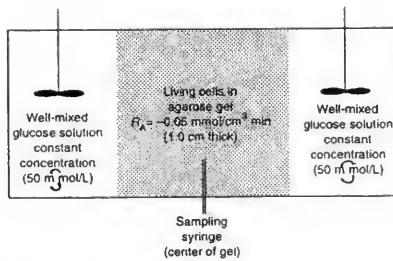
$$W_A = J_{A2} A = (5.346 \times 10^{-4}) (8)$$

$$= 4.277 \text{ cm}^3/\text{s}$$

$$= 15.4 \text{ cm}^3/\text{DAY}$$

246

26.22



$$\nabla \cdot \vec{N}_A - R_A = 0$$

$$\frac{dN_{Az}}{dz} - R_A = 0$$

FOR NO BULK CONTRIBUTION

$$N_{Az} = -D_{AB} \frac{dC_A}{dz}$$

$$\frac{d}{dz} \left( -D_{AB} \frac{dC_A}{dz} \right) = R_A$$

$$d \left( -D_{AB} \frac{dC_A}{dz} \right) = R_A dz$$

$$-D_{AB} \frac{dC_A}{dz} = R_A z + C_1$$

$$-D_{AB} C_A = R_A \frac{z^2}{2} + C_1 z + C_2$$

$$\text{B.C. } C_A(0.5 \text{ cm}) = C_{A0}$$

$$\frac{dC_A}{dz}(0) = 0 \Rightarrow C_1 = 0$$

$$C_2 = -D_{AB}(C_{A0}) - \frac{R_A}{2}(0.5)^2$$

$$C_A = -\frac{1}{D_{AB}} \left[ R_A \frac{z^2}{2} + C_1 z + C_2 \right]$$

WITH VALUES SUBSTITUTED

$$C_A = C_{A0} - \frac{R_A}{D_{AB}} \left( \frac{z^2}{2} - 0.125 \right)$$

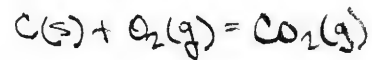
26.23 CYLINDRICAL GEOMETRY --

$$T = 1100 \text{ K} \quad P = 2 \text{ ATM}$$

$$\delta(\text{B.L. Film}) = 5 \text{ mm} \quad L = 25 \text{ cm}$$

$$d = 2 \text{ cm}$$

$$\nabla \cdot \vec{N}_A = \frac{1}{r} \frac{d}{dr} (r N_{Ar}) = 0 \quad r N_{Ar} \sim \text{CONST.}$$



$$O_2 \text{ IS A} \quad N_{Br} = -N_{Ar}$$

$$CO_2 \text{ IS B} \quad N_{Ar} = -C D_{AB} \frac{dy_A}{dr}$$

$$\frac{2\pi L r N_{Ar}}{W_A} \int_{r_1}^{r_2} \frac{dr}{r} = -2\pi L C D_{AB} \int_0^{0.21} dy_A$$

$$W_A \ln \frac{r_2}{r_1} = -2\pi L C D_{AB} (0.21)$$

$$W_A = -\frac{2\pi L C D_{AB} (0.21)}{\ln(r_2/r_1)}$$

$$C = \frac{P}{RT} = \frac{2.026 \times 10^5}{(8.314)(1100)} = 22.1 \text{ mol/m}^3 = 2.21 \times 10^{-5} \text{ mol/cm}^3$$

$$D_{AB} = 0.175 \left( \frac{1}{2} \right) \left( \frac{1100}{273} \right)^{3/2} = 0.708 \text{ cm}^2/\text{s}$$

$$\text{AT } t=0 \quad r_2/r_1 = 1.5$$

SOLVING FOR  $W_A$  AT  $t=0$ 

$$W_A = -1.28 \times 10^{-3} \text{ mol/s}$$

FOR  $t > 0 \sim r_1$  DECREASES

$$W_A = -\frac{2\pi L C D_{AB} (0.21)}{\ln(r_2/r_1)}$$

26.23 CONTINUED -

FOR SOLID C: RATE OF DEPLETION =  $\frac{\rho}{M} \frac{dV}{dt}$

PER MOLE:  $W_A = -\frac{\rho}{M} \frac{dV}{dt}$

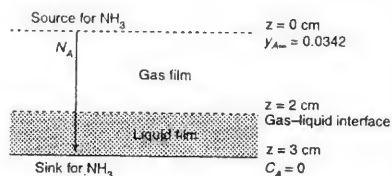
$$\frac{2\pi r L C_{DAB} (0.21)}{\ln\left[\frac{(r+0.5)}{r}\right]} = \frac{\rho}{M} (2\pi r L) \frac{dr}{dt}$$

INTEGRATING BETWEEN  $r=r_i$  @  $t=0$   
 $r=0$  @  $t$

THE SOLUTION ~ A BIT MESSY ~

$$t = 18000 \text{ s (5 h)}$$

26.24



EQUILIBRIUM  
DATA →

$P_A$ (mmHg)	5.0	10.0	15.0	20.0	25.0	30.0
$c_A$ (mol/m <sup>3</sup> )	6.1	11.9	20.0	32.1	53.6	84.8

NH<sub>3</sub> (A) DIFFUSES, IN SERIES, THROUGH  
GAS & LIQUID LAYERS

THROUGH GAS:  $N_{A2} = \frac{C D_{AB}}{\delta_L} \ln\left(\frac{1-y_{Ai}}{1-y_{A1}}\right)$

LIQUID:  $N_{A2} = \frac{D_L}{\delta_L} (C_{Ai} - C_{As})$

$$D_{AB} = 0.198 \left(\frac{298}{273}\right)^{3/2} = 0.245 \text{ cm}^2/\text{s}$$

$$D_L = 1.77 \times 10^{-5} \text{ cm}^2/\text{s}$$

$$N_{A2} = N_{AL}$$

$$\frac{C D_{AB}}{\delta_L} \ln\left(\frac{1-y_{Ai}}{1-y_{A1}}\right) = \frac{D_L}{\delta_L} (C_{Ai} - C_{As})$$

26.24 CONTINUED -

$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{(8314)(298)} = 42.3 \text{ mol/m}^3$$

$$= 4.23 \times 10^{-5} \text{ mol/cm}^3$$

INSERTING VALUES:

$$0.257 \ln \frac{1-y_{Ai}}{0.9658} = C_{Ai}$$

VALUES OF  $y_{Ai}$ ,  $C_{Ai}$  MUST AGREE  
WITH PLOT OF DATA --

TRIAL & ERROR IS NECESSARY --

~ RESULT IS  $p_{AL} = 25.88 \text{ mm}$

$$~ y_{Ai} = \frac{25.88}{760} = 0.0339$$

$$C_{Ai} = 5.58 \times 10^{-5} \text{ mol/cm}^3$$

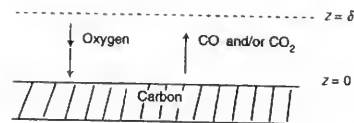
WITH THESE VALUES --

$$N_{A2} = \frac{(1.77 \times 10^{-5})(5.58 \times 10^{-5})}{1}$$

$$= 9.88 \times 10^{-10} \text{ mol/cm}^2 \cdot \text{s}$$

26.25

CONSTITUENT A IS O<sub>2</sub>



$$\nabla \cdot \vec{N}_A = \frac{d}{dz} N_{A2} = 0 \quad N_{A2} \text{ -- CONSTANT}$$

$$N_{A2} = -C D_{AB} \frac{dy_A}{dz} + y_A (N_{A2} + N_{B2})$$

REACTION AT SURFACE IS  $C + O_2 = 2CO$

$$~ 2N_{A2} = -N_{B2}$$

$$y_A (N_{A2} + N_{B2}) = y_A N_{A2} (-1)$$

26.25 CONTINUED -

$$N_{Az}(1+y_A) = -C_{DAB} \frac{dy_A}{dz}$$

SEPARATING VARIABLES & INTEGRATING!

FROM  $z=0$  TO  $\delta$   $y_A$  FROM 0 TO 0.21

$$N_{Az} = -\frac{C_{DAB}}{\delta} \ln 1.21 \quad (a)$$

IF REACTION AT SURFACE IS

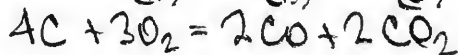


$$\text{THEN } y_A(N_{Az} + N_{Bz}) = 0$$

& SOLUTION IS

$$N_{Az} = -\frac{C_{DAB}}{\delta} (0.21) \quad (b)$$

IF REACTION AT SURFACE IS



$$\text{THEN } N_{Bz} = N_{Cz} = -\frac{2}{3} N_{Az}$$

$$y_A(N_{Az} + N_{Bz} + N_{Cz}) = y_A N_{Az} \left(-\frac{1}{3}\right)$$

FICK'S LAW EXPRESSION BECOMES

$$N_{Az} = -C_{DAB} \frac{dy_A}{dz} - y_A \frac{N_{Az}}{3}$$

$$N_{Az} \left(1 + \frac{y_A}{3}\right) = -C_{DAB} \frac{dy_A}{dz}$$

& SOLUTION IS

$$\begin{aligned} N_{Az} &= -\frac{C_{DAB}}{\delta} [3 \ln 1.07] \\ &= \frac{C_{DAB}}{\delta} (0.203) \quad (c) \end{aligned}$$

26.26

Time (h)	Measured SiO <sub>2</sub>	film thickness (μm)
	(100) Si	(111) Si
1	0.049	0.070
2	0.078	0.105
4	0.124	0.154
7	0.180	0.212
16	0.298	0.339

SYSTEM CONSIDERED WAS EVALUATED IN TEXT - EXAMPLE 2.

$$\delta^2 = \frac{2M_B D_{AB} C_{AS} t}{S_B}$$

FROM DATA IN TABLE -

$$\frac{d\delta^2}{dt} = \frac{2M_B D_{AB} C_{AS}}{S_B} = A$$

$$D_{AB} = \frac{AS_B}{2M_B C_{AS}}$$

A EVALUATED FROM DATA VARIES FROM 0.0049 TO 0.00718 ~ TAKE MIDDLE VALUE (CONDITION @  $t=4h$ )

$$\begin{aligned} A &= 0.00593 \mu m^2/h \\ &= 1.646 \times 10^{-14} cm^2/s \end{aligned}$$

$$\begin{aligned} D_{AB} &= \frac{(1.646 \times 10^{-14})(2.27)}{2(60)(9.68 \times 10^{-8})} \\ &= 3.24 \times 10^{-9} cm^2/s \end{aligned}$$

26.27 CYLINDRICAL GEOMETRY -

$$\nabla \cdot \vec{N}_A = \frac{1}{r} \frac{d}{dr} (r N_{Ar}) = 0 \quad r N_{Ar} = \text{CONST.}$$

$$\begin{aligned} A &\sim O_2 \\ B &\sim CO \end{aligned} \quad N_A = -C_{DAB} \frac{dy_A}{dr} + y_A (N_{Ar} + N_{Br})$$

REACTION IS  $2C + O_2 = 2CO$

$$N_{Br} = -2N_{Ar}$$

26.27 CONTINUED -

$$N_{Ar} = -\frac{C_{DAB}}{1+y_A} \frac{dy_A}{dr}$$

$$\underbrace{2\pi L r N_{Ar}}_{W_A} \int_{r_1}^{r_2} \frac{dr}{r} = -2\pi L C_{DAB} \int_0^{0.14} \frac{dy_A}{1+y_A}$$

$$W_A = -\frac{2\pi L C_{DAB}}{\ln(r_2/r_1)} \ln(1.14)$$

$$C = \frac{P}{RT} = \frac{1 \text{ atm}}{(82.04)(1145)} = 1.065 \times 10^{-5} \text{ mol/cm}^3$$

$$D_{AB} = 1.0 \times 10^{-5} \text{ m}^2/\text{s} = 0.10 \text{ cm}^2/\text{s}$$

SUBSTITUTING NUMERICAL VALUES -

$$W_A = -4.099 \times 10^{-5} \text{ mol/s} \quad (a)$$

$$W_B = W_{CO} = -2W_A = \underline{8.20 \times 10^{-5} \text{ mol/s}}$$

FOR CONCENTRATION PROFILE:

$$\frac{d}{dr} r N_{Ar} = \frac{d}{dr} \left[ -r \frac{C_{DAB}}{1+y_A} \frac{dy_A}{dr} \right] = 0$$

INTEGRATE ONCE:

$$\frac{r}{1+y_A} \frac{dy_A}{dr} = C_1$$

$$\text{AGAIN: } \ln(1+y_A) = C_1 \ln r + C_2$$

$$\text{B.C. } y_A(r_1=0.5) = 0$$

$$y_A(r_2=1.5) = 0.14$$

$$\text{SOLVING: } C_1 = 0.304 \quad C_2 = 0.212$$

Now - For  $r=1$

26.27 CONTINUED -

$$\ln(1+y_A) = C_1 \ln(r) + C_2$$

$$y_A = e^{0.212} - 1 = \underline{0.236}$$

26.28 PROBLEM STATEMENT REFERS TO EXAMPLE 4 IN CHAPTER

FOR SPHERICAL GEOMETRY -

$$\frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) = -k_1 C_A$$

WITH PURE DIFFUSION:

$$N_{Ar} = -D_{AB} \frac{dc_A}{dr}$$

$$\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{dc_A}{dr}) = -k_1 C_A$$

$$\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{dc_A}{dr}) = \frac{k_1}{D_{AB}} C_A \quad (1)$$

$$\text{LETTING } y = C_A r \sim \frac{dy}{dr} = C_A + r \frac{dC_A}{dr}$$

$$\text{WE HAVE: } r^2 \frac{dC_A}{dr} = r \frac{dy}{dr} - y \quad (2)$$

SUBSTITUTING (2) INTO (1) WE HAVE

$$\frac{d^2 y}{dr^2} - \frac{k_1}{D_{AB}} y = 0$$

SOLUTION IS

$$y = C_1 \cosh\left(\sqrt{\frac{k_1}{D_{AB}}} r\right) + C_2 \sinh\left(\sqrt{\frac{k_1}{D_{AB}}} r\right)$$

$$\text{B.C. } y(0) = 0 \quad y(R) = C_{A0}$$

$$\therefore C_1 = 0 \quad C_2 = \frac{C_{A0} R}{\sinh\left(\sqrt{\frac{k_1}{D_{AB}}} R\right)}$$

26.28 CONTINUED -

$$\frac{C_A}{C_{A0}} = \frac{R}{r} \frac{\sinh\left(\frac{\sqrt{k_1/D_{AB}} r\right)}{\sinh\left(\frac{\sqrt{k_1/D_{AB}} R\right)} \quad (a)$$

TO EVALUATE  $N_{Ar}$  - MUST KNOW  $\frac{dC_A}{dr}$  FROM (a)

$$\frac{dC_A}{dr} = \frac{C_{A0} R}{\sinh\left(\frac{\sqrt{k_1/D_{AB}} R\right)} \times \left[ -\frac{1}{r^2} \sinh\left(\frac{\sqrt{k_1/D_{AB}} r\right) + \frac{\sqrt{k_1/D_{AB}}}{r} \cosh\left(\frac{\sqrt{k_1/D_{AB}} r\right) \right]$$

EVALUATING AT  $r = R$ :

$$\left. \frac{dC_A}{dr} \right|_r = \frac{C_{A0}}{R} + C_{A0} \frac{\sqrt{k_1/D_{AB}}}{R} \coth\left(\frac{\sqrt{k_1/D_{AB}} R\right)$$

∴ FINALLY -

$$N_{Ar} = \frac{D_{AB} C_{A0}}{R} \left[ 1 - R \frac{\sqrt{k_1/D_{AB}}}{R} \coth\left(\frac{\sqrt{k_1/D_{AB}} R\right) \right]$$

FROM EXAMPLE 4:  $D_{AB} = 2 \times 10^{-10} \text{ m}^2/\text{s}$

$$R = 0.002$$

$$C_{A0} = 0.02 \text{ mol/m}^3$$

$$k_1 = 0.019 \text{ s}$$

SUBSTITUTING:

$$N_{Ar} = 1.02 \times 10^{-12} \text{ mol/m}^2 \cdot \text{s}$$

26.29 FLAT CATALYTIC SURFACE:

$$\frac{dN_{Az}}{dz} - k_1 y_B = 0 \quad (1)$$

$$\frac{dN_{Bz}}{dz} + k_1 y_B = 0 \quad (2)$$

$$\text{ADDING: } \frac{d}{dz} (N_{Az} + N_{Bz}) = 0 \quad \{ N_{Bz} = -N_{Az} \}$$

$$N_{Az} = -C_{DAB} \frac{dy_B}{dz}$$

EQU (1) BECOMES:

$$C_{DAB} \frac{d^2 y_B}{dz^2} - k_1 y_B = 0$$

$$\text{OR } \frac{d^2 y_B}{dz^2} - \frac{k_1}{C_{DAB}} y_B = 0$$

$$\text{LETTING } b^2 = k_1 / C_{DAB}$$

$$\frac{d^2 y_B}{dz^2} - b^2 y_B = 0$$

∴ SOLUTION IS

$$y_B = C_1 \cosh bz + C_2 \sinh bz$$

$$\text{B.C. } y(0) = y_{B0} \quad y(\delta) = 1$$

$$\text{GIVING } C_1 = y_{B0}$$

$$C_2 = \frac{1 - y_{B0} \cosh b\delta}{\sinh b\delta}$$

DOING THE MATH:

$$N_{Az} = b C_{DAB} \left[ \frac{1 - y_{B0} \cosh b\delta}{\sinh b\delta} \right]$$

26.30 SAME CONFIGURATION AS IN  
~~PROB~~ 26.29 EXCEPT IN FILM  
 $A \xrightarrow[k_1]{k'_1} B \quad R_A = k_1 y_B - k'_1 y_A$

FICK'S LAW:  $N_{A2} = -CD_{AB} \frac{dy_A}{dz}$

CONSERVATION OF MASS:

$$\frac{dN_{A2}}{dz} - k_1 y_B + k'_1 y_A = 0$$

$$\therefore -CD_{AB} \frac{d^2 y_A}{dz^2} - k_1 (1 - y_A) + k'_1 y_A = 0$$

WITH A LITTLE ALGEBRA WE GET

$$\frac{d^2 y_A}{dz^2} - \frac{k_1 + k'_1}{CD_{AB}} y_A = -\frac{k_1}{CD_{AB}}$$

DEFINING  $M^2 = \frac{k_1 + k'_1}{CD_{AB}}$

$$N^2 = k_1 / CD_{AB}$$

OUR EQN FOR  $y_A(z)$  IS

$$\frac{d^2 y_A}{dz^2} - M^2 y_A = -N^2$$

SOLN IS:

$$y_A = C_1 \cosh Mz + C_2 \sinh Mz + \frac{N^2}{M^2}$$

B.C.  $y_A(0) = y_{A0}$

$$y_A(\delta) = 0$$

~ DOING THE MATH ~

$$C_1 = y_{A0} - \frac{N^2}{M^2}$$

26.30 CONTINUED -

$$C_2 = \frac{\left(\frac{N^2}{M^2} - y_{A0}\right) \cosh M\delta - \frac{N^2}{M^2}}{\sinh M\delta}$$

## CHAPTER 27

### 27.1 - SEMI INFINITE BODY OF LIQUID

- THIS CASE IS DISCUSSED IN TEXT  
EQN (27-9) APPLIES

$$\frac{p_A - p_{A0}}{p_{As} - p_{A0}} = 1 - \operatorname{erf}\left(\frac{z}{2\sqrt{D_{AB}t}}\right)$$

FOR  $O_2$  IN  $H_2O$  - WILKE-CHANG,  
EQN. (24-52) APPLIES

$$D_{AB} = \frac{7.4 \times 10^{-8} (\phi_B \mu_B)^{1/2} T}{(V_B)^{0.6} \mu_B}$$

VALUES:  $\phi_B = 2.26$   $\mu_B = 18$

$T = 283$   $V_B = 25.6$

$\mu_B = 0.01394$  cp

$$D_{AB} = 1.469 \times 10^{-3} \text{ cm}^2/\text{s}$$

$$\text{@ } t = 3600 \text{ s} - 2\sqrt{D_{AB}t} = 4.60$$

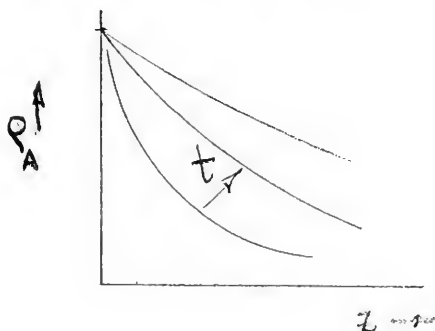
$$\text{@ } 36000 - = 14.54$$

$$\text{@ } 360000 - = 46.10$$

EQN - @ 3600 s -

$$\frac{p_A - 2}{9 - 2} = 1 - \operatorname{erf}\left(\frac{z}{46.10}\right)$$

LONG. PROFILES APPEAR AS  $\beta$



### 27.2 $O_2$ DISSOLVING INTO POLYMER FILM

$$C_{As} = 3.16(1.5) = 4.74 \text{ g mol/m}^3$$

FOR  $t = 10 \text{ s}$  - VERY SHORT PENETRATION -  
EQN (27-11) APPLIES

$$\begin{aligned} N_{A2} &= \sqrt{\frac{D_{AB}}{\pi t}} (C_{As} - C_{A0}) \\ &= \left[ \frac{1 \times 10^{-11}}{\pi (10)} \right]^{1/2} (4.74 - 0.39) \\ &= 2.45 \text{ g mol/m}^2 \cdot \text{s} \quad a) \end{aligned}$$

FOR  $C_A = 3 \text{ g mol/m}^3$  @  $z = 4 \text{ mm}$

$$\frac{C_A - C_{A0}}{C_{As} - C_{A0}} = 1 - \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{3 - 0.39}{4.74 - 0.39} = 0.6 = 1 - \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}} = 0.4$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 0.372$$

$$\begin{aligned} t &= 2.89 \times 10^6 \text{ s} \\ &= 802.8 \text{ h} \\ &= 33.4 \text{ DAYS} \end{aligned}$$

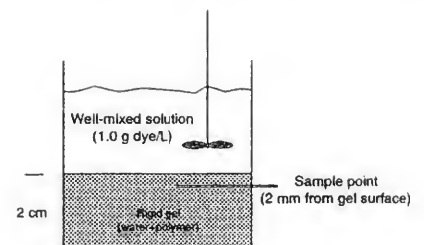
### 27.3

$$C_{A0} = 0$$

$$C_{As} = 1.0$$

FOR  $t = 24 \text{ h}$

$$C_A = 0.203 \text{ @ } z = 2 \text{ mm}$$



27.3 CONTINUED -

$$\frac{C_A S - C_A}{C_A S - C_{A0}} = \text{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{1.0 - 0.203}{1.0} = 0.797 = \text{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\text{For } t = 24 \text{ h} = 86400 \text{ s}$$

$$z = 0.2 \text{ cm}$$

$$D_{AB} = 1.43 \times 10^{-7} \text{ cm}^2/\text{s} \quad a)$$

ASSUMPTIONS:

1. SURFACE CONCENTRATION CONSTANT  $\sim C_A S(t)$
2. ONE DIRECTIONAL DIFFUSION
3. CONSTANT  $D_{AB}$  b)

USING WILKE-CAMM EQR:  
(24-54)

$$D_{AB/T_2} = D_{AB/T_1} \frac{\mu_{B1} T_2}{\mu_{B2} T_1}$$

$$= 1.43 \times 10^{-7} \left( \frac{9.93 \times 10^{-4} (313)}{6.58 \times 10^{-4} (293)} \right)$$

$$= 2.30 \times 10^{-7} \text{ cm}^2/\text{s} \quad (c)$$

27.4

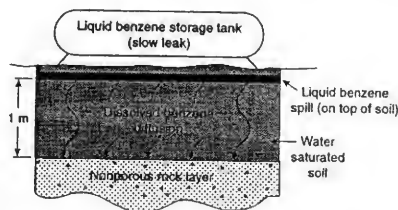
$$T = 293 \text{ K}$$

$$C_A^* = 24 \text{ mol/m}^3$$

$$C_{A0} = 0$$

$$t = 72 \text{ h}$$

$$= 2.59 \times 10^5 \text{ s}$$



$$D_{A \text{ EFF}} = 1 \times 10^{-9} \text{ m}^2/\text{s}$$

27.4 CONTINUED -

$$\frac{z}{2\sqrt{D_{AB}t}} = \frac{0.105}{2\sqrt{(1 \times 10^{-9})(2.59 \times 10^5)}}$$

$$= 1.553$$

$$\text{erf}(1.553) = 0.972 = \frac{C_A S - C_A}{C_A S - C_{A0}}$$

$$C_A = 0.672 \text{ mol/m}^3$$

27.5  $H_2$  INTO  $Fe$

$$D_{AB} = 1.24 \times 10^{-11} \text{ cm}^2/\text{s}$$

$$C_{A0} = 0$$

$$C_A @ 0.1 \text{ cm} = 1.76 \times 10^{-7} \text{ mol/g Fe}$$

$$C_{AS} = 2.2 \times 10^{-7} \text{ mol/g Fe}$$

$$T = 373 \text{ P} = 1 \text{ ATM}$$

$$\frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \text{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{2.2 - 1.76}{2.2 - 0} = 0.2 = \text{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 0.179$$

$$t = 6.27 \times 10^5 \text{ s} = 174 \text{ h}$$

27.6 HERBICIDE INTO SOIL -

$$D_{AB} = 1 \times 10^{-8} \text{ m}^2/\text{s} \quad t = 1800 \text{ s}$$

$$C_{AS} = 1 \quad C_{A0} = 0 \quad C_A = 0.001$$

$$\frac{C_A - C_{A0}}{C_{AS} - C_{A0}} = \frac{0.001}{1} = 1 - \text{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

27.6 CONTINUED -

$$\operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}} = 0.999$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 2.25$$

$$z = 2(2.25) \left[ (1 \times 10^{-3}) (1800) \right]^{1/2}$$

$$= \underline{6.037 \text{ m}}$$

27.7 BORON DIFFUSING INTO Si

$$C_B = 5 \times 10^{20} \text{ ATOMS/cm}^3$$

$$C_{A2} = 0.17 \times 10^{20} \text{ " @ } z = 2 \times 10^{-7} \text{ m}$$

$$C_{A0} = 0 \quad t = 1800 \text{ s}$$

$$\frac{C_{AS} - C_{A2}}{C_{AS} - C_{A0}} = \frac{(5 - 0.17) \times 10^{20}}{5 \times 10^{20}} = 0.966$$

$$= \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

FROM APPENDIX L  $\frac{z}{2\sqrt{D_{AB}t}} = 1.5$

$$D_{AB} = \left( \frac{2 \times 10^{-7}}{3} \right)^2 (1800)^{-1}$$

$$= 2.469 \times 10^{-18} \text{ m}^2/\text{s}$$

AS STATED -  $D_{AB} = D_0 e^{-Q_0/RT}$

$$\ln \frac{D_{AB}}{D_0} = -\frac{Q_0}{RT}$$

$$T = \frac{Q_0}{R \ln D_0/D_{AB}}$$

$$= \frac{2.74 \times 10^5}{(8.314) \ln \frac{1.9 \times 10^{-6}}{2.469 \times 10^{-18}}}$$

$$= \underline{1204 \text{ K}}$$

27.8 CARBON DIFFUSING INTO STEEL

$$W_{AS} = 0.007 \left\{ \frac{0.007 - W_A}{0.007 - 0.002} \right\} = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$W_{A0} = 0.002$$

$$\frac{0.007 - W_A}{0.005} = \operatorname{erf} \left[ \frac{z}{2(1 \times 10^{-11})(3600)} \right]$$

$$W_A = 0.007 - 0.005 \operatorname{erf} \left[ \frac{z}{3.7 \times 10^{-4} \text{ m}} \right]$$

for  $z = 0.01 \text{ cm}$

$$\operatorname{erf} [ ] = \operatorname{erf} (0.264) = 0.291$$

$$W_A = 0.007 - 0.005(0.291)$$

$$= \underline{0.55 \text{ wt \% C}}$$

for  $z = 0.02 \text{ cm}$

$$\operatorname{erf} [ ] = \operatorname{erf} (0.528) = 0.545$$

$$W_A = 0.007 - 0.005(0.528) = \underline{0.427 \text{ wt \% C}}$$

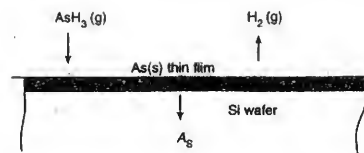
for  $z = 0.04 \text{ cm}$

$$\operatorname{erf} [ ] = \operatorname{erf} (1.056) = 0.866$$

$$W_A = 0.007 - 0.005(0.866) = \underline{0.267 \text{ wt \% C}}$$

27.9

AS INTO Si:



$$\frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{z}{2\sqrt{D_{AB}t}} = \frac{2 \times 10^{-4}}{2 \left[ (5 \times 10^{-13}) (3600) \right]^{1/2}} = 2.357$$

# 27.9 CONTINUED

$$\text{erf}(2.357) = 0.9990 = \frac{2 \times 10^{21} - C_A}{2 \times 10^{21} - 1 \times 10^{12}}$$

$$C_A = 2.0 \times 10^{18} \text{ ATOMS/cm}^3$$

$$\begin{aligned} N_{A2} &= \left[ \frac{D_{AB}}{\pi t} \right]^{1/2} (C_{AS} - C_{AO}) \\ &= \left[ \frac{5 \times 10^{-13}}{\pi (3600)} \right]^{1/2} (2 \times 10^{21} - 1 \times 10^{12}) \\ &= 1.33 \times 10^{13} \text{ ATOMS/cm}^2 \cdot \text{s} \end{aligned}$$

# 27.10 SAME SITUATION AS Prob 27.9

$$C_{AS} = 2 \times 10^{21} \text{ ATOMS/cm}^3$$

$$C_{AO} = 1 \times 10^{12} \text{ "}$$

$$C_A = 2 \times 10^{17} \text{ "}$$

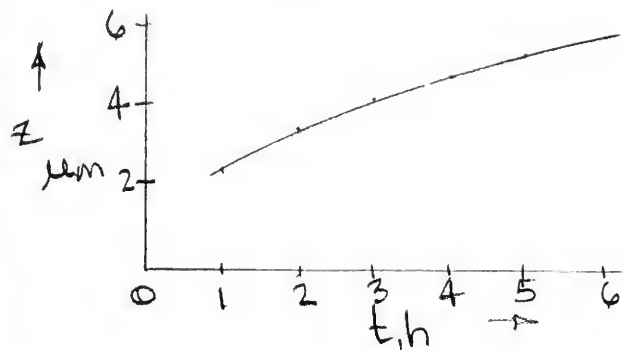
$$\begin{aligned} \text{erf} \frac{z}{2\sqrt{D_{AB}t}} &= \frac{2 \times 10^{21} - 2 \times 10^{17}}{2 \times 10^{21} - 1 \times 10^{12}} \\ &= 0.9999 \end{aligned}$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 2.8$$

$$\begin{aligned} z &= 5.6 (5 \times 10^{-13})^{1/2} t^{1/2} \\ &= 3.960 \times 10^{-6} t^{1/2} \end{aligned}$$

$t(s)$	$t^{1/2}$	$z(\text{cm}) \times 10^4$
3600	60	2.346
7200	84.8	3.36
10800	103.9	4.115
14400	120	4.75
18000	134.2	5.31
21600	147.0	5.82

# 27.10 - CONTINUED -



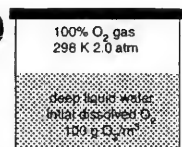
$z(t^{1/2})$  IS OBVIOUSLY LINEAR  
WITH SLOPE =  $0.0396 \mu\text{m/s}^{1/2}$

# 27.11 $\text{O}_2(\text{A})$ INTO $\text{H}_2\text{O}(\text{B})$

$$T = 298 \text{ K} \quad P = 2 \text{ ATM}$$

$$C_{AO} = 109/\text{m}^3$$

$$\begin{aligned} C_{AS} &= 2.10/0.8 = 2.5 \text{ mol/m}^3 \\ &= 2.5(32) = 80 \text{ g/m}^3 \end{aligned}$$



Sealed tank

$$\frac{C_{AS} - C_A}{C_{AS} - C_{AO}} = \frac{80 - 20}{80 - 10} = 0.857 = \text{erf} \left[ \frac{z}{2\sqrt{D_{AB}t}} \right]$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 1.038$$

$$\begin{aligned} t &= \left[ \frac{z}{2(1.038)} \right]^2 \left[ \frac{1}{D_{AB}} \right] \\ &= \left[ \frac{0.3}{2(1.038)} \right]^2 \left[ \frac{1}{2.1 \times 10^{-5}} \right] \\ &= 994 \text{ s} = 16.6 \text{ m} \end{aligned}$$

27.12 A DIFFUSING INTO SEMI-INFINITE MEDIUM

$$C_{AS} = 2 \text{ mol/m}^3$$

$$C_{AO} = 0$$

$$C_A = 0.2 \text{ " @ } z = 5 \text{ mm}$$

$$\frac{C_{AS} - C_A}{C_{AS} - C_{AO}} = \frac{2 - 0.2}{2} = 0.9 = \text{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 1.165$$

$$t = \left[ \frac{z}{2(1.165)} \right]^2 \left( D_{AB}^{-1} \right)$$

$$= \left[ \frac{0.5}{2(1.165)} \right]^2 (1 \times 10^{-6})$$

$$= 46053 \text{ s} = 12.79 \text{ h}$$

27.13 REFER TO PROB 27.4 --

$C_6H_6$  DIFFUSING INTO  $H_2O$  SATURATED SOIL.

ANALYTICAL SOLN GIVEN BY EQ (27-16):

$$\frac{C_A - C_{AS}}{C_{AO} - C_{AS}} = \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi z}{L}\right) e^{-\left(\frac{n\pi}{L}\right)^2 X_D}$$

$$\text{FOR } n \text{ ODD ; } X_D = D_{AB}t/x_1^2$$

$$\text{IN THE PRESENT CASE: } x_1 = \frac{L}{2} = 1 \text{ m}$$

CALCULATIONS MADE USING SPREAD SHEET -- R.H. COLUMN

PROCEDURE IS TO GUESS A VALUE OF  $t$  & SOLVE CONTINUOUSLY UNTIL  $C_A = 1 \text{ g/m}^3$  @  $z = x_1$

27.13 CONTINUED --

EXCEL SPREADSHEET.

$$T = 273 \text{ K}$$

$$M_A = 78$$

$$D_{AB} = 1 \times 10^{-9} \text{ m}^2/\text{s}$$

$$C_{AS} = 24.0 \text{ mol/m}^3$$

$$C_{AO} = 0$$

n	Term	Summation	% Change
1	1.16E+00	1.160E+00	
3	-1.84E-01	9.763E-01	18.8
5	2.50E-02	1.001E+00	2.5
7	-1.92E-03	9.994E-01	0.2
9	7.67E-05	9.995E-01	0.0
11	-1.53E-06	9.995E-01	0.0
13	1.50E-08	9.995E-01	0.0
15	-7.19E-11	9.995E-01	0.0
17	1.67E-13	9.995E-01	0.0
19	-1.86E-16	9.995E-01	0.0
21	1.00E-19	9.995E-01	0.0
23	-2.59E-23	9.995E-01	0.0
25	3.21E-27	9.995E-01	0.0

$$\text{RESULT: } t = 3.763 \times 10^7 \text{ s} \\ = 10452 \text{ h} \\ = 435.5 \text{ DAYS}$$

27.14 REFER TO PROBLEM 27.4

FLUX EXPRESSION GIVEN AS EQN. (27-17).

$$N_{Az} = \frac{4D_{AB}(C_{AS} - C_{AO})}{L} \sum_{n=1}^{\infty} \cos\left(\frac{n\pi z}{L}\right) e^{-\frac{n^2\pi^2 D_{AB}t}{4x_1^2}}$$

$$\text{WITH } x_1 = \frac{L}{2} = 1 \text{ m} \quad N_{Az} = 0 \text{ @ } x_1 = 1$$

$$M_A(t) = W \int_0^t N_A(t) \Big|_{z=0} dt$$

27.14 CONTINUED -

SPREADSHEET FOR SUMMATION:

n	Term	Summation	% Change
1	1.17E-01	1.168E-01	
3	6.79E-02	1.847E-01	36.7
5	3.18E-02	2.164E-01	14.7
7	1.65E-02	2.330E-01	7.1
9	1.00E-02	2.430E-01	4.1
11	6.70E-03	2.497E-01	2.7
13	4.80E-03	2.545E-01	1.9
15	3.60E-03	2.581E-01	1.4
17	2.80E-03	2.609E-01	1.1
19	2.25E-03	2.631E-01	0.9
21	1.84E-03	2.650E-01	0.7
23	1.53E-03	2.665E-01	0.6
25	1.30E-03	2.678E-01	0.5
27	1.11E-03	2.689E-01	0.4
29	9.64E-04	2.699E-01	0.4
31	8.43E-04	2.707E-01	0.3
33	7.44E-04	2.715E-01	0.3
35	6.62E-04	2.721E-01	0.2
37	5.92E-04	2.727E-01	0.2
39	5.33E-04	2.733E-01	0.2
41	4.82E-04	2.737E-01	0.2
43	4.38E-04	2.742E-01	0.2
45	4.00E-04	2.746E-01	0.1
47	3.67E-04	2.749E-01	0.1
49	3.38E-04	2.753E-01	0.1
51	3.12E-04	2.756E-01	0.1

RESULT: For  $t = 2\gamma = 6.3 \times 10^7$  S  
 $M_A = 516$  grams

27.15. CONCENTRATION PROFILE IN A SLAB w/ NO SURFACE RESISTANCE IS EXPRESSED BY EQN (27-16),

$$\frac{C_A - C_{AS}}{C_{AO} - C_{AS}} = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi z}{L} e^{-\left(\frac{n\pi}{2}\right)^2 X_D}$$

NO OPD -

27.15 CONTINUED -

$$\text{MEAN CONCENTRATION } \bar{C} = \frac{\int_0^L C_A dz}{\int_0^L dz}$$

SUBSTITUTING:

$$\bar{C}_A = \frac{4}{\pi} (C_{AO} - C_{AS}) \left[ \int_0^L \frac{1}{n} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} \times \sin\left(\frac{n\pi z}{L}\right) dz + C_{AS} dz \right]$$

$$= -\frac{4}{\pi} (C_{AS} - C_{AO}) \left[ \sum_{n=1}^{\infty} \frac{L}{n^2 \pi} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} \times \cos\left(\frac{n\pi z}{L} + C_{AS} L\right) \right]$$

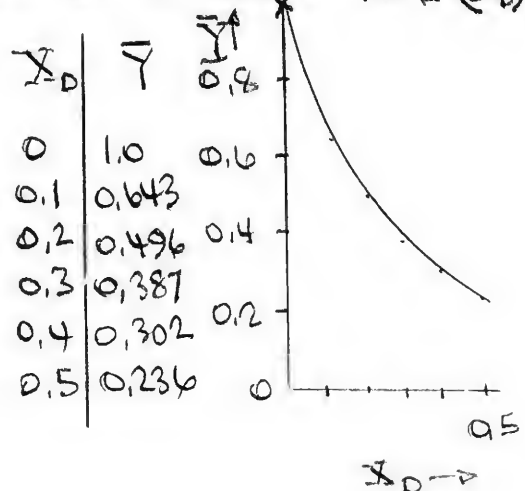
$$\frac{\bar{C}_A - C_{AS}}{C_{AO} - C_{AS}} = -\frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} [-1 - (-1)]$$

$$= \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} - n \text{ odd}$$

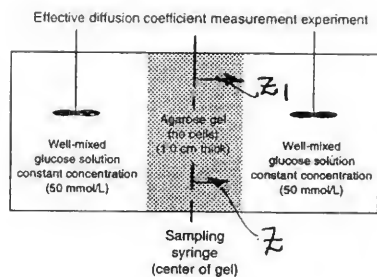
$$\text{FOR } \bar{Y} = \frac{\bar{C}_A - C_{AS}}{C_{AO} - C_{AS}}$$

$$\bar{Y} = 0.8106 \left[ e^{-\left(\frac{\pi}{2}\right)^2 X_D} + \frac{1}{9} e^{-\left(\frac{3\pi}{2}\right)^2 X_D} + \frac{1}{25} e^{-\left(\frac{5\pi}{2}\right)^2 X_D} + \dots \right]$$

DONOR THE CALCULATION FOR  $\bar{Y}(X_D)$ :



27.16



$$C_{A0} = 0$$

$$C_A|_{t=42h} = 485 \text{ mmol/L}$$

GOVERNING DIFFERENTIAL EQN:

$$\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2} \quad (a)$$

CHARTS WILL BE USED -

$$Y = \frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \frac{50 - 485}{50} = 0.03$$

FIGURE F.4

$$\text{At } z = 0 \quad X = \frac{D_{AB} t}{X_1^2} \approx 1.6$$

$$D_{AB} = \frac{1.6 (0.5)^2}{42 (3600)} = 2.64 \times 10^{-6} \text{ cm}^2/\text{s}$$

## 27.17 CYLINDRICAL GEOMETRY

$$D_{AB} = 4 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$L = 5 \text{ cm} \quad C_A' = 2.0 \text{ g/mol/m}^3$$

$$r = 0.5 \text{ cm} \quad K = 1.5 \frac{\text{cm}^3 \text{ FLUID}}{\text{cm}^3 \text{ ABSORBENT}}$$

$$C_A = K C_A'$$

$$C_{A0} = 0$$

$$C_{AS} = 1.5 C_A' = 3.0 \text{ g/mol/m}^3$$

$$C_A = 2.94 \text{ g/mol/m}^3 \text{ @ } X = 0.1 \text{ cm}$$

## 27.17 CONTINUED -

USING CHARTS - SINCE  $L \gg r$  WE  
ASSUME DIFFUSION IS ONLY SIGNIF-  
ICANT IN THE  $r$ -DIRECTION

$$Y = \frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \frac{3.0 - 2.94}{3.0} = 0.02$$

FIGURE F.2 @  $\frac{X}{X_1} = 0.2 \quad m = 0$ 

$$X \approx 0.75 = \frac{D_{AB} t}{r^2}$$

$$t = \frac{0.75 (0.5)^2}{4 \times 10^{-7}} = 4.69 \times 10^5 \text{ s}$$

$$= 130.3 \text{ h}$$

$$= 5.428 \text{ DAYS}$$

## 27.18 SPHERICAL GEOMETRY

$$D_{AB} = 1.5 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$\text{AT } r = 0 \quad C_A = 0.02$$

$$C_{A0} = 0.2$$

$$r_1 = 0.05 \text{ cm}$$

$$\frac{C_A - C_{AS}}{C_{A0} - C_{AS}} = 0.10$$

USING CHARTS - FIG F.9

$$n = \frac{r}{r_1} = 0 \quad m = 0$$

$$X = 0.3 = \frac{D_{AB} t}{r_1^2}$$

$$t = \frac{0.3 (0.05)^2}{1.5 \times 10^{-7}} = 5 \times 10^3 \text{ s}$$

$$= 1.389 \text{ h}$$

# 27.19 - TRANSIENT DRYING OF A SLAB

$$D_{AB} = 1.3 \times 10^{-4} \text{ cm}^2/\text{s}$$

$$@ t=0 \quad w = 15\% \text{ BY WT}$$

$$@ x=x_1, \quad w = 4\% \text{ BY WT}$$

$$\text{DESIRE } w @ x = \frac{x_1}{2} = 10\% \text{ BY WT.}$$

MOISTURE CONCENTRATIONS MUST BE EXPRESSED IN CONSISTENT TERMS ~

WT  $H_2O$  PER WT DRY SOLID ~

$$\therefore w'_0 = \frac{0.15}{1-0.15} = 0.1765 \frac{\text{g } H_2O}{\text{g D.S.}}$$

$$w'_s = \frac{0.04}{1-0.04} = 0.0417 \quad "$$

$$w'_\Delta = \frac{0.10}{1-0.10} = 0.111 \quad "$$

FOR CHART SOLN:

$$Y = \frac{0.111 - 0.0417}{0.1765 - 0.0417} = 0.515$$

$$n = \frac{x}{x_1} = 0.5$$

$$m = 0$$

$$\text{FIG F.7} - @ n=0.4 \quad \frac{D_{AB}t}{x_1^2} \approx 0.24$$

$$@ n=0.6 \quad " \approx 0.16$$

$$\therefore @ n=0.5 \quad \frac{D_{AB}t}{x_1^2} \approx 0.20$$

$$t = \frac{0.20(5)^2}{1.3 \times 10^{-4}} = \frac{38460 \text{ s}}{= 10.68 \text{ h}}$$

# 27.20

AL DIFFUSES INTO Si



$$T = 1250 \text{ K} \quad t = 10 \text{ h} = 3.6 \times 10^4 \text{ s}$$

$$\text{FIG 24.6} \sim D_{AB} \approx 1.1 \times 10^{-13} \text{ cm}^2/\text{s}$$

$$x_1 = 0.5 \text{ } \mu\text{m}$$

$$\text{CONDITION SOUGHT IS } w_A @ \frac{x}{x_1} = 0.5$$

CHART SOLUTION ISN'T POSSIBLE SINCE B.C. ON TOP Si SURFACE IS UNKNOWN

PRESUMING Si THICKNESS IS LARGE COMPARED TO PENETRATION DEPTH - CONSIDER THIS A SEMI-INFINITE SITUATION:

$$\frac{w_{AS} - w_A}{w_{AS} - w_{A0}} = \text{erf} \frac{z}{2\sqrt{D_{AB}t}} \quad \left\{ \text{eq (27-10)} \right\}$$

$$\frac{z}{2[D_{AB}t]^{1/2}} = \frac{0.25 \times 10^{-4}}{2[(1.1 \times 10^{-13})(3.6 \times 10^4)]^{1/2}} \approx 0.1986$$

$$\text{erf}(0.1986) \approx 0.2212$$

$$= \frac{0.01 - w_A}{0.01} \quad w_A = 0.00779 \approx \underline{\underline{0.779\%}}$$

## 27.21 SPHERICAL GEOMETRY

$$D_{AB} = 2 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$r_i = 0.25 \text{ cm}$$

$$C_{AS} = 0.1 (150) = 15 \text{ mol/m}^3$$

$$C_A(r=0) = 12 \text{ mol/m}^3$$

$$\frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \frac{15 - 12}{15} = 0.2$$

$$n = \frac{x}{x_i} = 0 \quad m = 0$$

$$\text{Fig F.9} \quad X_0 = \frac{D_{AB} t}{r_i^2} \approx 0.25$$

$$t = \frac{0.25 (0.25)^2}{2 \times 10^{-6}} = \underline{\underline{7812 \text{ s}}} \\ = \underline{\underline{2.17 \text{ h}}}$$

## 27.22 RECTANGULAR BLOCK - EDGES SEALED -

$$C_{A0} = 64 \text{ mg/cm}^3$$

$$C_{AS} = 0$$

$$D_{AB} = 3 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$V = (1 \text{ cm})(0.652 \text{ cm}) L$$

$L = \text{OTHER DIMENSION}$

$$\frac{41.7}{64} = (1)(0.652) L$$

$$L = 1 \text{ cm}$$

$$\frac{D_{AB} t}{x_i^2} = \frac{(3 \times 10^{-7})(96)(3600)}{(0.652/2)^2}$$

$$= 0.976$$

## 27.22 CONTINUED -

$$n = 0 \quad m = 0$$

$$\text{Fig F.7} \quad Y \approx 0.11$$

$$\frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \frac{0 - C_A}{0 - 64} = 0.11 \quad \underline{\underline{\dot{C}_A = 7.04 \text{ mg/cm}^3}}$$

## 27.23 CYLINDER: $r = 1.25 \text{ cm}$ $L = 80 \text{ cm}$

$$C_{A0} = 30 \text{ wt\%} = \frac{0.3}{1 - 0.3} = 0.429 \text{ g}_A/\text{g}_{D.S.}$$

$$C_{AS} = 1 \text{ wt\%} = \frac{0.01}{1 - 0.01} = 0.0101 \quad "$$

SINCE  $L \gg r$  - VIRTUALLY ALL DIFFUSION IS IN  $r$ -DIRECTION

$$C_A(r=0) = 18 \text{ wt\%} = \frac{0.18}{1 - 0.18} = 0.2195$$

$$\text{AT } t = 36000 \text{ s} \quad (10 \text{ h})$$

$$Y = \frac{w'_A - w_{AS}}{w'_{A0} - w_{AS}} = \frac{0.219 - 0.0101}{0.429 - 0.0101} \approx 0.50$$

$$\text{FOR } n = m = 0 \quad X = \frac{D_{AB} t}{r^2} = 0.2$$

$$\frac{D_{AB}}{r^2} = \frac{0.2}{10 \text{ h}}$$

$$\text{AFTER } 15 \text{ h} \quad X = \frac{D_{AB}}{r^2} (15) = 0.3$$

$$\text{Fig F.8} \quad Y \approx 0.29 = \frac{w'_A - 0.0101}{0.429 - 0.0101}$$

$$w'_A = 0.1316$$

$$\text{wt\%} = \frac{w_A}{1 - w_A} = 0.1316 \quad \underline{\underline{w_A = 11.6\%}}$$

# 27.24 SPHERICAL GEOMETRY

$$r = 0.1 \text{ cm}$$

$$\text{For H}_2\text{O in Air} - D_{AB} = 0.260 \text{ cm}^2/\text{s}$$

$$@ 298 \text{ K, 1 atm}$$

$$w_{A0} = 0$$

$$w_A(r=0) = 0.9 w_{AS}$$

$$Y = \frac{w_{AS} - w_A}{w_{AS} - w_{A0}} = \frac{1 - 0.9}{1} = 0.1$$

$$\text{Fig F.9} - n=m=0$$

$$X_0 = \frac{D_{AB} t}{r^2} \approx 0.3$$

$$t = \frac{0.3(0.1)^2}{0.260} = \underline{\underline{0.0115 \text{ s}}}$$

# 27.25 RECTANGULAR SOLID

$$10 \text{ cm} \times 10 \text{ cm} \times 45 \text{ cm}$$

$$\text{H}_2\text{O DIFFUSES: } D_{AB} = 1.04 \times 10^{-5} \text{ cm}^2/\text{s}$$

$$C_{A0} = 45 \text{ wt\%}, w_{A0} = \frac{0.45}{1-0.45} = 0.818$$

$$C_{AS} = 15 \text{ " } w_{AS} = \frac{0.15}{1-0.15} = 0.176$$

$$C_A = 25 \text{ " } w_A = \frac{0.25}{1-0.25} = 0.333$$

$$Y = \frac{w_{AS} - w_A}{w_{AS} - w_{A0}} = \frac{0.176 - 0.333}{0.176 - 0.818} = 0.244$$

$$= Y_1 Y_2 = Y_3^2 \quad \text{SINCE SIDES HAVE SAME DIMENSIONS}$$

$$\therefore Y_3 = (0.244)^{1/2} = 0.494$$

# 27.25 CONTINUED

USING Fig F.7

$$n=m=0 \quad X_0 \approx 0.39 = \frac{D_{AB} t}{x_1^2}$$

$$t = \frac{0.39(5)^2}{1.04 \times 10^{-5}} = \underline{\underline{9.375 \times 10^5 \text{ s}}}$$

$$= \underline{\underline{260.4 \text{ h}}}$$

$$= \underline{\underline{10.85 \text{ DAYS}}}$$

IF ALL DIFFUSION IS FROM ENDS:

$$Y = 0.244$$

$$X_0 \approx 0.72 = \frac{D_{AB} t}{x_1^2}$$

$$t = \frac{0.72(22.5)^2}{1.04 \times 10^{-5}} = \underline{\underline{3.50 \times 10^7 \text{ s}}}$$

$$= \underline{\underline{9736 \text{ h}}}$$

$$= \underline{\underline{405.6 \text{ DAYS}}}$$

## CHAPTER 28

28.1 FOR  $O_2$  DIFFUSING IN AIR  
@ 300 K, 1 ATM

$$D_{AB}(273K) = 0.175 \text{ cm}^2/\text{s}$$

$$D_{AB}(300K) = 0.175 \left( \frac{300}{273} \right)^{3/2} = 0.202 \text{ cm}^2/\text{s}$$

$$@ 300 \text{ K } - y_{H_2} = 0.1569 \text{ cm}^2/\text{s}$$

$$Sc = \frac{0.1569}{0.202} = \underline{0.777} \quad (a)$$

FOR  $O_2$  IN  $H_2O$  @ 300 K

$$D_{AB} = 1.5 \times 10^{-9} \text{ m}^2/\text{s}$$

$$D_{H_2O} = 0.880 \times 10^{-6} \text{ m}^2/\text{s}$$

$$Sc = \frac{0.880 \times 10^{-6}}{1.5 \times 10^{-9}} = \underline{587} \quad (b)$$

28.2  $SiH_4$  IN  $He$  900 K  
(A) (B) 100 Pa

$$y_{SiH_4} = 0.01$$

$$D_{AB} @ 298 \text{ K}, 101.3 \text{ kPa} = 0.518 \text{ cm}^2/\text{s}$$

$$D_{AB,T,P} = (0.518) \left( \frac{P_1}{P_2} \right) \left( \frac{T_2}{T_1} \right)^{3/2} \left( \frac{\Omega_{D1}}{\Omega_{D2}} \right)$$

$$\text{VALUES: } E_{AB}/K = 46.06$$

$$@ 298 \text{ K } E_{AB}/KT = 6.470 \quad \Omega_D = 0.802$$

$$@ 900 \text{ K } E_{AB}/KT = 19.54 \quad \Omega_D = 0.1668$$

$$D_{AB,T,P} = (0.518) \left( \frac{1.013 \times 10^5}{1100} \right) \left( \frac{900}{298} \right)^{3/2} \times \frac{0.802}{0.1668}$$

28.2 CONTINUED-

$$D_{AB,T,P} = 3.31 \times 10^3 \text{ cm}^2/\text{s}$$

$$\begin{aligned} D_{H_2} @ 900 \text{ K} &= 6 \times 10^{-3} \text{ ft}^2/\text{s} \left( \frac{0.3048 \text{ m}}{\text{ft}} \right)^2 \\ &= 5.574 \times 10^{-4} \text{ m}^2/\text{s} \\ &= 5.574 \text{ cm}^2/\text{s} \end{aligned}$$

$$Sc = \frac{5.574}{3310} = \underline{0.001684}$$

28.3  $Cl_2$  IN  $SiCl_4$  (l)  
(A) (B)

FOR  $D_{AB}$  - USE WILKE-CHANG Eq.  
Eq. (24-52)

$$D_{AB} = \frac{7.4 \times 10^{-8} (M_B \phi_B)^{1/2} T}{V_A^{0.6} \mu_B}$$

$$\text{VALUES: } \phi_B = 1.0 \quad M_B = 170$$

$$\mu_B = 5.2 \times 10^{-4} \text{ kg/m.s}$$

$$= 0.52 \text{ cP}$$

$$V_A = 48.4$$

$$\text{SUBSTITUTING: } D_{AB} = 5.395 \times 10^{-5} \text{ cm}^2/\text{s}$$

$$\eta = \frac{\mu}{g} = \frac{5.2 \times 10^{-5} \text{ g/m.s}}{1.47 \text{ g/cm}^3} = 3.54 \times 10^{-3} \text{ cm}^2/\text{s}$$

$$Sc = \frac{3.54 \times 10^{-3}}{5.395 \times 10^{-5}} = \underline{65.6}$$

$$\begin{aligned}
 28.4 \quad St &= k_c / v_p \\
 &= \frac{k_c L}{D_{AB}} \frac{D}{L v_p} \frac{D_{AB}}{D} \\
 &= \underline{\underline{Sh / Re Sc}}
 \end{aligned}$$

$$\begin{aligned}
 Re &= \frac{v_p L}{D_{AB}} \\
 &= \frac{v_p L}{D} \frac{D}{D_{AB}} = \underline{\underline{Re Sc}}
 \end{aligned}$$

28.5

VARIABLE	SYMBOL	Dim.
MASS TX COEF	$k_c$	$L t^{-1}$
LENGTH	$L$	$L$
VELOCITY	$v$	$L t^{-1}$
VISCOSITY	$\mu$	$M L^{-1} t^{-1}$
DIFFUSIVITY	$D_{AB}$	$L^2 t^{-1}$
DENSITY	$\rho$	$M L^{-3}$

$$L = n - r = 6 - 3 = 3 \text{ } \pi \text{ groups}$$

CORE -  $D_{AB}, \rho, L$

$$\begin{aligned}
 \pi_1 &= D_{AB}^a \rho^b L^c k_c \\
 &= \left(\frac{L^2}{t}\right)^a \left(\frac{M}{L^3}\right)^b L^c \frac{L}{t}
 \end{aligned}$$

$$\begin{aligned}
 M: \quad 0 &= b \\
 L: \quad 0 &= 2a - 3b + c + 1 \\
 t: \quad 0 &= -a - 1
 \end{aligned}$$

$$a = -1 \quad b = 0 \quad c = 1$$

$$\underline{\underline{\pi_1 = k_c L / D_{AB} = Sh}}$$

28.5 CONTINUED -

$$\pi_2 = D_{AB}^d \rho^e v^f = \left(\frac{L^2}{t}\right)^d \left(\frac{M}{L^3}\right)^e L^f \frac{L}{t}$$

$$\begin{aligned}
 M: \quad 0 &= e \\
 L: \quad 0 &= 2d - 3e + f + 1 \\
 t: \quad 0 &= -d - 1
 \end{aligned}$$

$$d = -1 \quad e = 0 \quad f = 1$$

$$\underline{\underline{\pi_2 = v L / D_{AB}}}$$

$$\pi_3 = D_{AB}^g \rho^h \mu^i = \left(\frac{L^2}{t}\right)^g \left(\frac{M}{L^3}\right)^h L^i \frac{M}{L t}$$

$$\begin{aligned}
 M: \quad 0 &= h + i \\
 L: \quad 0 &= 2g - 3h + i - 1 \\
 t: \quad 0 &= -g - 1 \\
 g &= -1 \quad h = -1 \quad i = 0
 \end{aligned}$$

$$\underline{\underline{\pi_3 = \frac{\mu}{\rho D_{AB}} = Sc}}$$

$$\text{NOTE THAT } \pi_2 / \pi_3 = \frac{v \rho L}{\mu} = Re$$

28.6

VARIABLE	SYMBOL	DIMENSIONS
MASS	$M$	$M$
DIAMETER	$D$	$L$
<del>SURFACE TENSION</del>	<del><math>\sigma</math></del>	<del><math>L/t^2</math></del>
DENSITY ( $L$ )	$\rho_L$	$M/L^3$
VISCOSITY ( $L$ )	$\mu_L$	$M/Lt$
VELOCITY	$v$	$L/t$
DENSITY ( $g$ )	$\rho_g$	$M/L^3$
VISCOSITY ( $g$ )	$\mu_g$	$M/Lt$

$$L = n - r = 9 - 3 = 6 \text{ } \pi \text{ groups}$$

CORE -  $\rho_L, \mu_L, D$

28.6 CONTINUED -

$$\pi_1 = \rho_L^a \mu_L^b D^c U = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{Lt}\right)^b L^c \frac{L}{t}$$

$$\begin{aligned} M: 0 &= a+b & a &= 1 \\ L: 0 &= -3a-b+c+1 & c &= 1 \\ t: 0 &= -b-1 & b &= -1 \end{aligned}$$

$$\pi_1 = \rho_L D U / \mu_L = Re$$

$$\pi_2 = \rho_L^a \mu_L^b D^c g = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{Lt}\right)^b L^c \frac{L}{t^2}$$

$$\begin{aligned} M: 0 &= a+b & a &= 2 \\ L: 0 &= -3a-b+c+1 & c &= 3 \\ t: 0 &= -b-2 & b &= -2 \end{aligned}$$

$$\pi_2 = \frac{\rho_L^2 D^3 g}{\mu_L^2} = \frac{D^3 g}{\nu^2}$$

$$\pi_3 = \rho_L^a \mu_L^b D^c \sigma = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{Lt}\right)^b L^c \frac{M}{t^2}$$

$$\begin{aligned} M: 0 &= a+b+1 & a &= 1 \\ L: 0 &= -3a-b+c & c &= 1 \\ t: 0 &= -b-2 & b &= -2 \end{aligned}$$

$$\pi_3 = \frac{\rho_L D \sigma}{\mu_L^2}$$

$$\pi_4 = \rho_L^a \mu_L^b D^c \mu_g$$

$$\sim \text{By INSPECTION} - \pi_4 = \mu_L / \mu_g$$

$$\pi_5 = \rho_L^a \mu_L^b D^c \rho_g$$

$$\sim \text{By INSPECTION} \quad \pi_5 = \rho_g / \rho_L$$

$$\pi_6 = \rho_L^a \mu_L^b D^c M = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{Lt}\right)^b L^c M$$

$$\begin{aligned} M: 0 &= a+b+1 & a &= -1 \\ L: 0 &= -3a-b+c & c &= -3 \\ t: 0 &= -b & b &= 0 \end{aligned}$$

$$\pi_6 = \frac{M}{\rho_L D^3}$$

28.7 VARIABLE SYMBOL DIMENSIONS

MASS TX COEFF.	$k_c$	$L/t$
VELOCITY	$U$	$L/t$
PIPE DIAMETER	$D_i$	$L$
" "	$D_o$	$L$
DENSITY	$\rho$	$M/L^3$
VISCOSITY	$\mu$	$M/Lt$
DIFFUSIVITY	$D_{AB}$	$L^2/t$

$$L = n - r = 7 - 3 = 4 \text{ } \pi \text{ GROUPS}$$

CONST -  $D_{AB}, \rho, D_o$

$$\pi_1 = D_{AB}^a \rho^b D_o^c k_c = \left(\frac{L^2}{t}\right)^a \left(\frac{M}{L^3}\right)^b L^c \frac{L}{t}$$

$$\begin{aligned} M: 0 &= b & b &= 0 \\ L: 0 &= 2a-3b+c+1 & c &= 1 \\ t: 0 &= -a-1 & a &= -1 \end{aligned}$$

$$\pi_1 = k_c D_o / D_{AB} = Sh$$

$$\pi_2 = D_{AB}^a \rho^b D_o^c U = \left(\frac{L^2}{t}\right)^a \left(\frac{M}{L^3}\right)^b L^c \frac{L}{t}$$

SAME FORM AS  $\pi_1$  -

$$\pi_2 = D_o U / D_{AB}$$

28.7 CONTINUED --

$$\pi_3 = D_{AB}^a \rho^b \mu^c = \left(\frac{L^2}{t}\right)^a \left(\frac{M}{L^3}\right)^b L^c \frac{M}{Lt}$$

$$M: 0 = b + 1 \quad b = -1$$

$$L: 0 = 2a - 3b + c - 1 \quad c = 0$$

$$t: 0 = -a - 1 \quad a = -1$$

$$\pi_3 = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}} = Sc$$

$$\pi_4 = D_{AB}^a \rho^b \mu^c D_i$$

$$\sim \text{By Inspection} - \pi_4 = \frac{D_i}{D_0}$$

$$\text{NOTE THAT } \frac{\pi_2}{\pi_3} = \frac{D_0 \nu \rho}{\mu} = Re$$

28.8 VARIABLE SYMBOL DIMENSION

CONCENTRATION DIFFERENCE	$C_{A0} - C_{A\infty}$	$M/L^3$
OVERALL " RADIUS	$R$	$L$
REFERENCE RADIUS	$R$	$L$
DIFFUSIVITY	$D_{AB}$	$L^2/t$
MASS TX. COEF	$k_c$	$L/t$
TIME	$t$	$t$

$$L = N - r = 7 - 3 = 4$$

COEFF --  $C_{A0} - C_{A\infty}, R, D_{AB}$

$$\pi_1 = (C_{A0} - C_{A\infty})^a R^b D_{AB}^c (C_A - C_{A\infty})$$

$$\text{BY INSPECTION} - \pi_1 = \frac{C_A - C_{A\infty}}{C_{A0} - C_{A\infty}}$$

28.8 CONTINUED

$$\pi_2 = (C_{A0} - C_{A\infty})^a R^b D_{AB}^c v$$

$$\sim \text{By Inspection} - \pi_2 = \frac{v}{R}$$

$$\pi_3 = (C_{A0} - C_{A\infty})^a R^b D_{AB}^c k_c = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L^2}{t}\right)^c \frac{L}{t}$$

$$M: 0 = a$$

$$a = 0$$

$$L: 0 = -3a + b + 2c + 1$$

$$b = 1$$

$$t: 0 = -c - 1$$

$$c = -1$$

$$\pi_3 = k_c R / D_{AB}$$

$$\pi_4 = (C_{A0} - C_{A\infty})^a R^b D_{AB}^c t = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L^2}{t}\right)^c t$$

$$M: 0 = a$$

$$a = 0$$

$$L: 0 = -3a + b + 2c$$

$$b = -2$$

$$t: 0 = -c + 1$$

$$c = 1$$

$$\pi_4 = D_{AB} t / R^2$$

28.9 B.L. EQUATIONS:

$$\text{LAMINAR: } \frac{k_c x}{D_{AB}} = 0.332 Re_x^{1/2} Sc^{1/3}$$

$$\text{TURBULANT: } \frac{k_c x}{D_{AB}} = 0.0292 Re_x^{4/5} Sc^{1/3}$$

$$Re_{x, \text{tr}} = 2 \times 10^5$$

$$\left. \begin{array}{l} \text{FRACTION OF} \\ \text{MASS TX} \\ \text{WHICH IS} \\ \text{LAMINAR} \end{array} \right\} = \frac{N_{AL}}{N_{AL} + N_{AT}} = \frac{\bar{k}_{cL}}{\bar{k}_{cL} + \bar{k}_{ct}}$$

28.9 CONTINUED

$$\bar{k}_{cl} = \frac{1}{L} \int_0^{L_{tr}} k_{cx} dx$$

$$= \frac{1}{L} (0.332) \left( \frac{U_x}{\nu} \right)^{1/2} Sc^{1/3} \int_0^{L_{tr}} x^{1/2} dx$$

$$= \frac{0.664}{L} Re_{tr}^{1/2} Sc^{1/3}$$

$$\bar{k}_{ct} = \frac{1}{L} \int_{L_{tr}}^L k_{ctx} dx$$

$$= \frac{1}{L} (0.0292) \left( \frac{U_x}{\nu} \right)^{4/5} Sc^{1/3} \int_{L_{tr}}^L x^{4/5} dx$$

$$= \frac{0.0365}{L} (Re_L - Re_{tr})^{4/5} Sc^{1/3}$$

LAMINAR FRACTION =  $\frac{0.664 Re_{tr}^{1/2}}{0.664 Re_{tr}^{1/2} + 0.0365 (Re_L - Re_{tr})^{4/5}}$

$$Re_{tr}^{1/2} = (2 \times 10^5)^{1/2} = 447.2$$

$$Re_{tr}^{4/5} = (2 \times 10^5)^{4/5} = 17411$$

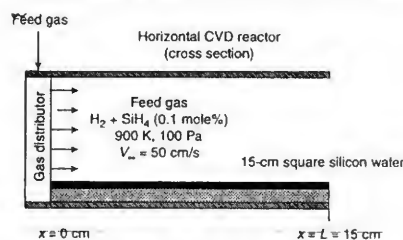
$$Re^{4/5} = (3 \times 10^6)^{4/5} = 151950$$

SUBSTITUTING & SOLVING

LAMINAR FRACTION = 0.057 = 5.7%

28.10

AT SURFACE



$T = 900 \text{ K}$   $P = 100 \text{ Pa}$   $U_m = 50 \text{ cm/s}$   
 $L = 15 \text{ cm}$   $D_{AB} = 0.4036 \times 10^{-4} \text{ cm}^2/\text{s}$   
 $y_{A,p} = 0.001$

28.10 CONTINUED

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.8 \times 10^{-4}}{2.67 \times 10^{-8}} \left( \frac{1}{0.4036 \times 10^{-4}} \right) = 1.67$$

$$Re = \frac{(50)(15)(2.67 \times 10^{-8})}{1.8 \times 10^{-4}} = 0.1125$$

(LAMINAR)

$$Sh = \frac{\bar{k}_{cl} L}{D_{AB}} = 0.664 Re^{1/2} Sc^{1/3} = 0.264$$

$$k_c = \frac{0.4036 \times 10^{-4}}{15} (0.264) = 71.1 \text{ cm}^2/\text{s}$$

$$c = \frac{P}{RT} = \frac{100 / (1.0135 \times 10^5)}{(82.06)(900)} = 1.336 \times 10^{-8} \text{ mol/cm}^3$$

$$C_{A,p} = 0.001 (1.336 \times 10^{-8}) = 1.336 \times 10^{-11}$$

$$W_A = N_A A = k_c (C_{A,p} - C_{A,s}) (15)(15)$$

$$= (71.1)(1.336 \times 10^{-11})(225)$$

$$= 2.136 \times 10^{-7} \text{ mol/s}$$

$$= 1.28 \times 10^{-5} \text{ mol/m}$$

THICKEST Si LAYER WILL OCCUR WHERE  $Re_x$  IS LARGEST

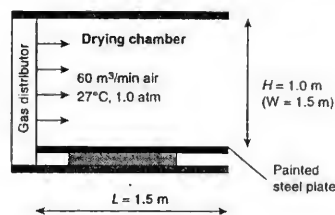
~ AT  $x=0$

28.11

$$T = 300 \text{ K}$$

$$P = 1 \text{ ATM}$$

$$P_{A,s} = 0.137 \text{ ATM}$$



$$U = \frac{\dot{V}}{(W)(H)} = \frac{60}{(1)(1.5)(60)} = 0.67 \text{ m/s}$$

$$= 67 \text{ cm/s}$$

28.11 CONTINUED

$$D_{AB} = 0.0962 \left( \frac{300}{299} \right)^{3/2} = 0.0972 \text{ cm}^2/\text{s}$$

$$\nu = 1.569 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Sc = \frac{(1.569 \times 10^{-5} \times 10^4)}{0.0972} = 1.614$$

$$Re = \frac{UL}{\nu} = \frac{(67)(150)}{0.1569} = 6.41 \times 10^4 \text{ (LAMINAR)}$$

$$k_c = 0.664 \frac{(0.0972)}{150} (6.41 \times 10^4)^{1/2} (1.614)^{1/3}$$

$$= 0.128 \text{ cm/s}$$

$$C_{AS} = \frac{P_A^0}{RT} = \frac{0.137}{(82.06)(300)} = 5.65 \times 10^{-6} \text{ mol/cm}^3$$

$$W_A = N_A A = k_c (C_{AS} - C_{AS}) (A)$$

$$= (0.128)(5.65 \times 10^{-6} - 0)(150)(150)$$

$$= 0.0160 \text{ mol/s}$$

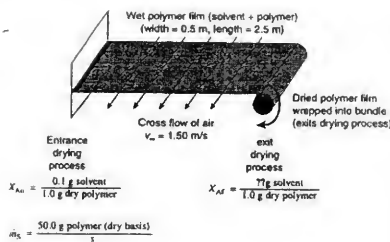
$$= 4500 \text{ g/h}$$

28.12

$$T = 293 \text{ K}$$

$$P = 1 \text{ ATM}$$

$$U = 150 \text{ cm/s}$$



$$D_{AB} = 0.080 \text{ cm}^2/\text{s}$$

$$P_A^0 = 0.16 \text{ ATM}$$

$$\nu_{\text{AIR}} = 0.15 \text{ cm}^2/\text{s}$$

$$Sc = \frac{0.15}{0.080} = 1.875$$

28.12 CONTINUED

$$Re = \frac{UL}{\nu} = \frac{150(50)}{0.15} = 50,000 \text{ (LAMINAR)}$$

$$k_c = 0.664 \frac{D_{AB}}{L} Re^{1/2} Sc^{1/3}$$

$$= 0.664 \frac{0.080}{50} (5 \times 10^4)^{1/2} (1.875)^{1/3}$$

$$= 0.293 \text{ cm/s}$$

$$W_A = N_A A = k_c \Delta C_A (2WL)$$

$$C_A^0 = \frac{P_A^0}{RT} = \frac{0.16}{(82.06)(293)} = 6.65 \times 10^{-6} \text{ mol/cm}^3$$

$$W_A = (0.293)(6.65 \times 10^{-6} - 0)(2 \times 50 \times 250)$$

$$= 0.0487 \text{ mol/s}$$

$$= (0.0487)(86) = 4.19 \text{ g/s (b)}$$

$$Sh = \frac{k_c W}{D_{AB}} = \frac{(0.293)(50)}{0.080} = 183.1 \text{ (c)}$$

$$\text{INPUT} = \frac{0.1 \text{ g SOLVENT}}{\text{g DRY SOL.}} \left( \frac{50 \text{ g DRY SOL.}}{S} \right)$$

$$= 5 \text{ g/s SOLVENT}$$

$$\text{OUTPUT} = 4.19 \text{ g/s} + \dot{m} \text{ g/s (IN POLYMER)}$$

$$\dot{m} = 5 - 4.19 = 0.81 \text{ g/s}$$

$$X = \frac{0.81}{50} = 0.0162 \frac{\text{g SOLVENT}}{\text{g DRY SOLID}} \text{ (c)}$$

28.13 ACETONE (A) IN AIR (B)

$$T = 298 \text{ K}$$

$$P = 1.013 \times 10^5 \text{ Pa} \quad p_A^\circ = 3.066 \times 10^4 \text{ Pa}$$

$$U = 600 \text{ cm/s} \quad L = 100 \text{ cm}$$

$$D_{AB} = 0.93 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{AT } 298 \text{ K} \quad D_{AIR} = 1.55 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Sc = \frac{1.55 \times 10^{-5}}{0.93 \times 10^{-5}} = 1.67$$

$$Re_y = \frac{(6)(0.4)}{1.55 \times 10^{-4}} = 1.548 \times 10^5 \quad (\text{LAMINAR})$$

$$k_{ex}^x = 0.332 Re_y^{1/2} Sc^{1/3}$$

$$k_c = \frac{0.93 \times 10^{-5}}{0.4} (0.332) (1.548 \times 10^5)^{1/2} \times (1.67)^{1/3}$$

$$= 3.6 \times 10^{-3} \text{ m/s} \quad (a)$$

$$\text{For } L = 1 \text{ m}$$

$$Re_L = \frac{(6)(1)}{1.55 \times 10^{-5}} = 3.87 \times 10^5$$

TURBULENT FOR  $Re_x > 2 \times 10^5$

ASSUMING B.L. IS

LAMINAR FOR  $0 < Re_x < 2 \times 10^5$

TURBULENT FOR  $2 \times 10^5 < Re_x$

$$k_c = 0.604 \frac{D_{AB}}{L} Re_L^{1/2} Sc^{1/3} + 0.0365 \frac{D_{AB}}{L} Sc^{1/3} \left[ Re_L^{4/5} - Re_{tr}^{4/5} \right]$$

28.13 CONTINUED -

$$Re_{tr} = 2 \times 10^5$$

$$k_{CL} = 3.87 \times 10^{-3}$$

SUBSTITUTING & SOLVING:

$$k_c = 8.15 \times 10^{-3} \text{ m}^2/\text{s}$$

$$W_A = k_c A (C_{AS} - C_{AO})$$

$$C_{AS} = \frac{p_A^\circ}{RT} = \frac{3.066 \times 10^4}{(8.314)(298)} = 12.37 \text{ mol/m}^3$$

$$W_A = (8.15 \times 10^{-3})(12.37 - 0)(1)$$

$$= 0.101 \text{ mol/s}$$

$$= (0.101)(58) = \underline{\underline{5.86 \text{ g/s}}}$$

28.14 GAS STREAM CONTAINING

CO (A)

O<sub>2</sub> (B)

CO<sub>2</sub> (C)

$$y_A = 0.009$$

$$y_B = 0.001$$

$$y_C = 0.99$$

$$T = 300 \text{ K}$$

$$P = 1 \text{ atm}$$

$$y'_A = \frac{0.009}{0.999} = 0.00901$$

$$y'_C = \frac{0.99}{0.999} = 0.991$$

$$D_{AB} = 0.213 \text{ cm}^2/\text{s}$$

$$D_A = 0.158 \text{ cm}^2/\text{s}$$

$$D_{AC} = 0.155 \text{ "}$$

$$D_B = 0.159 \text{ "}$$

$$D_{BC} = 0.166 \text{ "}$$

$$D_C = 0.0832 \text{ "}$$

28.14 CONTINUED -

$$D_{B-MIN} = \frac{1}{\frac{y'_A}{D_{BA}} + \frac{y'_C}{D_{BC}}}$$

SUBSTITUTING & SOLVING:

$$D_{B-MIN} = 0.166 \text{ cm}^2/\text{s}$$

USE VISCOSITY  $\sim D_C$  - THE DOMINANT COMPONENT

$$Sc = \frac{D}{D_{AB}} = \frac{0.0832}{0.166} = 0.501 \quad (a)$$

$$Re = \frac{UL}{D} = \frac{(1200)(300)}{0.0832} = 4.327 \times 10^6$$

{ VERY MUCH INTO TURBULENT REGIME }

PRESUMING: B.L. FLOW TO BE

LAMINAR FOR  $0 < Re < 2 \times 10^5$

TURBULENT  $2 \times 10^5 < Re$

$$Re = 0.664 \frac{D_{AB}}{L} Sc^{1/3} Re_{tr}^{1/2} + 0.0365 \frac{D_{AB}}{L} Sc^{1/3} \left[ Re_L^{4/5} - Re_{tr}^{4/5} \right]$$

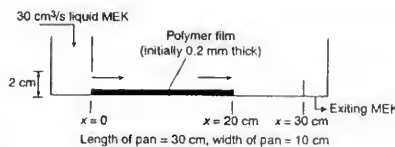
SUBSTITUTING VALUES & SOLVING:

$$Re = 3.277 \text{ cm/s} \quad (c)$$

TURBULENT EFFECTS DOMINATE (b)

28.15

SOLUTE (A)  
INTO MEK (B)



28.15 CONTINUED -

$$C_{AD} = 0$$

$$D_{AB} = 3 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$D = 6 \times 10^{-3} \text{ m}$$

$$\rho_{\text{SOLID}} = 1.05 \text{ g/cm}^3 \quad \rho_L = 0.80 \text{ g/cm}^3$$

$$p_A^* = 0.04 \text{ g/cm}^3 \quad \dot{V} = 30 \text{ cm}^3/\text{s}$$

$$U = \frac{30}{(1.98)(10)} = 1.515 \text{ cm/s}$$

{ UNIT DEPTH }

$$Sc = \frac{6 \times 10^{-3}}{3 \times 10^{-6}} = 2000$$

$$Re = \frac{(1.515)(10)}{6 \times 10^{-3}} = 5050 \quad \{ \text{LAMINAR} \}$$

$$Re = 0.664 \frac{D_{AB}}{L} Re_L^{1/2} Sc^{1/3}$$

$$= 88.94 \times 10^{-6} \text{ cm/s}$$

$$n_A = k_c (p_A^* - p_{A,0})$$

$$= (88.94 \times 10^{-6})(0.04)$$

$$= 3.558 \times 10^{-6} \text{ g/cm}^2 \cdot \text{s} \quad (a)$$

$$W_A = n_A A$$

$$= (3.558 \times 10^{-6})(10 \times 20)$$

$$= 7.116 \times 10^{-4} \text{ g/s}$$

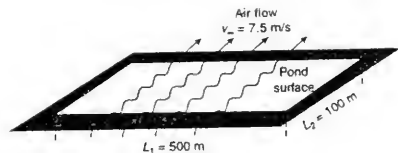
$$m_{\text{SOLID}} = (0.02)(10)(20)(1.05)$$

$$= 4.20 \text{ g}$$

$$t = \frac{4.20}{7.116 \times 10^{-4}} = 5902 \text{ s}$$

$$= 1.64 \text{ h} \quad (b)$$

2816



METHYLENE CHLORIDE (A) IN AIR (B)

$$T = 293 \text{ K} \quad P = 1 \text{ atm}$$

$$D_{AB} = 0.085 \text{ cm}^2/\text{s} \quad U = 7.5 \text{ m/s}$$

$$\alpha = 0.15 \text{ cm}^2/\text{s}$$

$$Sc = \frac{0.15}{0.085} = 1.765$$

$$Re = \frac{UL}{\alpha} = \frac{(7.5)(100)}{0.15 \times 10^{-4}} = 5 \times 10^7$$

PRESUMING B.L. FLOW TO BE

LAMINAR FOR  $0 < Re < 2 \times 10^5$ TURBULENT FOR  $2 \times 10^5 < Re$ 

$$\bar{k}_c = 0.664 \frac{D_{AB}}{L} Sc^{1/3} Re_L^{1/2} + 0.0365 \frac{D_{AB}}{L} Sc^{1/3} \left[ Re_L^{4/5} - Re_{tr}^{4/5} \right]$$

$$Re_{tr} = 2 \times 10^5 \quad Re_L = 5 \times 10^7$$

SUBSTITUTING &amp; SOLVING

$$\bar{k}_c = 0.5372 \text{ cm/s} \quad (b)$$

$$\text{For } Re = 2 \times 10^5 = \frac{UL_{tr}}{\alpha}$$

$$L_{tr} = \frac{(2 \times 10^5)(0.15 \times 10^{-4})}{7.5}$$

$$= 0.4 \text{ m} \quad (a)$$

THIS IS THE EXTENT OF THE LAMINAR B.L.

$$Sc(L) = \frac{0.010}{1.07 \times 10^{-5}} = 934 \quad (c)$$

2817 LUBRICATING OIL (A) IN AIR (B)

$$T = 386 \text{ K} \quad P = 1 \text{ atm} \quad U = 50 \text{ m/s}$$

$$x_{tr} = 0.097 \text{ m} \quad D_{AB} = 0.040 \text{ cm}^2/\text{s}$$

$$\mu = 2.23 \times 10^{-5} \text{ kg/m.s}$$

$$\rho = 0.917 \text{ kg/m}^3$$

$$P_A^0 = 0.20 \text{ Pa}$$

$$\alpha = \frac{2.23 \times 10^{-5}}{0.917} = 2.43 \times 10^{-5} \text{ m}^2/\text{s} = 0.243 \text{ cm}^2/\text{s}$$

$$Sc = \frac{\alpha}{D_{AB}} = \frac{0.243}{0.040} = 6.075 \quad (a)$$

$$\bar{k}_c = 0.664 \frac{D_{AB}}{L} Sc^{1/3} Re_L^{1/2} + 0.0365 \frac{D_{AB}}{L} Sc^{1/3} \left[ Re_L^{4/5} - Re_{tr}^{4/5} \right]$$

$$Re_{tr} = \frac{(5000)(9.7)}{0.243} \quad Re_L = \frac{(5000)(200)}{0.243}$$

$$\approx 2 \times 10^5 \quad = 4.115 \times 10^6$$

SUBSTITUTING VALUES INTO  $\bar{k}_c$  EXPRESSION:

$$\bar{k}_c = 2.48 \text{ cm/s} \quad (b)$$

AT  $x = 120 \text{ cm}$ 

$$Re_x = \frac{(5000)(120)}{0.243} = 2.469 \times 10^6$$

$$k_{cx} = \frac{0.0292 D_{AB}}{x} Sc^{1/3} Re_x^{1/2} = \frac{0.0292(0.040)(6.075)^{1/3}(2.469 \times 10^6)^{1/2}}{120}$$

$$= 2.307 \text{ cm/s} \quad (c)$$

28.18 for  $Re_x = 70,000 = \frac{xU}{\nu}$

$$x = 70,000 \frac{\nu}{U}$$

$$\frac{k_c x}{D_{AB}} = 0.332 Re_x^{1/2} Sc^{1/3}$$

$$k_c = \frac{0.332 (70,000)^{1/2} Sc^{1/3} D_{AB} U}{70,000} \\ = 0.00125 U Sc^{-2/3} \quad (a)$$

for  $Re_L = 70,000 = \frac{LU}{\nu}$

$$L = 70,000 \frac{\nu}{U}$$

$$\frac{Re L}{D_{AB}} = 0.664 Re_L^{1/2} Sc^{1/3}$$

$$= \frac{0.664 (70,000)^{1/2} Sc^{1/3} D_{AB} U}{70,000} \\ = 0.0025 U Sc^{-2/3} \quad (b)$$

for  $Re_x = 7 \times 10^5$

$$x = 7 \times 10^5 \frac{\nu}{U}$$

$$\frac{k_c x}{D_{AB}} = 0.0292 Re_x^{4/5} Sc^{1/3}$$

$$k_c = \frac{0.0292 (7 \times 10^5)^{4/5} Sc^{1/3} D_{AB} U}{(7 \times 10^5)} \\ = 0.00198 U Sc^{-2/3} \quad (c)$$

28.19 - VON KARMAN BL ANALYSIS

GIVEN  $U = \alpha + \beta y^{1/7}$

B.C.  $U(y=0) = 0 \quad \alpha = 0$

$U(y=\delta) = U_\infty \quad \beta = \frac{U_\infty}{\delta^{1/7}}$

$$\therefore U_x = U_\infty \left( \frac{y}{\delta} \right)^{1/7}$$

Given  $C_A - C_{A\infty} = \eta + \xi y^{1/7}$

@  $y=0 \quad C_A - C_{A\infty} = C_{AS} - C_{A\infty}$

@  $y=\delta_c \quad C_A - C_{A\infty} = 0$

$$\eta = C_{AS} - C_{A\infty}$$

$$\xi = -\frac{\eta}{\delta_c^{1/7}} = -\frac{C_{AS} - C_{A\infty}}{\delta_c^{1/7}}$$

Now -  $\frac{C_A - C_{A\infty}}{C_{AS} - C_{A\infty}} = 1 - \left( \frac{y}{\delta_c} \right)^{1/7}$

FOR (28-29)

$$\frac{d}{dx} \int_0^{\delta_c} (C_A - C_{A\infty}) U dy = k_c (C_{AS} - C_{A\infty})$$

DIVIDE BY  $(C_{AS} - C_{A\infty}) U_\infty$

$$\frac{d}{dx} \int_0^{\delta_c} \left( \frac{C_A - C_{A\infty}}{C_{AS} - C_{A\infty}} \right) \frac{U}{U_\infty} dy = \frac{k_c}{U_\infty}$$

EVALUATING THE INTEGRAL

$$\int_0^{\delta_c} \left[ 1 - \left( \frac{y}{\delta_c} \right)^{1/7} \right] \left( \frac{y}{\delta_c} \right)^{1/7} dy$$

$$\int_0^{\delta_c} \left[ \left( \frac{y}{\delta_c} \right)^{1/7} - \frac{y^{2/7}}{\delta_c^{1/7} \delta_c^{1/7}} \right] dy$$

$$\left[ \frac{7}{8} \frac{y^{8/7}}{\delta_c^{1/7}} - \frac{7}{9} \frac{y^{9/7}}{\delta_c^{1/7} \delta_c^{1/7}} \right]_0^{\delta_c}$$

28.19 CONTINUED

INTEGRAL EVALUATION - BETWEEN  $0 \leq \delta_c$

$$\frac{7}{8} \frac{\delta_c^{8/7}}{\delta^{1/7}} - \frac{7}{9} \frac{\delta_c^{8/7}}{\delta^{1/7}} = \frac{7}{92} \frac{\delta_c^{8/7}}{\delta^{1/7}}$$

BACK INTO GOVERNING EQN.:

$$\frac{d}{dy} \left[ \frac{7}{92} \frac{\delta_c^{8/7}}{\delta^{1/7}} \right] = \frac{k_c}{u_p}$$

LETTING  $\delta_c = 1$  -  $\delta_c = \delta$

$$\frac{7}{92} \frac{d\delta_c}{dy} = \frac{k_c}{u_p}$$

SINCE WE KNOW  $\delta = \frac{0.371 x}{Re_x^{1/2}}$

$$\frac{d\delta_c}{dy} = \frac{d\delta}{dy} = \frac{0.371}{(u_p/\nu)^{1/2}} \left( \frac{4}{5} x^{-1/2} \right) = 0.297 Re_x^{-1/2}$$

WE NOW HAVE

$$\frac{7}{92} (0.297 Re_x^{-1/2}) = k_c / u_p$$

FINALLY  $k_c = 0.0289 u_p Re_x^{-1/2}$

28.20  $u_x = a + by$

B.C.  $u_x(0) = 0$

$u_x(\delta) = u_p$

$$u_x = u_p (y/\delta)$$

$$C_A = a + by$$

B.C.  $C_A = C_{AS}$  @  $y=0$

$C_A = C_{AP}$  @  $y=\delta$

28.20 CONTINUED

$$\frac{C_A - C_{AS}}{C_{AP} - C_{AS}} = \frac{y}{\delta}$$

ANOTHER PHYSICAL SITUATION THAT A PROFILE SHOULD PROVIDE IS

$$\frac{dC_A}{dy} = 0 \text{ @ } y=0$$

THE LINEAR MODEL DOES YIELD THE RESULT & WILL NOT SATISFY ALL OF THE PHYSICAL REQUIREMENTS

28.21 FOR A SPHERICAL PELLET ( $d = 1 \text{ cm}$ )

$$Nu = 0.37 Re_d^{0.6} Pr^{1/3}$$

FROM DATA PROVIDED IN PROBLEM STATEMENT:

$$Re_d = \frac{u_p d}{\nu} = \frac{(1)(0.01)}{1.569 \times 10^{-5}} = 637$$

$$Pr = \frac{\nu}{\alpha} = \frac{1.569 \times 10^{-5}}{2.216 \times 10^{-5}} = 0.708$$

$$h = Nu \frac{k}{d} = \frac{0.37 (637)^{0.6} (0.708)^{1/3} (0.02442)}{0.01}$$

$$= 41.64 \text{ W/m}^2 \quad (a)$$

AT MASS TX ANALOGY:

$$\frac{h}{\rho c_p u} Pr^{2/3} = \frac{k_c}{u} Sc^{2/3}$$

$$k_c = \frac{h}{\rho c_p} \left( \frac{Pr}{Sc} \right)^{2/3}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.569 \times 10^{-5}}{9.62 \times 10^{-6}} = 1.63$$

28.21 CONTINUED -

$$k_c = \frac{41.64}{(1177)(1.006)} \left( \frac{0.708}{1.63} \right)^{2/3}$$

$$= 0.020 \text{ m/s} \quad (b)$$

$$N_A = k_c (C_{AS} - C_{AP})$$

$$C_{AS} = \frac{P_A^0}{RT} = \frac{1.27 \times 10^4}{(8.314)(300)} = 5.09 \text{ mol/m}^3$$

$$C_{AP} = 0$$

$$N_A = (0.020)(5.09) = 0.102 \text{ mol/m}^2 \cdot \text{s} \quad (c)$$

28.22 - GIVEN IN PROBLEM STATEMENT

$$\frac{h d_1}{k} = 0.031 Re_{d1}^{0.8} Pr^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14} \left( \frac{d_2}{d_1} \right)^{0.15}$$

HEAT & MASS TX ARE RELATED BY

$$\frac{h}{\rho C_p U} Pr^{2/3} = \frac{k_c}{U} Sc^{2/3}$$

SUBSTITUTING & SOLVING:

$$Sh = \frac{k_c d_1}{D_{AB}}$$

$$= 0.031 Re_{d1}^{0.8} Sc^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14} \left( \frac{d_2}{d_1} \right)^{0.15}$$

28.23 AS GIVEN:

$$Nu = \frac{h d_p}{k} = 0.37 Re_{dp}^{0.6} Pr^{1/3}$$

& FROM CHILTON - COLBURN ANALOGY

$$\frac{h}{\rho C_p U} Pr^{2/3} = \frac{k_c}{U} Sc^{2/3}$$

28.23 CONTINUED

COMBINING RELATIONS -

$$Sh = \frac{k_c d_p}{D_{AB}} = 0.37 Re_{dp}^{0.6} Sc^{1/3}$$

FOR SLOW FLOW - NO BULK CONTRIBUTION!

$$\text{STEADY STATE: } \frac{d}{dr} (r^2 N_{Ar}) = 0$$

$$N_{Ar} = -D_{AB} \frac{dC_A}{dr}$$

$$r^2 N_{Ar} \int_R^{r_0} \frac{dr}{r^2} = -D_{AB} \int_{C_{AS}}^{C_{AP}} dC_A$$

$$r^2 N_{Ar} \left( -\frac{1}{r} \right) \bigg|_R^{r_0} = D_{AB} (C_{AS} - C_{AP})$$

$$\text{AT } r=R: N_{Ar} = \frac{D_{AB}}{R} (C_{AS} - C_{AP}) = k_c \Delta C_A$$

$$k_c = \frac{D_{AB}}{R} = \frac{2 D_{AB}}{d_p}$$

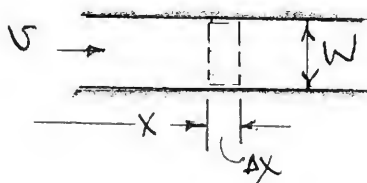
$$\text{GIVING: } Sh = \frac{k_c d_p}{D_{AB}} = 2$$

MODIFIED EQUATION IS, THUS

$$Sh = 2 + 0.37 Re_{dp}^{0.6} Sc^{1/3}$$

28.24

FOR FLOW IN A CHANNEL BETWEEN 2 PLANES (PER UNIT DEPTH)



28.24 CONTINUED -

MASS BALANCE FOR CONTROL VOLUME

$$C_A U W|_x + 2 k_c (C_A - C_A^s) \Delta x = C_A U W|_{x+\Delta x}$$

$$\frac{C_A|_{x+\Delta x} - C_A|_x}{\Delta x} = \frac{2 k_c (C_A^s - C_A)}{W U}$$

IN LIMIT AS  $\Delta x \rightarrow 0$

$$\frac{dC_A}{dx} = \frac{2 k_c}{W U} (C_A^s - C_A)$$

LET  $\theta = C_A - C_A^s \sim \frac{dC_A}{dx} = \frac{d\theta}{dx}$

$$\frac{d\theta}{dx} = -\frac{2 k_c}{W U} \theta$$

$$\int_{\theta_0}^{\theta_L} \frac{d\theta}{\theta} = -\frac{2 k_c}{W U} \int_0^L dx$$

$$\ln \frac{\theta_L}{\theta_0} = -\frac{2 k_c L}{W U}$$

$$\frac{\theta_L}{\theta_0} = \exp\left(-\frac{2 k_c L}{W U}\right)$$

$$\frac{C_{AL} - C_A^s}{C_{AO} - C_A^s} = \exp\left(-\frac{2 k_c L}{U W}\right)$$

FOR  $C_{AO} = 0$

$$C_{AL} = C_A^s \left[ 1 - \exp\left(-\frac{2 k_c L}{U W}\right) \right]$$

NAPHTHALENE IN AIR:

$T = 273 \text{ K}$   $P = 1.013 \times 10^5 \text{ Pa}$

$P_A^o = 1 \text{ Pa}$   $D_{AB} = 5.14 \times 10^{-6} \text{ m}^2/\text{s}$

$Sc = 2.5$   $\lambda = Sc D_{AB} = 1.32 \times 10^{-5}$

$U = 15 \text{ m/s}$

28.24 CONTINUED

$$C_A^s = \frac{P_A}{RT} = \frac{1}{(8.314)(273)} = 4.406 \times 10^{-4} \text{ mol/m}^3$$

USING REYNOLDS ANALOGY

$$Re = \frac{D_{EQUIV} U}{\nu}$$

$$D_{EQUIV} = \frac{4 (1)(W)}{\pi} = 2W$$

$$Re = \frac{2 (0.0075)^2 (15)}{1.32 \times 10^{-5}} = 1.7 \times 10^4$$

FIG (13.1)  $f_f = C_f^* = 0.0064$

$$\frac{k_c}{U} = \frac{0.0064}{2} = 0.0032$$

$$C_{AL} = (4.406 \times 10^{-4}) \left[ 1 - \exp\left(-2 (0.0032) \left(\frac{0.1}{0.0075}\right)\right) \right] = 3.60 \times 10^{-5} \text{ mol/m}^3 \quad (a)$$

USING THE VON KARMAN ANALOGY:

$$\frac{k_c}{U} = \frac{C_f/2}{1 + 5 (C_f/2)^{1/2} \left[ Sc - 1 + \ln\left(1 + \frac{5}{6} Sc\right) \right]}$$

$C_f/2 = 0.0032$   $Sc = 2.5$

$$\frac{k_c}{U} = 0.00184$$

$$C_{AL} = (4.406 \times 10^{-4}) \left[ 1 - \exp\left(-2 (0.00184) \left(\frac{0.1}{0.0075}\right)\right) \right] = 2.11 \times 10^{-5} \text{ mol/m}^3 \quad (b)$$

CHILTON COLBURN:  $\frac{k_c}{U} = \frac{C_f}{2} Sc^{-2/3}$

$k_c/U = 0.00174$

GIVING  $C_{AL} = 2.0 \times 10^{-5} \text{ mol/m}^3 \quad (c)$

28.24 CONTINUED -

PARTS (a), (b) & (c) COMPARE CONCENTRATIONS AT/NEAR STARTING CONDITIONS - BEFORE NAPHTHALENE SHEETS HAVE CHANGED DIMENSIONS.

AFTER EXTENDED TIME -

ORIGINAL VOL. OF NAP

$$= (10)(10)(0.25) = 25 \text{ cm}^3$$

WHEN  $\frac{1}{2}$  OF VOLUME HAS BEEN SUBLIMED -  $12.5 \text{ cm}^3$  REMAIN &  $12.5 \text{ cm}^3$  ARE GONE -

NEW CHANNEL WIDTH =  $0.00875 \text{ m}$

AT AVERAGE CONDITIONS -

$$D_{\text{DOWN}} = 2W = 2(0.00875) = 0.0175 \text{ m}$$

$$Re = \frac{(0.0175)(15)}{1.32 \times 10^{-5}} = 1.95 \times 10^4$$

$$C_f = f_f' = 0.0064 \quad \frac{C_f}{2} = 0.0032$$

$\therefore$  AT AVERAGE CONDITIONS THE ANSWERS TO PARTS (a), (b) & (c) BECOME -

$$\text{REYNOLDS} \quad C_{AL} = 3.34 \times 10^{-5} \text{ mol/m}^3$$

$$\text{VON KARMAN} = 1.95 \times 10^{-5} \text{ "}$$

$$\text{C-COLBURN} = 1.84 \times 10^{-5} \text{ "}$$

THESE ARE PROBABLY MORE REPRESENTATIVE VALUES -

28.24 CONTINUED

TOTAL NAPHTHALENE LOST -

$$= (12.5 \text{ cm}^3)(1.145 \text{ g/cm}^3) \left( \frac{\text{mol}}{128.1 \text{ g}} \right)$$

$$= 0.1117 \text{ mol}$$

$$W_A = C_{AL} U A$$

$$= C_{AL} (15 \text{ m/s})(0.1 \text{ m})(0.00875 \text{ m})$$

$$= 0.0122 (C_{AL}) \text{ mol/s}$$

$$t = \frac{0.1117}{0.0122 C_{AL}}$$

USING CORRECTED RESULTS FOR  $C_{AL}$

$$\begin{aligned} \text{REYNOLDS: } t &= \frac{0.1117}{(0.0122)(3.34 \times 10^{-5})} \\ &= 2.744 \times 10^5 \text{ s} \\ &= \underline{\underline{76.2 \text{ h}}} \end{aligned}$$

$$\text{VON-KARMAN: } t = \underline{\underline{130.6 \text{ h}}}$$

$$\text{C-COLBURN: } t = \underline{\underline{138.4 \text{ h}}}$$

28.25 SPHERICAL DROP IN AIR -

$$\begin{aligned} D_{\text{AIR}} &= 1.5689 \times 10^{-5} \text{ m}^2/\text{s} & P_{\text{AIR}} &= 1.177 \text{ kg/m}^3 \\ \kappa_{\text{AIR}} &= 2.2156 \times 10^{-5} \text{ " } & k &= 2.64 \times 10^{-2} \text{ W/m} \cdot \text{K} \\ D_{AB} &= 2.63 \times 10^{-5} \text{ " } & c_p &= 1006 \text{ kJ/kg} \cdot \text{K} \end{aligned}$$

$$T_s = 290 \text{ K} \quad \lambda = 2461 \text{ J/g}$$

$$P_w^0 = 1940 \text{ Pa} \quad T_p = 310 \text{ K}$$

28,25 CONTINUED --

ENERGY BALANCE --

$$\left\{ \begin{array}{l} \text{HT. TO DROP} \\ \text{BY CONVECTION} \end{array} \right\} = \left\{ \begin{array}{l} \text{HT. LOST BY} \\ \text{EVAPORATION} \end{array} \right\}$$

$$h(T_D - T_S) = \lambda k_c (C_{AS} - C_{AP}) M$$

USING CHILTON-COLBURN ANALOGY

$$\frac{k_c}{u_D} Sc^{2/3} = \frac{h}{8 c_p u_D} Pr^{2/3}$$

$$\frac{h}{k_c} = \left( \frac{Sc}{Pr} \right)^{2/3} 8 c_p$$

$$C_{AS} - C_{AP} = \frac{h}{k_c} \frac{(T_D - T_S)}{\lambda M}$$

$$= \left( \frac{Sc}{Pr} \right)^{2/3} \frac{8 c_p (T_D - T_S)}{\lambda M}$$

$$= \left( \frac{0.60}{0.708} \right)^{2/3} \frac{(1177)(1.006)(20)}{2461(18)}$$

$$= 0.478 \text{ mol/m}^3$$

$$C_{AS} = \frac{P^0}{RT} = \frac{1940}{(8.314)(298)} = 0.805 \text{ mol/m}^3$$

$$C_{AP} = 0.805 - 0.478 = 0.326 \text{ mol/m}^3$$

28,26 - THIS IS THE SAME PHYSICAL PROCESS AS IN TEXT EXAMPLE 6

$$T_D = \frac{\lambda T_S}{8 c_p} \left( \frac{Pr}{Sc} \right)^{1/3} (C_{AS} - C_{AP}) + T_S$$

28,26 CONTINUED --

$$T_S = 298 \text{ K}$$

$$C_{AIR} = 1 \text{ J/g} \cdot \text{K}$$

$$\mu = 1.84 \times 10^{-4} \text{ g/cm} \cdot \text{s}$$

$$\lambda = 2.45 \text{ kJ/g}$$

$$\rho = 1.17 \times 10^3 \text{ g/cm}^3$$

$$P_w^0 = 1300 \text{ Pa}$$

$$k = 2.62 \times 10^{-4} \text{ W/cm} \cdot \text{K}$$

$$u_D = 0.22 \text{ m/s}$$

$$D_{AB} = 3 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Pr = \frac{\mu c_p}{k} = \frac{(1.84 \times 10^{-4})(1)}{2.62 \times 10^{-4}} = 0.70$$

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{(1.84 \times 10^{-4})}{(1.17 \times 10^3)(3 \times 10^{-5})} = 0.524$$

$$C_{AS} = \frac{P}{RT} = \frac{1300}{(8.314)(298)} = 0.525 \text{ mol/m}^3$$

$$C_{AP} = 0$$

$$T_D = \frac{(2450)(18)}{(1.17 \times 10^3)(1)} \left( \frac{0.70}{0.524} \right)^{1/3} (0.525 \times 10^{-6}) + 298$$

$$= 21.8 + 298 = \underline{\underline{319.8 \text{ K}}}$$

28,27 H<sub>2</sub>O (A) INTO AIR (B)

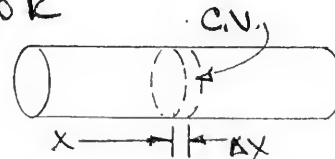
$$T = 310 \text{ K}$$

$$D = 0.15 \text{ m}$$

$$P = 1.013 \times 10^5 \text{ Pa}$$

$$D/\epsilon = 10^4$$

$$T_S = 290 \text{ K}$$



MASS BALANCE FOR C.V. SHOWN:

28.27 CONTINUED -

$$C_A \sqrt{\frac{\pi D^2}{4}} \Big|_x + k_c (C_{AS} - C_A) \pi D \Delta x =$$

$$C_A \sqrt{\frac{\pi D^2}{4}} \Big|_{x+\Delta x}$$

$$\frac{C_A|_{x+\Delta x} - C_A|_x}{\Delta x} = \frac{4 k_c}{D \sqrt{\pi}} (C_{AS} - C_A)$$

IN LIMIT AS  $\Delta x \rightarrow 0$

$$\frac{dC_A}{dx} = \frac{4 k_c}{D \sqrt{\pi}} (C_{AS} - C_A)$$

$$\text{LET } \theta = C_A - C_{AS} \quad - \quad \frac{d\theta}{dx} = \frac{dC_A}{dx}$$

$$\frac{d\theta}{dx} = - \frac{4 k_c}{D \sqrt{\pi}} \theta$$

$$\int_{\theta_0}^{\theta_L} \frac{d\theta}{\theta} = - \frac{4 k_c}{D \sqrt{\pi}} \int_0^L dx$$

$$\ln \frac{\theta_L}{\theta_0} = - \frac{4 k_c}{D \sqrt{\pi}} L$$

$$\frac{C_{AL} - C_{AS}}{C_{AO} - C_{AS}} = e^{-\frac{4 k_c}{D \sqrt{\pi}} L}$$

CHILTON-COLBURN ANALOGY

$$\frac{k_c}{\sqrt{\pi}} Sc^{2/3} = \frac{C_F}{2} = \frac{f}{2}$$

$$Sc = \frac{\nu}{D_{AB}}$$

$$\nu = 1.569 \times 10^{-5} \text{ m}^2/\text{s} @ T_f = 300\text{K}$$

28.27 CONTINUED -

$$D_{AB} = \frac{2.634}{1.013 \times 10^5} \left( \frac{300}{298} \right)^{3/2} = 2.626 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.569 \times 10^{-5}}{2.626 \times 10^{-5}} = 0.597$$

$$Re = \frac{DV}{\nu} = \frac{(0.15)(1.5)}{1.569 \times 10^{-5}} = 1.43 \times 10^4$$

$$\text{FIG 13.1} \quad f_F \approx 0.0066$$

$$\frac{k_c}{\sqrt{\pi}} = \frac{0.0066}{2} (0.597)^{-2/3} = 4.65 \times 10^{-3}$$

$$\frac{C_{AL} - C_{AS}}{C_{AO} - C_{AS}} = e^{-\frac{4(6)}{0.15} (4.65 \times 10^{-3})} = 0.474$$

$$C_{AS} = \frac{P^*}{RT} = \frac{1895}{(8.314)(290)} = 0.786 \text{ mol/m}^3$$

$$C_{AL} = C_{AS} (1 - 0.474) = 0.786 (0.526) = \underline{\underline{0.413 \text{ mol/m}^3}}$$

28.28 SAME PHYSICAL SITUATION AS IN PROB 28.27

$$\ln \frac{C_{AL} - C_{AS}}{C_{AO} - C_{AS}} = - \frac{4 k_c}{D \sqrt{\pi}} L$$

$$Re = \frac{DV}{\nu} = \frac{(0.025)(15)}{1.415 \times 10^{-5}} = 2.65 \times 10^4$$

$$\text{FIG 13.1} \quad f_F = C_F = 0.0058$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.415 \times 10^{-5}}{540 \times 10^{-6}} = 2.62$$

28,28 CONTINUED

USE GILTON-COLBURN ANALOGY

$$\frac{k_c}{U} = \frac{C_b/2}{Sc^{1/3}} = \frac{0.0058/2}{2.62^{1/3}}$$
$$= 0.00153$$

$$\ln \left[ \frac{C_{AL} - C_{AS}}{C_{AO} - C_{AS}} \right] = - \frac{4(0.00153) L}{0.025}$$
$$= -0.245 L$$

$$C_{AS} = \frac{P^0}{RT} = \frac{3}{(8.314)(283)} = 1.275 \times 10^{-3} \text{ mol/m}^3$$

$$C_{AL} = 4.75 \times 10^{-4} \text{ mol/m}^3 \quad C_{AO} = 0$$

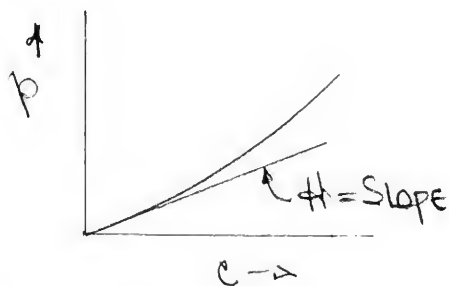
$$\ln \frac{4.75 - 12.75}{-12.75} = -0.466$$

$$L = \frac{0.466}{0.245} = \underline{\underline{1.90 \text{ m}}}$$

# CHAPTER 29 -

## 29.1 EQUILIBRIUM DATA - $\text{Cl}_2$ IN $\text{H}_2\text{O}$

$p, \text{Cl}_2$	$\text{kg/m}^3$	$\text{mol/m}^3$
666	0.438	6.17
1330	0.575	8.10
4000	0.937	13.20
6600	1.210	
13200	1.773	



A CAREFUL PLOT WILL YIELD  
 $H \approx 62 \text{ Pa}/(\text{mol/m}^3)$

## 29.2 EQUILIBRIUM DATA FOR TCE IN $\text{H}_2\text{O}$

$p, \text{TCE}$	$C$
ATM	$\text{mol/m}^3$
0.000	0
0.050	5.0
0.150	15.0
0.200	20.0

PLOT & OBSERVATION WILL  
SHOW LINEAR BEHAVIOR -

$$H = \frac{\Delta p}{\Delta C} = 0.010 \text{ atm}/(\text{mol/m}^3)$$

## 29.3 BENZENE (B) - 49 moles TOLUENE (T) - 21 moles $\Sigma = 70$

AT 363 K,  $P = 1.013 \times 10^5 \text{ Pa}$

$$P_B = 1.344 \times 10^5 \text{ Pa}$$

$$P_T = 5.38 \times 10^5 \text{ Pa}$$

	$X$	$P_i \times 10^{-5}$	$P_i \times 10^{-5}$
B	$X_B$	1.344	$1.344 X_B$
T	$1 - X_B$	5.38	$5.38 (1 - X_B)$

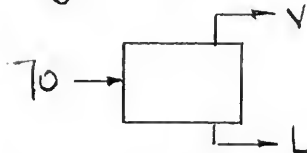
$$P_B + P_T = 1.013 \times 10^5$$

$$1.344 \times 10^5 X_B + 5.38 \times 10^5 (1 - X_B) = 1.013 \times 10^5$$

$$X_B = 0.589 \quad X_T = 0.411 \quad (a)$$

$$y_B = \frac{1.344 \times 10^5}{1.013 \times 10^5} (0.589) = 0.783$$

$$y_T = 1 - y_B = 0.217$$



MASS BALANCE -

$$\text{TOTAL: } T_0 = V + L$$

$$\begin{aligned} B: 49 &= 0.783V + 0.589L \\ &= 0.783(T_0 - L) + 0.589L \end{aligned}$$

$$L = 29.95 \text{ mol} \quad (b)$$

29.4 BASIS - 100 kg H<sub>2</sub>O

	kg	M	mol	X <sub>i</sub>
O <sub>2</sub>	2 × 10 <sup>-3</sup>	32	6,25 × 10 <sup>-5</sup>	1,126 × 10 <sup>-5</sup>
H <sub>2</sub> O	100	18	5,55	

$$p_{O_2} = y_{O_2} P = 0,21 (1,013 \times 10^5) = 0,2127 \times 10^5$$

$$p_{O_2}^* = H x_{O_2} = (4,06 \times 10^9) (1,126 \times 10^{-5})$$

$$= 4,57 \times 10^4$$

As  $p_{O_2}^* > p_{O_2}$

SOLUTION WILL LOSE O<sub>2</sub> (a)

$$p_{O_2} = H x_{O_2}$$

$$2,127 \times 10^4 = 4,06 \times 10^9 x_{O_2}$$

$$x_{O_2} = 5,24 \times 10^{-6}$$

FROM TABLE: TOTAL MOLES = 5,55

IN EQUILIBRIUM SOLUTION:

$$z_{O_2} = (5,24 \times 10^{-6}) (5,55)$$

$$= 2,91 \times 10^{-5} \text{ kg mol}$$

BY MASS:

$$(2,91 \times 10^{-5} \text{ kg mol}) (32)$$

$$100 \text{ kg H}_2\text{O}$$

$$= 9,3 \times 10^{-4} \text{ kg O}_2 / 100 \text{ kg H}_2\text{O} \quad (b)$$

29.5

$$\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{H k_G}$$

$$p_A = H c_A = H C x_A$$

$$C = (1 \text{ g/cm}^3) (10^6 \text{ cm}^3/\text{m}^3) \left( \frac{\text{kg}}{1000 \text{ g}} \right) \left( \frac{\text{kg mol}}{18 \text{ kg}} \right)$$

$$= 55,56 \text{ kg mol/m}^3$$

$$H' = \frac{4,06 \times 10^9}{55,56} = 7,3 \times 10^7 \frac{\text{Pa}}{\text{kg mol/m}^3}$$

$$\frac{1}{K_L} = \frac{1}{2,15 \times 10^{-5}} + \frac{1}{(9,28 \times 10^8) (7,3 \times 10^7)}$$

$$= 4,65 \times 10^5 + 0,148 \approx 4,65 \times 10^5$$

$$K_L = 2,15 \times 10^{-5} \text{ m/s}$$

ALL (100%) OF RESISTANCE IS IN GAS PHASE

29.6 INTERPHASE TRANSPORT

ClO<sub>2</sub> - AIR - H<sub>2</sub>O

$$P = 1,5 \text{ ATM} \quad H = 7,7 \times 10^{-4} \text{ ATM/(g mol/m}^3)$$

$$y_A = 0,0401 \quad \rho_L = 992,3 \text{ kg/m}^3$$

$$x_A = 0,00040$$

AT EQUILIBRIUM:

$$p_A = H c_A$$

$$= y_A P = 0,04 (1,5) = 0,06 \text{ ATM}$$

$$c_A = y_A C = \frac{0,0004 (992,3)}{18}$$

$$= 0,022 \text{ kg mol/m}^3 = 22,0 \text{ g mol/m}^3$$

$$c_A^* = \frac{p_A}{H} = \frac{0,06}{7,7 \times 10^{-4}} = 77,9 \text{ g mol/m}^3$$

29.6 CONTINUED -

SINCE  $C_A^* > C_A$  - ABSORPTION (a)

$$\text{maximum } C_A = C_A^* = 779 \text{ g mol/m}^3 \text{ (b)}$$

$$k_x = 1.0 \text{ g mol/m}^2 \cdot \text{s}$$

$$k_g = 0.010 \text{ g mol/m}^2 \cdot \text{s} \cdot \text{atm}$$

$$k_y = k_g P = 0.010(1.5) = 0.015 \text{ g mol/m}^2 \cdot \text{s}$$

$$\frac{1}{k_y} = \frac{1}{k_g} + \frac{H}{k_x}$$

$$p_A = H C_A$$

$$\frac{p_A}{P} = y_A = \frac{C_A H}{P} x_A = H' x_A$$

$$H' = \frac{C_A H}{P} = \frac{(55.13)(7.7 \times 10^{-4})}{1.5}$$

$$= 0.0283$$

$$\frac{1}{k_y} = \frac{1}{0.015} + \frac{0.0283}{1.0}$$

$$k_y = 0.015 \text{ g mol/m}^2 \cdot \text{s} \text{ (c)}$$

$$N_A = k_y (y_{A,i} - y_A^*)$$

$$y_{A,i} = 0.06$$

$$y_A^* = \frac{p_A^*}{P} = \frac{7.7 \times 10^{-4}}{1.5} \text{ (22)}$$

$$= 0.0113$$

$$N_A = (0.015)(0.06 - 0.0113)$$

$$= 7.30 \times 10^{-4} \text{ g mol/m}^2 \cdot \text{s} \text{ (d)}$$

$$29.7 \quad N_A = K_L (C_A^* - C_{A,L})$$

$$C_A^* = \frac{p_{A,g}}{H} = \frac{1.013 \times 10^4}{1.674 \times 10^3} = 6.05 \text{ kg mol/m}^3$$

$$N_A = (1.26 \times 10^{-6})(6.05 - 4) = 2.58 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s}$$

$$\frac{1/k_L}{1/k_L} = \frac{K_L}{k_L} = 0.53 \text{ (d)}$$

$$k_L = \frac{K_L}{0.53} = \frac{1.26 \times 10^{-6}}{0.53} = 2.38 \times 10^{-6} \text{ (a)}$$

{UNITS ARE  $\text{kg mol/m}^2 \cdot \text{s} \cdot (\text{mol/m}^3)$ }

$$N_A = (2.58 \times 10^{-6}) = 2.38 \times 10^{-6} (C_{A,i} - 4)$$

$$C_{A,i} = 5.08 \text{ kg mol/m}^3 \text{ (c)}$$

$$N_A = k_g (p_{A,i} - p_{A,i})$$

$$p_{A,i} = H C_{A,i} = (1.674 \times 10^3)(5.08) = 8.50 \times 10^3 \text{ Pa}$$

$$k_y = \frac{N_A}{p_{A,i} - p_{A,i}}$$

$$= \frac{2.58 \times 10^{-6}}{1.013 \times 10^4 - 0.850 \times 10^4}$$

$$= 1.58 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa} \text{ (b)}$$

# 28,8 STRIPPING TCA FROM WASTEWATER

$$T = 293 \text{ K} \quad P = 1,25 \text{ ATM}$$

$$H' = 400 \text{ ATM} \quad p_A = H' x_A$$

$$p_A = H' x_A$$

$$y_A = \frac{H'}{P} x_A \quad \frac{H'}{P} = \frac{400}{1,25} = 320 \frac{\Delta y}{\Delta x}$$

$$p_A = \frac{H'}{C} c_A \quad H = \frac{H'}{C} \quad (a)$$

$$C = \frac{P}{RT} = \frac{1,25}{(0,08206)(293)} = 0,0520 \frac{\text{kg mol}}{\text{m}^3}$$

$$H = \frac{400}{0,0520} = 7690 \frac{\text{atm}}{(\text{kg mol}/\text{m}^3)} \quad (b)$$

$$\text{GIVEN} - k_c = 0,01 \text{ mol/s}$$

$$k_g = \frac{k_c}{RT} = \frac{0,01}{(0,08206)(293)} = 4,16 \times 10^{-4} \frac{\text{kg mol}}{\text{m}^2 \cdot \text{s} \cdot \text{atm}} \quad (c)$$

$$N_A = k_c \Delta c_A = C k_c \frac{\Delta c_A}{C} = k_y \Delta y_A$$

$$k_y = C k_c = (0,0520)(0,01) = 5,2 \times 10^{-4} \frac{\text{kg mol}}{\text{m}^2 \cdot \text{s} \cdot \Delta y} \quad (d)$$

$$k_x = C k_L \quad (e)$$

$$\left\{ \text{IN LIQUID} \right\} C = \rho_w = \frac{998,2}{18} = 55,46 \frac{\text{kg mol}}{\text{m}^3}$$

$$k_x = (55,46)(0,01) = 0,5546 \frac{\text{kg mol}}{\text{m}^2 \cdot \text{s}} \quad (d)$$

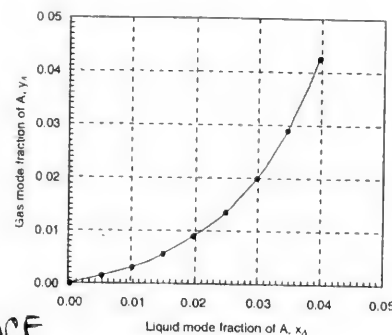
# 29,9

$$T = 300 \text{ K}$$

$$P = 2 \text{ ATM}$$

$$y_A = 0,01$$

$$x_A = 0,035$$



80% OF RESISTANCE IS IN LIQUID

$$N_A = k_y (x_{AP} - x_{Ai}) - K_x (x_{AP} - x_A^*)$$

$$\frac{1/k_y}{1/K_y} = \frac{K_y}{k_y} = 0,8 = \frac{x_{AP} - x_{Ai}}{x_{AP} - x_A^*} = \frac{0,035 - x_{Ai}}{0,035 - 0,021}$$

$$x_{Ai} = 0,035 - 0,8(0,014) = 0,0238 \quad (a)$$

FROM EQUILIBRIUM DIAGRAM ABOVE

$$y_{Ai} = 0,0122 \quad (a)$$

$$\frac{1/k_y}{1/K_y} = \frac{K_y}{k_y} = 0,2$$

$$K_y = 0,2(1,25) = 0,25 \frac{\text{g mol}}{\text{m}^2 \cdot \text{s} \cdot \Delta y} \quad (b)$$

$$K_x = \frac{K_y}{P} = \frac{0,25}{2} = 0,125 \frac{\text{g mol}}{\text{m}^2 \cdot \text{s} \cdot \text{atm}} \quad (c)$$

$$\frac{K_y}{P} = \frac{K_c}{RT} \sim K_c = \frac{K_y RT}{P}$$

$$K_c = 0,25 \frac{(82,06)(300)}{2} = 3077 \text{ m}^2/\text{s}$$

29.10 SOLUTE A REMOVED FROM GAS STREAM

$$T = 300 \text{ K} \quad x_{A1} = 0.01$$

$$P = 2 \text{ ATM} \quad y_{A1} = 0.035$$

EQUILIBRIUM:  $y_A = 0.3 x_A$

$$\frac{1/k_y}{1/k_y} = \frac{k_y}{k} = 0.6$$

$$\frac{y_{A1} - y_{A1}}{y_{A1} - y_A^*} = 0.6$$

$$y_{A1} - y_A^*$$

$$y_A^* = 0.3 x_A = 0.3(0.01) = 0.003$$

$$\frac{0.035 - y_{A1}}{0.035 - 0.003} = 0.6$$

$$0.035 - 0.003$$

$$y_{A1} = 0.0158$$

$$x_{A1} = \frac{y_{A1}}{0.3} = 0.0527 \quad (a)$$

$$k_y = 0.6 k_x = 0.6(1.25) = 0.75 \text{ g mol / m}^2 \cdot \text{s} \cdot \Delta y \quad (b)$$

$$\left\{ \begin{array}{l} \frac{1}{k_x} = \frac{1}{H k_y} + \frac{1}{k_x} \\ \frac{1}{k_y} = \frac{1}{k_y} + \frac{H'}{k_x} \end{array} \right\} \rightarrow \frac{1}{k_x} = \frac{H'}{k_y}$$

$$k_x = \frac{k_y}{H'} = \frac{0.75}{0.3} = 2.5 \text{ g mol / m}^2 \cdot \text{s} \cdot \Delta x \quad (c)$$

29.11 AERATION OF H<sub>2</sub>O

$$T = 293 \text{ K} \quad y_{A1} = 0.21$$

$$P = 2 \text{ ATM}$$

$$\rho_{H_2O} = 1000 \text{ kg / m}^3$$

$$H' = 40,100 \text{ ATM} \Delta y / \Delta x$$

$$p_{O_2} = 0.21(2) = 0.42 \text{ ATM}$$

$$= (40,100) x_{O_2}$$

$$x_{O_2}^* = \frac{0.42}{40,100} = 1.047 \times 10^{-5} \frac{\text{mol O}_2}{\text{mol H}_2\text{O}} \quad (a)$$

$$C_A^* = (1.047 \times 10^{-5}) \left( \frac{1000}{18} \right)$$

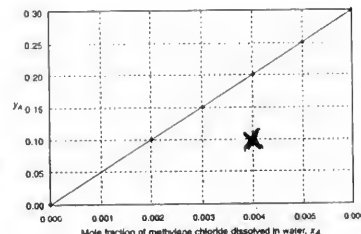
$$= 5.82 \times 10^{-4} \text{ kg mol / m}^3 \quad (b)$$

AS SYSTEM PRESSURE INCREASES

$x^* \propto C_A^*$  WILL INCREASE (c)

29.12

EQUILIBRIUM FOR SPECIES A IN AIR & SPECIES A DISSOLVED IN H<sub>2</sub>O



$$T = 293 \text{ K}$$

$$P = 2.20 \text{ ATM}$$

$$\rho_{H_2O} = 992.3 \text{ kg / m}^3$$

$$y_{A1} = 0.10$$

$$x_{A1} = 0.0040$$

$$k_y = 0.010 \text{ g mol / m}^2 \cdot \text{s}$$

$$k_x = 0.125 \text{ "}$$

THIS IS A STRIPPING PROCESS a)

29.12 CONTINUED -

$$y_A = H' x_A'$$

$$p_A = \frac{H' P}{C} x_{AC}^* = H C_A^* \quad H = \frac{H' P}{C}$$

$$C = \frac{992.3}{18} = 55.13 \text{ kg mol/m}^3$$

$$H' = 50 \quad \text{--- FROM DIAGRAM}$$

$$H = \frac{50 (2.2)}{55.13} = 1.996 \text{ Atm / (kg mol/m}^3) \quad (b)$$

$$y_A = H' x_A' \quad 0.10 = 50 x_A^* \\ x_A^* = 0.002$$

$$C_A^* = x_A^* C = 0.002 (55.13) \\ = 0.110 \text{ kg mol/m}^3$$

$$C_{AP} = x_{AP} C = (0.004) (55.13) \\ = 0.220 \text{ kg mol/m}^3$$

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{H' k_y} = \frac{1}{0.125} + \frac{1}{(50)(0.01)} \\ = 8 + 2 = 10$$

$$K_x = 0.10 \text{ g mol/m}^2 \cdot \text{s} \quad (c)$$

$$K_L = \frac{K_x}{C} = \frac{0.10}{55.13} = 1.81 \times 10^{-6} \text{ m/s}$$

$$N_A = K_L (C_{AP} - C_A^*) \\ = (1.81 \times 10^{-6}) (0.220 - 0.110) \\ = 8.91 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s}$$

29.12 CONTINUED -

$$N_A = (8.91 \times 10^{-6}) = k_x (x_{AP} - x_{Ai}) \\ = (0.125) (0.004 - x_{Ai}) \\ x_{Ai} = \frac{3.93 \times 10^{-3}}{\quad} \quad (d)$$

$$y_{Ai} = H' x_{Ai} = 50 (3.93 \times 10^{-3}) = 0.196$$

29.13 HEXANE (A) ABSORBED FROM AIR

$$T = 273 \text{ K}$$

$$P = 1.5 \text{ Atm}$$

$$\rho_L = 0.80 \text{ g/cm}^3$$

$$M_L = 180$$

$$y_{AP} = 0.015 \text{ Atm}$$

$$x_{AP} = 0.05$$

$$k_y = 0.02 \text{ kg mol/m}^2 \cdot \text{s}$$

$$k_x = 0.01 \quad "$$

$$C = \frac{\rho_L}{M} = \frac{0.80}{180} = 4.44 \times 10^{-3} \text{ g mol/cm}^3$$

$$p_A = 0.15 x_A^*$$

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{H' k_y} = \frac{1}{0.01} + \frac{1}{(0.15)(0.02)}$$

$$K_x = 2.31 \times 10^{-3} \text{ kg mol/m}^2 \cdot \text{s}$$

$$K_L = K_x / C = \frac{2.31 \times 10^{-3} (10)^{-3}}{4.44 \times 10^{-3} (10)^4}$$

$$= 0.52 \times 10^{-7} \text{ cm/s} \quad (a)$$

$$k_y (y_{AP} - y_{Ai}) = k_x (x_{Ai} - x_{AP})$$

$$0.02 (0.015 - y_{Ai}) = 0.01 (x_{Ai} - 0.05)$$

$$\text{Also: } y_{Ai} = \frac{p_{Ai}}{P} = \frac{0.15}{P} x_{Ai}$$

$$y_{Ai} = \frac{0.15}{1.5} = 0.10 x_{Ai}$$

29.13 CONTINUED -

COMBINING THESE EXPRESSIONS -

$$x_{Ai} = \underline{0.0667} \quad (b)$$

$$y_{Ai} = 0.1 x_{Ai} = \underline{0.00667}$$

29.14.  $SO_2$  ABSORBED INTO  $H_2O$

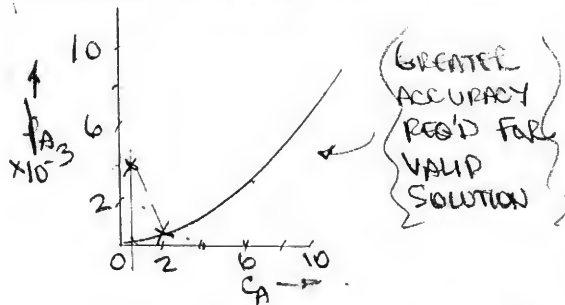
$$p_{A0} = 4 \times 10^3 \text{ Pa}$$

$$C_{A0} = 0.55 \text{ kg mol/m}^3$$

$$k_g = 3.95 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$k_L = 1.1 \times 10^{-4} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

EQUILIBRIUM DATA - GIVEN IN  
PROBLEM STATEMENT -



$$= \frac{k_L}{k_g} = \frac{1.1 \times 10^{-4}}{3.95 \times 10^{-9}} = 2.78 \times 10^{-4}$$

FROM PLOT -  $p_{Ai} \approx 213 \text{ Pa}$  (a)

$$C_{Ai} \approx 0.69 \text{ kg mol/m}^3$$

$$N_A = k_g (p_{A0} - p_{Ai})$$

$$= 3.95 \times 10^{-9} (4000 - 213)$$

$$= 1.496 \times 10^{-5} \text{ kg mol/m}^2 \cdot \text{s}$$

29.14 CONTINUED -

FROM EQUILIBRIUM PLOT -

FOR  $C_{AL} = 0.55$   $p_A^* \approx 164$

$$K_g = \frac{N_A}{p_{A0} - p_A^*} = \frac{1.496 \times 10^{-5}}{4000 - 164}$$

$$= 3.9 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

§ ALSO FROM EQUILIBRIUM PLOT -

FOR  $p_{A0} = 4000$   $C_A^* \approx 6.9$

$$K_L = \frac{N_A}{C_A^* - C_{AL}} = \frac{1.496 \times 10^{-5}}{6.9 - 0.55}$$

$$= 2.36 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

SUMMARY :

$$k_g = 3.95 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$k_L = 1.1 \times 10^{-4} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

$$K_g = 3.9 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$K_L = 2.36 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

$$p_{A0} - p_{Ai} = 3787 \text{ Pa}$$

$$C_{Ai} - C_{AL} = 0.14 \text{ kg mol/m}^3$$

$$p_{A0} - p_A^* = 3836 \text{ Pa}$$

$$C_A^* - C_{AL} = 6.35 \text{ kg mol/m}^3$$

$$\frac{1/k_g}{1/K_g} = \frac{K_g}{k_g} = \frac{3.9 \times 10^{-9}}{3.95 \times 10^{-9}} = 0.987$$

$\sim \underline{98.7\%}$  OF RESISTANCE IS IN GAS  $\phi$  (c)

29.15  $\text{Cl}_2$  FROM GAS STREAM INTO LIQUID

$$P = 1.013 \times 10^5 \text{ Pa} \quad y_{\text{A}\infty} = 0.002$$

$$k_y = 1.0 \text{ kg mol/m}^2 \cdot \text{h} \cdot \Delta y$$

$$k_x = 10 \text{ kg mol/m}^2 \cdot \text{h} \cdot \Delta x$$

$$H = 6.13 \times 10^4 \text{ Pa/(kg mol/m}^3\text{)}$$

$$C_{\text{AL}} = 2.6 \times 10^{-3} \text{ kg mol/m}^3$$

$$p_{\text{Ai}} = H C_{\text{Ai}} \quad y_{\text{Ai}} = \frac{H C}{P} x_{\text{A}} = H' x_{\text{A}}$$

$$C_{\text{L}} = \frac{1000}{18} = 55.55 \text{ kg mol/m}^3$$

$$H' = \frac{H C}{P} = \frac{(6.13 \times 10^4)(55.55)}{1.013 \times 10^5} = 33.6$$

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{H' k_y}$$

$$= \frac{1}{10} + \frac{1}{33.6(1)}$$

$$K_x = 7.71 \text{ kg mol/m}^2 \cdot \text{h} \cdot \Delta x \quad (a)$$

$$x_{\text{AL}} = \frac{C_{\text{AL}}}{C} = \frac{2.6 \times 10^{-3}}{55.55} = 4.68 \times 10^{-5}$$

$$x_{\text{A}}^* = \frac{y_{\text{A}\infty}}{H'} = \frac{0.002}{33.6} = 5.95 \times 10^{-5}$$

$$N_{\text{A}} = K_x (x_{\text{A}}^* - x_{\text{AL}}) = 7.71 (5.95 - 4.68) \times 10^{-5}$$

$$= 9.79 \times 10^{-5} \text{ kg mol/m}^2 \cdot \text{h} \quad (b)$$

$$N_{\text{A}} = k_x (x_{\text{Ai}} - x_{\text{AL}})$$

$$9.79 \times 10^{-5} = 10 (x_{\text{Ai}} - 4.68 \times 10^{-5})$$

$$x_{\text{Ai}} = 5.66 \times 10^{-5} \quad (c)$$

29.15 CONTINUED -

$$y_{\text{AL}} = H' x_{\text{Ai}} = 33.6 (5.66 \times 10^{-5})$$

$$= 1.90 \times 10^{-3} \quad (c)$$

FRACTION OF RESISTANCE IN LIQUID  $\phi$

$$\frac{1/k_x}{1/K_x} = \frac{k_x}{K_x} = \frac{7.71}{10} = 0.771$$

$$= 77.1\% \quad (d)$$

29.16 COMPONENT A - FROM LIQ TO GAS

$$T = 290 \text{ K}$$

$$P = 1.013 \times 10^5 \text{ Pa}$$

$$p_{\text{Ag}} = 4000 \text{ Pa}$$

$$C_{\text{AL}} = 4 \text{ kg mol/m}^3$$

60% of RESISTANCE IS IN GAS PHASE

$$K_g = 2.46 \times 10^{-8} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$H = 1400 \text{ Pa/(kg mol/m}^3\text{)}$$

$$\frac{1/k_g}{1/K_g} = \frac{k_g}{K_g} = 0.6 \quad k_g = \frac{2.46 \times 10^{-8}}{0.6}$$

$$= 4.1 \times 10^{-8} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa} \quad (a)$$

$$p_{\text{A}}^* = H C_{\text{AL}} = 1400 (4) = 5600 \text{ Pa}$$

$$N_{\text{A}} = K_g (p_{\text{A}}^* - p_{\text{Ag}}) = (2.46 \times 10^{-8}) (5600 - 4000)$$

$$= 3.94 \times 10^{-5} \text{ kg mol/m}^2 \cdot \text{s}$$

$$= k_g (p_{\text{Ai}} - p_{\text{Ag}}) = (4.1 \times 10^{-8}) (p_{\text{Ai}} - 4000)$$

$$p_{\text{Ai}} = 4961 \text{ Pa} \quad (c)$$

29.16 CONTINUED -

$$C_{Ai} = \frac{P_{Ai}}{H} = \frac{4901}{1400} = 3.54 \text{ kg mol/m}^3$$

$$N_A = k_L (C_{AL} - C_{Ai})$$

$$3.94 \times 10^{-5} = k_L (4 - 3.54)$$

$$k_L = 8.56 \times 10^{-5} \text{ kg mol/m}^2 \cdot \text{s} \quad (b)$$

$$N_A = K_L (C_{AL} - C_A^*)$$

$$C_A^* = \frac{P_{AG}}{H} = \frac{4000}{1400} = 2.86$$

$$K_L = \frac{3.94 \times 10^{-5}}{4 - 2.86} \quad (d)$$

$$= 3.46 \times 10^{-5} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

29.17  $\text{Cl}_2$  FROM GAS PHASE INTO WATER

(EQUILIBRIUM DATA FOR THIS SYSTEM GIVEN IN PROB 29.1)

$$T = 293 \text{ K} \quad P_{AG} = 4 \times 10^4 \text{ Pa}$$

$$P = 1.013 \times 10^5 \text{ Pa} \quad C_{AL} = 1 \text{ kg/m}^3$$

75% OF RESISTANCE IS IN LIQUID -

$$\frac{P_{AG} - P_{Ai}}{P_{AG} - P_A^*} = 0.25$$

FROM PLOT OF PROB 29.1 DATA

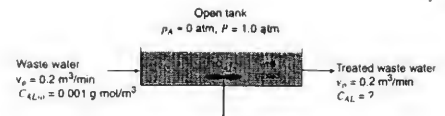
$$P_A^* = 4480 \text{ Pa}$$

$$\frac{40000 - P_{Ai}}{40000 - 4480} = 0.25 \quad \underline{P_{Ai} = 3.12 \times 10^4 \text{ Pa}}$$

$$\xi \text{ FROM PLOT} \quad \underline{C_{Ai} = 3.0 \text{ kg/m}^3}$$

29.18

SYSTEM  $\rightarrow$



$$\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{H k_g}$$

$$k_g = 0.01 \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{atm}$$

$$H = 10 \text{ atm/(kg mol/m}^3)$$

$$k_L = 5 \times 10^{-4} \text{ (kg mol/m}^2 \cdot \text{s)}$$

$$\frac{1}{K_L} = \frac{1}{5 \times 10^{-4}} + \frac{1}{(10)(0.01)} = 2010$$

$$K_L = 4.975 \times 10^{-4} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

FRACTION OF RESISTANCE IN LIQUID

$$= \frac{1/k_L}{1/K_L} = \frac{k_L}{K_L} = \frac{5 \times 10^{-4}}{4.975 \times 10^{-4}} = 0.995$$

$$= \underline{\underline{99.5\%}}$$

29.19  $\text{Tx}$  FROM BENZENE PHASE TO AQUEOUS  $\phi$

$$C_A' = 170 C_A''$$

(Bz) (Aq)

$$k_L' = 3.5 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

$$k_L'' = 2.5 \times 10^{-5} \text{ "}$$

$$\frac{1}{K_L'} = \frac{1}{k_L'} + \frac{A}{k_L''} = \frac{1}{3.5 \times 10^{-6}} + \frac{170}{2.5 \times 10^{-5}} \quad (a)$$

$$\underline{\underline{K_L' = 1.41 \times 10^{-7} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)}}$$

29.19 CONTINUED -

$$\frac{1}{K_L''} = \frac{1}{K_L} + \frac{1}{H'k_L}$$

$$= \frac{1}{2.5 \times 10^{-5}} + \frac{1}{(170)(3.5 \times 10^{-6})} \quad (b)$$

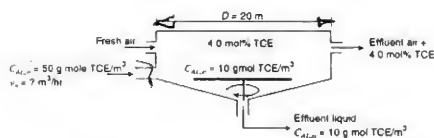
$$K_L'' = 2.40 \times 10^{-5} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

FRACTION OF RESISTANCE IN AIR FILM

$$\frac{1/k_L''}{1/K_L''} = \frac{K_L''}{k_L''} = \frac{2.40 \times 10^{-5}}{2.50 \times 10^{-5}} = 0.96$$

$$= 96\% \quad (c)$$

29.20



TCE TRANSFERRED FROM LIQUID TO GAS PHASE -

$$T = 293 \text{ K}$$

$$P = 1 \text{ ATM}$$

$$D = 20 \text{ m}$$

$$A = \frac{\pi}{4} (20)^2 = 314 \text{ m}^2$$

$$C_{A0} = 10 \text{ g mol/m}^3$$

$$y_{A0} = 0.04$$

$$k_x = 200 \text{ g mol/m}^2 \cdot \text{s}$$

$$k_y = 0.11 \text{ "}$$

$$H = 550 \text{ ATM/} \Delta x$$

$$C_L = 66 \text{ g mol/m}^3$$

$$p_A = H x_A$$

$$y_A = \frac{p_A}{P} = \frac{H}{P} x_A = H' x_A$$

$$H' = \frac{550}{1} = 550$$

29.20 CONTINUED -

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{H'k_y} = \frac{1}{200} + \frac{1}{(550)(0.11)}$$

$$K_x = 43.10 \text{ g mol/m}^2 \cdot \text{s}$$

$$K_L = \frac{K_x}{C} = \frac{43.10}{66} = 0.653 \text{ m/s} \quad (a)$$

$$N_A = K_x (x_{A0} - x_A^*)$$

$$x_{A0} = \frac{C_{A0}}{C} = \frac{10}{66} = 0.1515$$

$$y_{A0} = 550 x_A^*$$

$$x_A^* = \frac{0.04}{550} = 7.27 \times 10^{-5}$$

$$N_A = (43.10)(0.1515 - 7.27 \times 10^{-5})$$

$$= 6.53 \text{ g mol/m}^2 \cdot \text{s} \quad (b)$$

$$W_A = N_A \cdot A = 6.53 (314)$$

$$= 2050 \text{ g mol/s}$$

MASS BALANCE FOR LIQUID -

$$\dot{V}_0 C_A|_{in} = 2050 (3600) + \dot{V}_0 C_A|_{out}$$

$$\dot{V}_0 = \frac{2050 (3600)}{50 - 10}$$

$$= 1.845 \times 10^5 \text{ g mol/h}$$

$$= \frac{1.845 \times 10^5}{66} \text{ m}^3/\text{h}$$

$$= 2795 \text{ m}^3/\text{h} \quad (c)$$

29,21  $\text{NH}_3 \frac{1}{3} \text{H}_2\text{S}$  STRIPPED FROM  $\text{H}_2\text{O}$

FOR BOTH -

$$k_G = 3.20 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$k_L = 1.73 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

$$H_{\text{NH}_3} = 1.36 \times 10^3 \text{ Pa/(kg mol/m}^3)$$

$$H_{\text{H}_2\text{S}} = 8.81 \times 10^5 \quad "$$

$$\frac{1}{K_G} = \frac{1}{k_G} + \frac{H}{k_L}$$

$$\text{NH}_3: \frac{1}{K_G} = \frac{1}{3.20 \times 10^{-9}} + \frac{1.36 \times 10^3}{1.73 \times 10^{-9}}$$

$$\underline{K_G = 2.556 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}}$$

$$\text{H}_2\text{S}: \frac{1}{K_G} = \frac{1}{3.20 \times 10^{-9}} - \frac{8.81 \times 10^5}{1.73 \times 10^{-9}}$$

$$\underline{K_G = 1.95 \times 10^{-11} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}}$$

$$\frac{K_{G \text{ NH}_3}}{K_{G \text{ H}_2\text{S}}} = \frac{2.556}{1.95} = \underline{\underline{131 \text{ TO } 1}}$$

29,22  $\text{NH}_3$  ABSORBED

$$K_G = 3.12 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$C_{AL} = 4 \text{ kg mol/m}^3$$

$$P_{AG} = 3040 \text{ Pa}$$

$$P_{Ai} = (1360 \text{ Pa/(kg mol/m}^3)) C_{Ai}$$

75% OF RESISTANCE IS IN GAS PHASE

29,22 CONTINUED -

$$\frac{1/k_G}{1/K_G} = \frac{k_G}{K_G} = 0.75 = \frac{3.12 \times 10^{-9}}{K_G}$$

$$\underline{K_G = 4.16 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}} \quad (a)$$

$$\begin{aligned} K_L &= H K_G = (1360)(4.16 \times 10^{-9}) \\ &= 4.24 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3) \end{aligned} \quad (c)$$

25% OF RESISTANCE IN LIQUID PHASE

$$\begin{aligned} 0.25 &= \frac{K_L}{k_L} \quad k_L = \frac{4.24 \times 10^{-6}}{0.25} \\ & \quad (b) \\ k_L &= 16.96 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3) \end{aligned}$$

$$N_A = K_G (P_A^* - P_{AG})$$

$$P_A^* = H C_{AL} = (1360)(4) = 5440 \text{ Pa}$$

$$\begin{aligned} N_A &= (3.12 \times 10^{-9})(5440 - 3040) \\ &= 7.488 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s} \end{aligned}$$

$$= K_G (P_{Ai} - P_{AG})$$

$$= (4.16 \times 10^{-9})(P_{Ai} - 3040)$$

$$\underline{P_{Ai} = 4840 \text{ Pa}}$$

$$C_{Ai} = \frac{P_{Ai}}{H} = \frac{4840}{1360} = \underline{\underline{3.56 \text{ kg mol/m}^3}} \quad (d)$$

29.23

NH<sub>3</sub> REMOVAL

$$T = 303 \text{ K}$$

$$P = 2 \text{ atm}$$

$$C_L = 55.6 \text{ kg mol/m}^3$$

$$X_{AL} = 0.04$$

$$P_{AG} = 0.2 \text{ atm}$$

$$k_G = 1.0 \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{atm}$$

$$k_L = 0.045 \text{ m/s}$$

$$k_x = C_L k_L = (55.6)(0.045) \\ = 2.50 \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{atm} \quad (a)$$

VALUES FROM EQUILIBRIUM CURVE:

$$P_A = 0.02 \text{ atm} \quad X_A = 0.018$$

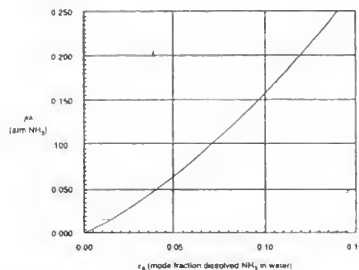
$$C_A = C X_A = 55.6(0.018) \\ = 1.0 \text{ kg mol/m}^3$$

$$H = P_A / C_A = 0.02 / 1 \\ = 0.02 \text{ atm / (kg mol/m}^3)$$

$$\frac{1}{K_g} = \frac{1}{k_G} + \frac{H}{k_L} = \frac{1}{1.0} + \frac{0.02}{0.045} \\ K_g = 0.692 \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{atm} \quad (c)$$

FRACTION OF RESISTANCE IN GAS  $\phi$ :

$$\frac{1/k_G}{1/k_G + H/k_L} = \frac{K_g}{K_g} = \frac{0.692}{1} = 0.692$$



29.23 CONTINUED -

FOR OPERATING POINT AT  $X_A = 0.04$ ,

$$P_A = 0.2 \quad - \quad P_A^* = 0.050 \text{ atm}$$

$$\frac{P_{AG} - P_{Ai}}{P_{AG} - P_A^*} = 0.692 = \frac{0.20 - P_{Ai}}{0.20 - 0.05}$$

$$P_{Ai} = 0.096 \text{ atm} \quad - \quad X_{Ai} = 0.067 \quad (\text{FROM CURVE})$$

$$y_{Ai} = \frac{P_{Ai}}{P} = \frac{0.096}{2} = 0.048 \quad (b)$$

$$N_A = K_G (P_{AG} - P_A^*) \\ = (0.692)(0.20 - 0.05) \\ = 0.104 \text{ kg mol/m}^2 \cdot \text{s} \quad (d)$$

29.24 ABSORPTION TOWER - SOLUTE (A)  
SOLVENT (B)

$$P_{AG} = 1.519 \times 10^4 \text{ Pa}$$

$$C_{AL} = 1.0 \times 10^{-3} \text{ kg mol/m}^3$$

$$N_A = 4 \times 10^{-5} \text{ kg mol/m}^2 \cdot \text{s}$$

$$k_G = 3.95 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$H = \frac{3.04 \times 10^3}{1 \times 10^{-3}} = 3.04 \times 10^6 \text{ Pa / (kg mol/m}^3)$$

$$N_A = k_G (P_{AG} - P_{Ai})$$

$$4 \times 10^{-5} = (3.95 \times 10^{-9})(1.59 \times 10^4 - P_{Ai})$$

$$P_{Ai} = 5070 \text{ Pa}$$

$$P_{AG} - P_{Ai} = 15190 - 5070 = 10120 \text{ Pa}$$

29,24 CONTINUED-

$$N_A = k_L (C_{Ai} - C_{AL})$$

$$C_{Ai} = \frac{p_{Ai}}{H} = \frac{5.07 \times 10^3}{3.04 \times 10^6}$$

$$= 1.67 \times 10^{-3} \text{ kg mol/m}^3$$

$$4 \times 10^{-5} = k_L (1.67 \times 10^{-3} - 1.0 \times 10^{-3})$$

$$k_L = 0.0597 \text{ m/s}$$

$$C_{AL} - C_{AL} = 1.67 \times 10^{-3} - 1.0 \times 10^{-3}$$

$$= 0.67 \times 10^{-3} \text{ kg mol/m}^3$$

$$\frac{1}{K_G} = \frac{1}{K_G} + \frac{H}{k_L} = \frac{1}{3.95 \times 10^9} + \frac{3.04 \times 10^6}{0.0597}$$

$$K_G = 3.29 \times 10^{-9} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$N_A = K_G (p_{AG} - p_A^*)$$

$$p_{AG} - p_A^* = \frac{4 \times 10^{-5}}{3.29 \times 10^{-9}}$$

$$= 1.216 \times 10^4 \text{ Pa}$$

$$\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{H K_G}$$

$$= \frac{1}{0.0597} + \frac{1}{(3.95 \times 10^9)(3.04 \times 10^6)}$$

$$K_L = 0.010 \text{ kg mol/m}^2 \cdot \text{s} \cdot (\text{kg mol/m}^3)$$

29,24 CONTINUED-

$$N_A = K_L (C_A^* - C_{AL})$$

$$C_A^* - C_{AL} = \frac{4 \times 10^{-5}}{0.01} = 4 \times 10^{-3} \text{ kg mol/m}^3$$

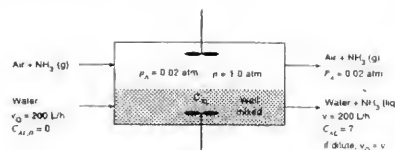
FRACTION OF RESISTANCE IN LIQUID  $\phi$

$$= \frac{1/k_L}{1/K_L} = \frac{k_L}{K_L} = \frac{0.010}{0.0597} = 0.167$$

$$(16.7\%)$$

29,25

NH<sub>3</sub> INTO H<sub>2</sub>O



$$T = 293 \text{ K}$$

$$P = 1 \text{ atm}$$

$$D = 4 \text{ m}$$

$$p_{AG} = 0.02 \text{ atm}$$

$$\dot{V} = 200 \text{ L/h} = 0.2 \text{ m}^3/\text{h}$$

$$H = 0.02 \text{ atm}/(\text{kg mol/m}^3)$$

$$k_G = 1.25 \text{ kg mol/m}^2 \cdot \text{h} \cdot \text{atm}$$

$$k_L = 0.05 \text{ m/h}$$

$$\frac{1}{K_G} = \frac{1}{k_G} + \frac{H}{k_L} = \frac{1}{1.25} + \frac{0.02}{0.05}$$

$$K_G = 0.833 \text{ kg mol/m}^2 \cdot \text{h} \cdot \text{atm} \quad (a)$$

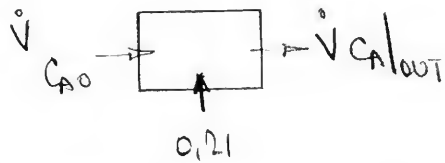
$$\frac{1/p_{AG}}{1/K_G} = \frac{k_G}{K_G} = \frac{0.0833}{1.25} = \frac{p_{AG} - p_{Ai}}{p_{AG} - p_A^*}$$

$$\frac{0.02 - p_{Ai}}{0.02 - 0} = 0.666 \quad p_{Ai} = 0.0067 \quad (b)$$

29,25 CONTINUED -

$$\begin{aligned}W_A &= N_A \cdot A_x = K_G (p_{A0} - p_A^*) A \\&= (0,833)(0,02-0) \left(\frac{\pi}{4}\right) (4)^2 \\&= 0,21 \text{ kg mol/h} \quad (c)\end{aligned}$$

MASS BALANCE:



$$\text{For NH}_3: \dot{V}(0) + 0,21 = \dot{V} C_{A,out}$$

$$C_{A,out} = \frac{0,21}{0,2} = 1,05 \text{ kg mol/m}^3 \quad (d)$$

# CHAPTER 30

## 30.1 SOLVENT EVAPORATING INTO AIR

$$\begin{aligned} T_s &= 313 \text{ K} \\ T_\infty &= 293 \text{ K} \\ T_f &= 303 \text{ K} \\ P_0 &= 1 \text{ atm} \\ P_s &= 0.05 \text{ atm} \\ C_{A,i} &= 0.001 \text{ mol/cm}^3 \end{aligned}$$

$$\begin{aligned} \text{AIR @ } 303 \text{ K: } \nu &= 0.158 \text{ cm}^2/\text{s} \\ \rho &= 1.17 \times 10^{-3} \text{ g/cm}^3 \end{aligned}$$

$$Re = \frac{L u_\infty}{\nu} = \frac{(20)(5.0)}{0.158} = 633 \quad \left\{ \begin{array}{l} \text{LAMINAR} \\ (a) \end{array} \right.$$

$$D_{AB} = 0.1 \left( \frac{303}{298} \right)^{3/2} = 1.025 \text{ cm}^2/\text{s}$$

$$Sc = \frac{0.158}{1.025} = 0.154 \quad (a)$$

$$\begin{aligned} Sh &= \frac{k_c L}{D_{AB}} = 0.664 Re_L^{1/2} Sc^{1/3} \\ &= 9.16 \quad (a) \end{aligned}$$

$$\bar{k}_c = \frac{9.16 (1.025)}{20} = 0.469 \text{ cm/s}$$

$$C = \frac{P}{RT} = \frac{1}{(82.06)(303)} = 4.02 \times 10^{-5} \text{ g mol/cm}^3$$

$$\begin{aligned} \bar{E}_y &= C \bar{k}_c = (4.02 \times 10^{-5})(0.469) \\ &= 1.89 \times 10^{-5} \text{ g mol/cm}^2 \cdot \text{s} \cdot \Delta y \quad (b) \end{aligned}$$

$$W_A = k_y (y_A^* - y_{A,\infty}) A_x$$

$$y_A^* = \frac{P_{A,0}}{P} = 0.05$$

$$\begin{aligned} W_A &= (1.89 \times 10^{-5})(0.05 - 0)(20 \times 10) \\ &= 1.89 \times 10^{-4} \text{ g mol/s} \end{aligned}$$

## 30.1 CONTINUED

$$\text{MOLES OF SOLVENT} = 8V$$

$$= (0.001)(20)(10)(0.01) = 0.002$$

$$t = \frac{0.002}{1.89 \times 10^{-4}} = 10.58 \text{ s} \quad (c)$$

## 30.2 NAPHTHALENE SUBLIMING INTO AIR

$$\begin{aligned} T_s &= 290 \text{ K} \\ T_\infty &= 310 \text{ K} \\ T_f &= 300 \text{ K} \\ P_A^0 &= 76 \text{ Pa} \end{aligned} \quad \left\{ \begin{array}{l} \text{LAMINAR} \\ \text{LAMINAR} \end{array} \right.$$

$$\nu = 1.569 \times 10^{-5} \text{ m}^2/\text{s} \quad \nu = 20 \text{ m/s}$$

$$D_{AB} = 5.61 \times 10^{-6} \left( \frac{300}{290} \right)^{3/2} = 5.90 \times 10^{-6} \text{ m}^2/\text{s}$$

$$At \ x=3 \text{ m} \quad Re_x = \frac{0.3(20)}{1.569 \times 10^{-5}} = 3.82 \times 10^5$$

$$Sc = \frac{1.569 \times 10^{-5}}{5.90 \times 10^{-6}} = 2.66$$

$$\begin{aligned} Re_x &= \frac{D_{AB}}{x} Re^{4/5} Sc^{1/3} \\ &= \frac{5.90 \times 10^{-6}}{0.3} (3.82 \times 10^5)^{4/5} (2.66)^{1/3} \\ &= 0.0232 \text{ m/s} \quad (a) \end{aligned}$$

$$\text{FROM } 0.5 \text{ m} < x < 0.75 \text{ m}$$

$$\bar{k}_c = \frac{0.0365 D_{AB} Sc^{1/3} [Re_{0.75}^{4/5} - Re_{0.5}^{4/5}]}{0.75 - 0.5}$$

$$Re_{0.75} = \frac{0.75(20)}{1.569 \times 10^{-5}} = 9.56 \times 10^5$$

$$Re_{0.5} = \frac{0.5(20)}{1.569 \times 10^{-5}} = 6.37 \times 10^5$$

30.2 CONTINUED-

SUBSTITUTING VALUES:

$$\bar{k}_c = 0.020 \text{ m/s}$$

$$W_A = N_A A = \bar{k}_c (C_{AS} - C_{AP}) A$$

$$C_{AS} = \frac{P}{RT} = \frac{26}{(8.314)(290)} = 0.0108 \text{ mol/m}^3$$

$$C_{AP} = 0$$

$$\begin{aligned} W_A &= (0.020)(0.0108)(0.25)(1) \\ &= 5.4 \times 10^{-5} \text{ mol/s} \\ &= \underline{\underline{0.1944 \text{ mol/h}}} \end{aligned}$$

30.3  $C_2H_5OH$  INTO AIR

$$T_f = \frac{289 + 303}{2} = 296 \text{ K} \quad P_{\infty} = 6.45 \times 10^{-2} \text{ atm}$$

$$D_{AB} = 1.32 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\nu = 1.53 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Re_L = \frac{U_{\infty} L}{\nu} = \frac{(3)(2)}{1.53 \times 10^{-5}} = 3.92 \times 10^5$$

TRICKY FLOW AS LAMINAR FOR  $Re \leq 2 \times 10^5$   
& TURBULENT FOR  $Re > 2 \times 10^5$

$$\bar{k}_c = \frac{D_{AB}}{L} \left[ 0.664 Re_L^{1/2} Sc^{1/3} + 0.0365 Sc^{1/3} (Re_L^{4/5} - Re_t^{4/5}) \right]$$

$$Re_t = 2 \times 10^5 \quad Re_L = 3.92 \times 10^5$$

SUBSTITUTING VALUES:

$$\bar{k}_c = 5.16 \times 10^{-3} \text{ m/s}$$

30.3 CONTINUED-

$$W_A = N_A A = \bar{k}_c (C_{AS} - C_{AP}) A$$

$$C_{AS} = \frac{P_A^0}{RT} = \frac{(6.45 \times 10^{-2})(1.013 \times 10^5)}{(8.314)(289)} = 2.72 \text{ mol/m}^3$$

$$\begin{aligned} W_A &= (5.16 \times 10^{-3})(2.72 - 0)(2 \times 4) \\ &= 0.112 \text{ mol/s} \\ &= \underline{\underline{0.112(46) = 5.15 \text{ g/s}}} \end{aligned}$$

30.4 MOLECULAR DIFFUSION THROUGH GRAVEL - THEN CONVECTIVE TRANSFER TO AIR -

$$T = 288 \text{ K}$$

$$U = 2 \text{ cm/s}$$

$$P_A^0 = 1039 \text{ Pa}$$

$$L = 10 \text{ m}$$

$$\text{GRAVEL DEPTH} = 1 \text{ m}$$

$$\text{THROUGH GRAVEL - } N_A = \frac{D_{AB}}{\delta} (C_{A1} - C_{A2})$$

$$\text{AT SURFACE } N_A = \bar{k}_c (C_{A2} - C_{AP})$$

$$Re_L = \frac{L U_{\infty}}{\nu} = \frac{(10)(0.02)}{1.46 \times 10^{-5}} = 1.37 \times 10^4$$

(LAMINAR)

$$\bar{k}_c = \frac{D_{AB}}{L} (0.664) Re_L^{1/2} Sc^{1/3}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.46 \times 10^{-5}}{5.72 \times 10^{-6}} = 2.55$$

$$\bar{k}_c = \frac{5.72 \times 10^{-6}}{10} (0.664) (1.37 \times 10^4)^{1/2} (2.55)^{1/3}$$

$$= 6.07 \times 10^{-5} \text{ m/s}$$

$$C_{A1} = \frac{P_A^0}{RT} = \frac{1039}{(8.314)(288)} = 0.434 \text{ mol/m}^3$$

30.4 CONTINUED -

AT STEADY STATE -

$$N_A = \frac{D_{AB}}{\delta} (C_{A1} - C_{A2}) = k_c (C_{A2} - 0)$$

$$\frac{5.72 \times 10^{-6}}{1} (0.0374 - C_{A2}) = 6.07 \times 10^{-5} C_{A2}$$

$$C_{A2} = 0.0374 \text{ mol/m}^3$$

$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{(8.314)(288)} = 42.31 \text{ mol/m}^3$$

$$y_{A2} = \frac{0.0374}{42.31} = 8.84 \times 10^{-4} \quad (a)$$

$$N_A = (6.07 \times 10^{-5})(0.0374) = 2.27 \times 10^{-6} \text{ mol/m}^2 \cdot \text{s}$$

FOR SAME CONFIGURATION & PROCESS  
BUT  $U_M = 50 \text{ cm/s}$

$$Re_L = \frac{(40)(50)}{1.46 \times 10^{-5}} = 3.42 \times 10^5$$

{ INTO TURBULENT  
FLOW REGIME }

FOR  $Re \leq 2 \times 10^5$  LAMINAR B.L.  
 $Re > "$  TURBULENT "

$$k_c = \frac{D_{AB} Sc^{1/3}}{L} \left[ 0.664 Re_{tr}^{1/2} + 0.0365 \left( Re_L^{4/5} - Re_{tr}^{4/5} \right) \right]$$

$$Re_{tr} = 2 \times 10^5$$

$$Re_L = 3.42 \times 10^5$$

SUBSTITUTING & SOLVING:

$$k_c = 4.98 \times 10^{-4} \text{ m/s}$$

30.4 CONTINUED -

$$\frac{5.72 \times 10^{-6}}{1} (C_{A1} - C_{A2}) = 4.98 \times 10^{-4} (C_{A2} - 0)$$

$$C_{A2} = 4.93 \times 10^{-3} \text{ mol/m}^3$$

$$y_{A2} = \frac{4.93 \times 10^{-3}}{42.31} = 1.16 \times 10^{-4} \quad (b)$$

$$N_A = (4.98 \times 10^{-4})(4.93 \times 10^{-3}) = 2.45 \times 10^{-6} \text{ mol/m}^2 \cdot \text{s}$$

$$\text{MPEC TX BIOT NO.} = \frac{k_c \delta}{D_{AB}} = \frac{(6.07 \times 10^{-5})(1)}{5.72 \times 10^{-6}} = 10.61 \quad \{ \text{CASE (a)} \}$$

$$\frac{1/k_c}{1/D_{AB}/\delta} = \frac{1}{10.61} = 0.094$$

~ 9.4% RESISTANCE IN FLOWING STREAM

$$\text{FOR CASE (b)} - Bi = \frac{k_c \delta}{D_{AB}} = 87.06$$

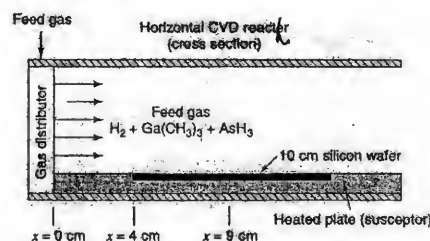
$$\frac{1}{87.06} = 0.0115$$

~ 1.15% RESISTANCE IN AIR STREAM

30.5 REFER TO CHAPTER - EXAMPLE 1 - FOR PROBLEM SPECIFICATIONS -

ACISINE (A)

TMG (B)



$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{(8.314)(800)} = 15.23 \text{ mol/m}^3$$

30.5 CONTINUED -

$$C_{AP} = C_{BP} = 0.001(15.23) \\ = 0.0152 \text{ mol/m}^3$$

For TM6 -  $T = 800 \text{ K}$

$$\nu_{H_2} = 5.686 \text{ cm}^2/\text{s} \quad \left\{ \begin{array}{l} Sc = 3.67 \\ D_{AB} = 1.65 \text{ cm}^2/\text{s} \end{array} \right.$$

At  $x = 4 \text{ cm}$ :  $Sh_x = k_c = N_A = 0$

$x = 9 \text{ cm}$  - SEE EXAMPLE 1 -

$$Sh_x = 8.375 \quad k_c = 0.0144 \text{ m/s}$$

$$N_B = 0.0144(0.0152) \\ = 2.19 \times 10^{-4} \text{ mol B/m}^2 \cdot \text{s}$$

$x = 14 \text{ cm}$

$$Re_x = \frac{\nu_{max} x}{D} = \frac{(100)(14)}{5.686} = 246$$

$$k_{cB} = \frac{D_{AB}}{x} \left[ 0.332 Re_x^{1/2} \left( \frac{Sc}{1 - \left( \frac{x}{L} \right)^{3/4}} \right)^{1/3} \right]$$

$$x/x = 4/14 \sim \text{OTHER QUANTITIES KNOWN}$$

SUBSTITUTING & SOLVING:

$$k_c = 0.01048 \text{ m/s}$$

$$N_B = (0.01048)(0.0152) = 1.59 \times 10^{-4} \text{ mol/m}^2 \cdot \text{s}$$

For A:

At  $x = 4 \text{ cm}$ :  $Sh_x = k_c = N_A = 0$

30.5 CONTINUED -

$$x = 9 \text{ cm} \quad Re_x = \frac{(100)(9)}{5.686} = 158.3$$

$$k_{cA} = \frac{3.17}{9} \left[ 0.332 (158.3)^{1/2} \left( \frac{1.784}{1 - (4/9)^{3/4}} \right)^{1/3} \right] \\ = 0.0232 \text{ m/s}$$

$$N_A = (0.0232)(0.0152) = 3.53 \times 10^{-4} \text{ mol/m}^2 \cdot \text{s}$$

At  $14 \text{ cm}$   $Re_x = \frac{(100)(14)}{5.686} = 246$

SAME FORMULA BUT  $x = 14$   $Re = 246$

$$k_{cA} = 0.0169 \text{ m/s}$$

$$N_A = (0.0169)(0.0152) = 2.57 \times 10^{-4} \text{ mol/m}^2 \cdot \text{s}$$

TO PRODUCE  $\frac{N_A}{N_B} = 1 \Rightarrow \frac{k_{cA}(C_{AS} - C_{AP})}{k_{cB}(C_{BS} - C_{BP})} = 1$

@  $9 \text{ cm}$ :  $\frac{k_{cA}}{k_{cB}} = \frac{0.0232}{0.0144} \frac{\Delta C_A}{\Delta C_B}$

$$= 1 \quad \text{IF} \quad \frac{\Delta C_A}{\Delta C_B} = 0.62$$

@  $14 \text{ cm}$ :  $\frac{\Delta C_A}{\Delta C_B} = \frac{k_{cB}}{k_{cA}} = \frac{0.01048}{0.0169} = 0.62$

FOR BOTH CASES -

$$\Delta C_B = C_{BS} - C_{BP} = 0.0152 - C_{BP}$$

$$\Delta C_A = C_{AS} - C_{AP} = 0.0152$$

SO  $C_{BP}$  SHOULD BE  $\frac{0.0152 - C_{BP}}{0.0152} = 0.62$

$$\text{OR } C_B = 0.00578 \text{ mol/m}^3$$

30.5 CONTINUED -

THICKNESS OF GaAs FILM AFTER 120S

- @  $x = 4 \text{ cm}$   $\delta = 0$

@  $x = 9 \text{ cm}$

GaAs DEPOSITED -

$$= (2.19 \times 10^{-4}) (144) (120) = 3.78 \text{ g/m}^2$$

$$P_s = 5.8 (100)^3 = 5.8 \times 10^6 \text{ g/m}^2$$

$$\delta = \frac{3.78}{5.68 \times 10^6} = 0.652 \times 10^{-6} \text{ m}$$

$$\sim \underline{\underline{0.652 \mu\text{m}}}$$

AT  $x = 14 \text{ cm}$

$$\delta = \frac{(1.59 \times 10^{-4}) (144) (120)}{5.68 \times 10^6}$$

$$= 0.474 \times 10^{-6} \text{ m} = \underline{\underline{0.474 \mu\text{m}}}$$

30.6 MASS TX FROM SPHERICAL SURFACE -

$$D = 1 \text{ cm} \quad P_A^0 = 1.17 \times 10^4 \text{ Pa}$$

$$T = 298 \text{ K} \quad M_A = 78$$

$$P = 1 \text{ ATM} \quad D_{AB} = 0.0962 \text{ cm}^2/\text{s}$$

$$\text{MASS OF SOLVENT} = (0.12 \text{ g/cm}^2) A$$

$$= 0.12 (\pi) (1)^2 = 0.377 \text{ g}$$

$$= 3.77 \times 10^{-4} \text{ kg}$$

$$W_A = N_A A = k_c (C_{AS} - C_{AP}) \pi D^2$$

$$D = \frac{\mu}{\rho} = \frac{1.85 \times 10^{-4}}{1.18 \times 10^{-3}} = 0.1568 \text{ cm}^2/\text{s}$$

$$D_{AB} = 0.0962 \text{ cm}^2/\text{s}$$

30.6 CONTINUED -

$$Sc = \frac{D}{D_{AB}} = \frac{0.1568}{0.0962} = 1.63$$

IN STAGNANT AIR -  $\frac{k_c D}{D_{AB}} = 2$

$$k_c = \frac{2(0.0962)}{1} = 0.1924 \text{ cm/s}$$

$$C_{AS} = \frac{P_A^0}{RT} = \frac{1.17 \times 10^4}{(8.314)(298)} = 4.72 \text{ mol/m}^3$$

$$W = (0.1924)(4.72 - 0) \pi (1) (78)(10^{-6})$$

$$= 2.225 \times 10^{-4} \text{ g/s} = 2.225 \times 10^{-7} \text{ kg/s}$$

$$t = \frac{3.77 \times 10^{-4}}{2.225 \times 10^{-7}} = \underline{\underline{1694 \text{ s}}}$$

$$= \underline{\underline{0.471 \text{ h}}} \quad (a)$$

FOR  $U_{\infty} = 1 \text{ m/s}$

$$Re = \frac{1(100)}{0.1568} = 638$$

$$Sh = \frac{k_c D}{D_{AB}} = 2 + 0.552 Re^{1/2} Sc^{1/3}$$

$$k_c = \frac{D_{AB}}{D} (\quad) = 1.77 \text{ cm/s}$$

$$W_A = 2.225 \times 10^{-4} \left( \frac{1.77}{0.1924} \right)$$

$$= 20.43 \times 10^{-7} \text{ kg/s}$$

$$t = \frac{3.77 \times 10^{-4}}{20.43 \times 10^{-7}} = \underline{\underline{184 \text{ s}}} \quad (b)$$

30.7 A DIFFUSING THROUGH STAGNANT B ~  $N_B = 0$

$$N_{Ar} = - \frac{C_{DAB}}{1-y_A} \frac{dy_A}{dr}$$

$$\nabla^2 N_A = \frac{1}{r^2} \frac{d}{dr} (r^2 N_{Ar}) = 0$$

$$\sim r^2 N_{Ar} = \text{CONSTANT}$$

$$N_{Ar} R^2 \int_{R/2}^R \frac{dr}{r^2} = C_{DAB} \int_{y_{AS}}^{y_{AR}} - \frac{dy_A}{1-y_A}$$

$$N_{Ar} R^2 \left[ \frac{1}{R} - 0 \right] = C_{DAB} \ln \left( \frac{1-y_{AR}}{1-y_{AS}} \right)$$

$$N_{Ar} R = \frac{C_{DAB}}{y_{BLM.}} (y_{AS} - y_{AR})$$

$$N_{Ar} \frac{D}{2} = \frac{C_{DAB}}{y_{BLM.}} (y_{AS} - y_{AR})$$

$$N_{Ar} = \frac{2 D_{AB}}{y_{BLM.}} (C_{AS} - C_{AR})$$

$$\Rightarrow k_c = \frac{2 D_{AB}}{(y_{BLM.}) D}$$

For  $y_{BLM.} \approx 1$  (DILUTE SOLN OF A)

$$\underline{\underline{Sh = \frac{k_c D}{D_{AB}} = 2}}$$

30.8 SPHERICAL PELLET IN CROSSFLOW

$$T = 293 \text{ K} \quad \nu = 9.95 \times 10^{-3} \text{ cm}^2/\text{s}$$

$$D = 1 \text{ cm} \quad D_{AB} = 1.2 \times 10^{-5} \text{ "}$$

$$U_{\infty} = 5 \text{ cm/s} \quad C_{AR} = 0$$

30.8 CONTINUED -

$$\frac{k_c D}{D_{AB}} = 2.0 + 0.552 Re^{1/2} Sc^{1/3}$$

$$Re = \frac{D U_{\infty}}{\nu} = \frac{(1)(5)}{9.95 \times 10^{-3}} = 502$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{9.95 \times 10^{-3}}{1.2 \times 10^{-5}} = 837$$

Solving for  $k_c$ :  $k_c = 0.00141 \text{ cm/s} \quad (2)$

$$V = \frac{\pi D^3}{6} \quad \frac{dV}{dt} = 2\pi D^2 \frac{dD}{dt}$$

$$W_A = \frac{P_A}{M_A} \frac{dV}{dt} = k_c (C_{AS} - C_{AR}^0) \pi D^2$$

$$\frac{P_A}{M_A} 2\pi D^2 \frac{dD}{dt} = \pi D^2 k_c C_{AS}$$

$$\frac{dD}{dt} = \frac{(0.00141)(7 \times 10^{-4})(110)}{2(2)}$$

$$= 2.71 \times 10^{-5} \text{ cm/s} = 0.0977 \text{ cm/h}$$

$$\underline{\underline{\frac{dP}{dt} = 0.195 \text{ cm/h} \quad (b)}}$$

For  $D = 0.5 \text{ cm}$

$$Re_D = 251 \quad k_c = 2.0 + 0.552 Re^{1/2} Sc^{1/3} = 0.00201 \text{ cm/s}$$

$$\frac{W_A|_{1.0}}{W_A|_{0.5}} = \frac{0.00141 \text{ cm/s} \pi (1)^2}{0.00201 \text{ cm/s} \pi (0.5)^2}$$

$$= 2.804 \quad (c) \left\{ \begin{array}{l} \text{INCREASE BY} \\ \text{THIS FACTOR} \end{array} \right\}$$

30.9 GLUCOSE (A) INTO AQUEOUS STREAM 30.10  $\text{Cl}_2$  (A) INTO LIQUID (B)

$$T = 298 \text{ K} \quad D = 0.3 \text{ cm}$$

$$V = 0.15 \text{ m/s} \quad \{\text{SPARKS}\}$$

$$D_{AB} = 6.9 \times 10^{-10} \text{ m}^2/\text{s}$$

FOR TX INTO A LIQUID STREAM:

$$Re_0 = \frac{Dv}{\nu}$$

$$\nu = \frac{0.00091 \text{ kg/m.s}}{997 \text{ kg/m}^3}$$

$$= 9.127 \times 10^{-7} \text{ m}^2/\text{s}$$

$$Re = \frac{(0.003)(0.15)}{9.127 \times 10^{-7}} = 493$$

$$Sc = \frac{9.127 \times 10^{-7}}{6.9 \times 10^{-10}} = 1322$$

$$Pe = Re Sc = 6.521 \times 10^5$$

Eqn. (30-8) APPLIES

$$\frac{k_L D}{D_{AB}} = 1.01 Pe^{1/3}$$

$$k_L = \frac{1.01 (6.521 \times 10^5)^{1/3} 6.9 \times 10^{-10}}{0.003}$$

$$= 2.0 \times 10^{-5} \text{ m/s}$$

$$k_L \sim \frac{1}{D} (D^{1/3} V^{1/3}) \sim \frac{V^{1/3}}{D^{2/3}}$$

FOR D INCREASING  $k_L$  DECREASES

FOR V "  $k_L$  INCREASES

LARGER EFFECT IS  $\Delta D$  (b)

BOBBLE:  $D = 0.002 \text{ m}$

$$\rho_B = 1.47 \text{ g/cm}^3 \quad H = 6.76 \text{ atm}/\Delta x_A$$

$$\mu_B = 5.2 \times 10^{-4} \text{ kg/m.s}$$

$$D_{AB} = 5.6 \times 10^{-5} \text{ cm}^2/\text{s}$$

FOR  $D < 2.5 \text{ mm}$  - Eq (30-14a) APPLIES -

$$k_c = \frac{D_{AB}}{D} \left[ 0.131 Gr_{AB}^{1/3} Sc^{1/3} \right]$$

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{5.2 \times 10^{-4}}{(1.47 \times 10^3)(5.6 \times 10^{-9})}$$

$$= 63.2$$

$$Gr = \frac{D^3 \rho_L (\rho_L - \rho_G) g}{\mu_L^2}$$

$$\rho_G = \frac{PM}{RT} = \frac{(1.013 \times 10^5)(71)}{(8.314)(298)} = 2.9 \text{ kg/m}^3$$

$$\rho_L = 1470 \text{ kg/m}^3$$

SUBSTITUTING VALUES:

$$k = 2.95 \times 10^{-4} \text{ m/s}$$

$$N_A = k_c (C_{AS} - C_{Ar})^0$$

$$C_{AS} = X_A C_L = \frac{P_A}{H} C_L = \frac{1}{6.76} C_L$$

$$C_L = \frac{(1470)(1000)}{169.8} = 8660 \text{ mol/m}^3$$

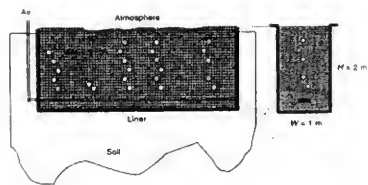
$$N_A = (2.95 \times 10^{-4}) \left( \frac{8660}{6.76} \right)$$

$$= 0.378 \text{ mol/m}^2.\text{s}$$

30.11

TOE (A)

BEING STRIPPED -



MASS BALANCE FOR A:

$$\left\{ \begin{array}{l} \text{RATE OF TX} \\ \text{FROM H}_2\text{O} \end{array} \right\} = \left\{ \begin{array}{l} \text{RATE OF DEPLETION} \\ \text{IN A}_2\text{O PHASE} \end{array} \right\}$$

$$N_A A_i = - \frac{dC_A}{dt} \quad \left\{ \text{PER m}^3 \right\}$$

$$K_L A_i C_A = - \frac{dC_A}{dt}$$

FOR LIQUID PHASE TX. CONTROLLING

$$k_L \approx K_L$$

$$- \frac{dC_A}{dt} = k_L A_i C_A$$

$$- \int_{C_{Ai}}^{C_A} \frac{dC_A}{C_A} = k_L A_i \int_0^t dt$$

$$\ln \left( \frac{C_{Ai}}{C_A} \right) = k_L A_i t \quad (a)$$

$$T = 293 K$$

$$M_A = 131.4$$

$$\mu_L = 9.93 \times 10^{-4} \text{ kg/m.s}$$

$$\rho_L = 998.2 \text{ kg/m}^3$$

$$\rho_G = 1.19 \text{ kg/m}^3$$

$$H = 9.97 \text{ Atm/(kg mol/m}^3)$$

$$D_{AB} = 8.9 \times 10^{-10} \text{ m}^2/\text{s}$$

$$Sc = \frac{9.95 \times 10^{-7}}{8.9 \times 10^{-10}} = 1117$$

$$\text{BUBBLE DIAM} \approx 0.005 \text{ m}$$

30.11 CONTINUED -

$$A_i \left( \text{PER m}^3 \right) = 0.015 \frac{\text{m}^3 \text{ AIR}}{\text{m}^3 \text{ H}_2\text{O}} \frac{6}{0.005} \frac{\text{m}^2}{\text{m}^3} = 18 \text{ m}^2/\text{m}^3$$

EQN (30-14b) APPLIES:

$$k_L = \frac{D_{AB}}{d_p} (0.42) Gr^{1/3} Sc^{1/2}$$

$$Gr = \frac{d_p^3 \rho_L g (\rho_L - \rho_G)}{\mu_L^2}$$

SUBSTITUTING VALUES:

$$k_L = 2.682 \times 10^{-4} \text{ m/s}$$

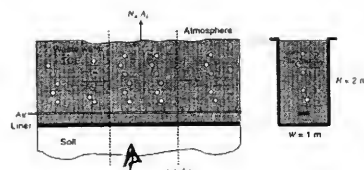
$$k_L A_i = (2.682 \times 10^{-4}) (18) = 0.00482 \text{ s}^{-1}$$

$$\ln \left( \frac{50}{0.005} \right) = 0.00482 t$$

$$t = 1911 \text{ s}$$

30.12

SAME SYSTEM AS IN PROB 13.11



MASS BALANCE FOR C.V. { CONSTITUENT A }

$$C_A A V|_z - C_A A V|_{z+\Delta z} = N_A A_i A \Delta z$$

DIVIDE BY  $A \Delta z$  & EVALUATE IN LIMIT  $\Delta z \rightarrow 0$ 

$$- V \frac{dC_A}{dz} = K_L A_i (C_A - C_A^*) \quad (a)$$

$$C_A^* = \frac{p_A}{H} = 0 \quad k_L = K_L \quad \left\{ \begin{array}{l} \text{LIQUID} \\ \text{PHASE} \\ \text{CONTROLS} \end{array} \right\}$$

30.12 CONTINUED -

$$-\int_{C_A}^{C_A^*} \frac{dC_A}{C_A} = \frac{k_L A_i}{V} \int_0^L dz$$

$$A_i = 18 \text{ m}^2/\text{m}^3 \quad \left\{ \begin{array}{l} \text{SEE SOLN TO} \\ \text{PROB 30.11} \end{array} \right\}$$

$$\ln \frac{C_{A0}}{C_A} = k_L A_i L$$

$$k_L = 2.682 \times 10^{-4} \text{ m/s}$$

{ SEE PROB 30.11 }  
FOR DETAILS

SUBSTITUTE VALUES & SOLVE -

$$L = 191.5 \text{ m}$$

30.13

THICKNESS OF  
COATING = 0.01 cm  
MASS OF COATING  
= 5 mg

$$\mu_L = 0.040 \text{ g/cm} \cdot \text{s} \quad \dot{V} = 10 \text{ cm}^3/\text{s}$$

$$\rho_L = 1.05 \text{ g/cm}^3$$

$$D_{AB} = 1 \times 10^{-6} \text{ cm}^2/\text{s} \quad M = 18$$

$$V = \frac{\dot{V}}{A} = \frac{10}{\pi/4 (1)^2} = 12.73 \text{ cm/s}$$

FOR A SINGLE CINDER - Eqn (30-16) -

$$\frac{k_L Sc^{0.56}}{V} = 0.281 Re^{-0.14}$$

30.13 CONTINUED

$$Sc = \frac{0.040}{(1.05)(1 \times 10^{-6})} = 3.8 \times 10^4$$

$$Re = \frac{(0.12)(12.73)(1.05)}{0.04} = 66.8$$

$$\text{SUBSTITUTING VALUES: } k_L = 0.00181 \text{ cm/s} \quad (a)$$

$$W = N_A A = k_L (C_A^* - C_{A0}) (\pi D L)$$

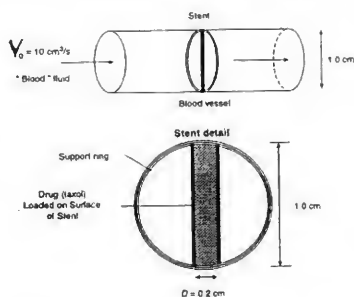
$$C_A^* = 2.5 \times 10^{-4} \text{ mg/cm}^3$$

$$W = (0.00181)(2.5 \times 10^{-4})(\pi)(0.2)(1) = 2.84 \times 10^{-7} \text{ mg/s}$$

$$t = \frac{5}{2.84 \times 10^{-7}} = 1.76 \times 10^7 \text{ s}$$

$$= 4890 \text{ h}$$

$$\approx 204 \text{ DAYS} \quad (b)$$



30.14

$$P_A^0 = 428 \text{ Pa}$$

$$M_A = 106$$

$$T_f = \frac{298 + 313}{2} = 305.5 \text{ K}$$

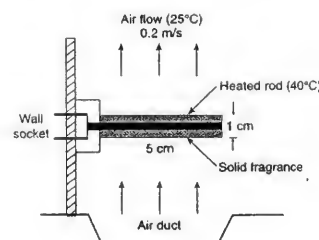
$$\rho_{AIR} = 1.156 \text{ kg/m}^3$$

$$D_{AB} = (0.08) \left( \frac{305.5}{313} \right)^{3/2} = 0.077 \text{ cm}^2/\text{s}$$

$$\text{INITIALLY: } Re = \frac{(1)(20)}{0.01621} = 1234$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{0.01621}{0.077} = 2.104$$

Eqn (30-16) APPLIES



30.14 CONTINUED -

$$k_G P_{Sc}^{0.56} = 0.281 \text{ Pa}^{-0.4}$$

$$P = 1.013 \times 10^5 \text{ Pa}$$

$$G_m = \frac{85}{M} = \frac{(1.156)(0.2)}{29} = 0.00797 \text{ kg mol/m}^2 \cdot \text{s}$$

SUBSTITUTING VALUES

$$k_G = 2.12 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$N_A = k_G (P_{AS} - P_{A\infty}) = (2.12 \times 10^{-6})(428) = 9.07 \times 10^{-7} \text{ kg mol/m}^2 \cdot \text{s}$$

$$\begin{aligned} W_A &= N_A A \\ &= (9.07 \times 10^{-7})(\pi)(0.9)(0.05) \\ &= 1.425 \times 10^{-9} \text{ kg mol/s} \\ &= (1.425 \times 10^{-9})(106) = 1.51 \times 10^{-7} \text{ kg/s} \\ &= \underline{\underline{0.544 \text{ g/h}}} \quad (a) \end{aligned}$$

WHEN A IS DEPLETED

$$D = 0.5 \text{ cm} \sim Re_D = 61.7$$

SAME PROCEDURE AS IN PART (a)

NEW VALUES:

$$k_G = 2.79 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

$$N_A = 1.19 \times 10^{-6} \text{ kg mol/m}^2 \cdot \text{s}$$

30.14 CONTINUED -

$$\begin{aligned} W_A &= (1.19 \times 10^{-6})(\pi)(0.005)(0.05) \\ &= 9.35 \times 10^{-10} \text{ kg mol/s} \end{aligned}$$

$$\begin{aligned} W_{A, \text{total}} &= \frac{(1.425 + 0.935) \times 10^{-9}}{2} \\ &= 1.18 \times 10^{-9} \text{ kg mol/s} \end{aligned}$$

TOTAL MASS OF A DEPLETED -

$$M = \frac{\pi(D_i^2 - D_f^2)(1.1)(0.4)}{106}$$

$$\begin{aligned} \{ 0.4 \text{ IS FRACTION OF A IN SOLID} \} \\ = 0.0122 \text{ g mol} \end{aligned}$$

$$t = \frac{0.0122}{(1.18 \times 10^{-9})(1000)} = \underline{\underline{10340 \text{ s}}} = \underline{\underline{2.87 \text{ h}}}$$

30.15 CONSTITUENT A INTO WATER (B)

CYLINDRICAL FILM -  $D_i = 1.8 \text{ cm}$

$$D_o = 2.0 \text{ "}$$

$$T = 293 \text{ K}$$

$$\dot{V}_{H_2O} = 314 \text{ cm}^3/\text{s} \quad U = \frac{314}{\pi/4(1.80)^2} = 123 \text{ cm/s}$$

$$D_{H_2O} = 0.00995 \text{ cm}^2/\text{s}$$

$$D_{AB} = 1.2 \times 10^{-9} \text{ m}^2/\text{s}$$

$$Sc = \frac{0.00995}{(1.2 \times 10^{-9})(100)^2} = 829$$

$$Re = \frac{(1.8)(123)}{0.00995} = 22,250$$

30.15 CONTINUED -

Eqn (30-18) APPLIES

$$k_L = \frac{D_{AB}}{D} (0.023) Re^{0.83} Sc^{1/3}$$

SUBSTITUTING VALUES:  $k_L = 0.00583 \text{ cm/s}$

WHEN SCALE HAS BEEN REMOVED.

$$D = 2 \text{ cm}$$

$$U = \frac{3.14}{\pi/4(2)^2} = 100 \text{ cm/s}$$

SAME PROCEDURE AS ABOVE -

NEW VALUES:

$$Re = 20,100$$

$$k_L = 0.00482 \text{ cm/s}$$

$$k_{L,AVG} = \frac{0.00583 + 0.00482}{2} = 0.005325 \text{ cm/s}$$

$$\begin{aligned} W_A &= k_L (C_{AS} - 0) (\pi D L) \\ &= (0.005325) (0.14 \times 10^{-6}) (\pi) (2) (100) \\ &= 4.68 \times 10^{-7} \text{ g mol/s} \end{aligned}$$

MASS OF  $\text{CaCO}_3$  REMOVED -

$$\begin{aligned} m &= \rho V = \frac{(2.7) \left( \frac{\pi}{4} \right) (2^2 - 1.8^2) (100)}{100} \\ &= 1.611 \text{ g mol} \end{aligned}$$

$$\begin{aligned} t &= \frac{1.611}{4.68 \times 10^{-7}} = 3.44 \times 10^6 \text{ s} \\ &= \underline{\underline{956 \text{ h}}} \end{aligned}$$

30.16

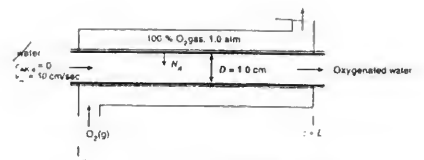
$\text{O}_2$  INTO  $\text{H}_2\text{O}$ :

$$T = 298 \text{ K}$$

$$P = 1 \text{ ATM}$$

$$\nu = 9.12 \times 10^{-3} \text{ cm}^2/\text{s}$$

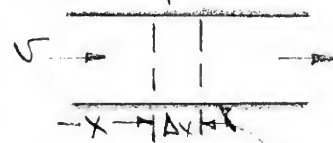
$$D_{AB} = 2.10 \times 10^{-5} \text{ " } \quad U = 50 \text{ cm/s}$$



$$L = 500 \text{ cm}$$

$$H = 0.78 \text{ atm}/(\text{mol}/\text{m}^3)$$

FOR MASS TX FROM CYLINDRICAL INTERFACE IN A PIPE:



MASS BALANCE FOR C.V. YIELDS

$$\ln \frac{C_{AS} - C_{A0}}{C_{AS} - C_{AL}} = \frac{4L}{D} \frac{k_L}{U}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{9.12 \times 10^{-3}}{2.10 \times 10^{-5}} = 434 \quad (a)$$

$$Re = \frac{1(50)}{9.12 \times 10^{-3}} = 5482$$

$$\text{Eqn (30-18)} \quad Sh = 0.023 Re^{0.83} Sc^{1/3}$$

SUBSTITUTING VALUES -  $Sh = 220 \quad (a)$

$$\begin{aligned} k_L &= \frac{D_{AB}}{D} Sh = \frac{2.10 \times 10^{-5}}{1} (220) \\ &= 0.00463 \text{ cm/s} \end{aligned}$$

$$\frac{4L}{D} \frac{k_L}{U} = 4 \frac{(500)(0.00463)}{1 \cdot 50} = 0.185$$

$$\ln \left[ \frac{1.28 - 0}{1.28 - C_{AL}} \right] = 0.185$$

$$C_{AL} = \underline{\underline{0.213 \text{ mol}/\text{m}^3}}$$

30.16 CONTINUED -

FOR  $C_A = 0.6 C_{AS}$

$$\ln \frac{C_{AS}}{C_{AS} - 0.6 C_{AS}} = \ln 2.5 = 0.916$$

$$0.916 = \frac{4L}{1} \frac{(0.00463)}{50}$$

$$L = 2470 \text{ cm} = \underline{24.7 \text{ m}}$$

30.17

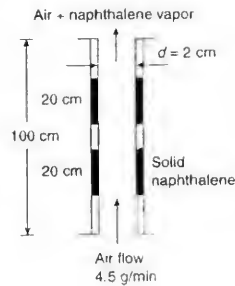
NAPHTHALENE - AIR

$T = 373 \text{ K}$   $P = 1 \text{ atm}$   
 $P_0 = 1 \text{ mm Hg}$

$D_{AB} = 0.086 \text{ cm}^2/\text{s}$

$\nu_B = 0.25 \text{ cm}^3/\text{s}$

$P_B = 9.5 \times 10^{-4} \text{ g/cm}^3$



MASS BALANCE FOR A IN X (UP) DIRECTION

$$C_A \nu \frac{\pi D^2}{4} \Big|_x + k_c (C_{AS} - C_A) \pi D \Delta x = C_A \nu \frac{\pi D^2}{4} \Big|_{x+\Delta x}$$

DO ALGEBRA & EVALUATE IN LIMIT AS  $\Delta x \rightarrow 0$

$$\frac{dC_A}{C_{AS} - C_A} = \frac{4}{D} \frac{k_c}{\nu} dx$$

LEFT-HAND-SIDE:

$$\int \frac{dC_A}{C_{AS} - C_A} = \int_{C_{A1}}^{C_{A2}} \frac{dC_A}{C_{AS} - C_A} + \int_{C_{A1}}^{C_{A2}} \frac{dC_A}{C_{AS} - C_{A2}}$$

$$= \ln \frac{C_{AS} - 0}{C_{AS} - C_{A1}} + \ln \frac{C_{AS} - C_{A1}}{C_{AS} - C_{A2}}$$

$$= \ln \frac{C_{AS}}{C_{AS} - C_{A2}}$$

30.17 CONTINUED

RIGHT-HAND-SIDE -

$$\frac{4}{D} \frac{k_c}{\nu} \int dx = \frac{4}{D} \frac{k_c}{\nu} \left[ \int_0^{20} dx + \int_x^{20+\Delta x} dx \right] = \frac{4}{D} \frac{k_c}{\nu} (40)$$

FINAL EXPRESSION IS

$$\ln \frac{C_{AS}}{C_{AS} - C_{A2}} = \frac{4}{D} \frac{k_c}{\nu} (40) \quad (a)$$

$$C = \frac{f}{RT} = \frac{1}{(82.06)(373)} = 3.267 \times 10^{-5} \text{ g mol/cm}^3$$

$$C_{AL} = y_{AC} = 0.0066 (C) = 2.15 \times 10^{-7} \text{ "}$$

$$C_{AS} = \frac{P_{AS}}{RT} = \frac{0.0316}{(82.06)(373)} = 4.29 \times 10^{-7} \text{ "}$$

$$\nu = 4.5 \text{ g/min} \left( \frac{\text{m}}{60 \text{ s}} \right) \left( \frac{\text{cm}^3}{9.5 \times 10^{-4} \text{ g}} \right) \left( \frac{4}{\pi} \right) (20 \text{ m})^2 = 25.13 \text{ cm/s}$$

SUBSTITUTING VALUES & SOLVING:

$$k_c = 0.220 \text{ cm/s} \quad (b)$$

$$Re = \frac{D \nu}{\mu} = \frac{(2)(25.13)}{0.25} = 201 \quad \{ \text{LAMINAR} \}$$

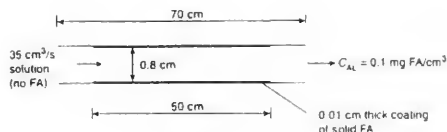
$$Sc = \frac{\nu}{D_{AB}} = \frac{0.25}{0.086} = 2.91$$

EQU (30-19) APPLIES:

$$k_c = \frac{D}{D_{AB}} (1.86) \left[ \frac{D}{L} Re Sc \right]^{1/3}$$

WITH VALUES SUBSTITUTED -  $k_c = 0.246 \text{ cm/s} \quad (c)$

30.18



A INTO SOLVENT -

$$C_A^* = 20 \text{ mg/cm}^3 \quad S_A = 1.10 \text{ g/cm}^3$$

$$D = 0.02 \text{ cm}^2/\text{s} \quad C_{AL} = 0.1 \text{ mg/cm}^3$$

$$S_{\text{soln}} = 1.04 \text{ g/cm}^3$$

USUAL MASS BALANCE FOR A

TRANSFERRING FROM TUBE WALL -

SEE PROB 30.17

$$\int_0^{C_A^*} \frac{dC_A}{C_A^* - C_A} = \frac{4 k_c}{D U} \int_0^L dx$$

$$\ln \frac{C_A^*}{C_A^* - C_A} = \frac{4 L k_c}{D U} \quad (a)$$

$$\ln \frac{20}{20 - 0.1} = 0.00501$$

$$= \frac{4(50) k_c}{0.8 U}$$

$$k_c = 2.005 \times 10^5 U$$

$$U = 35 \left( \frac{4}{\pi} \right) (0.8)^2 = 69.63 \text{ cm/s}$$

$$k_c = 1.389 \times 10^3 \text{ cm/s}$$

$$Re = \frac{DU}{\eta} = \frac{(0.8)(69.63)}{0.02} = 2793$$

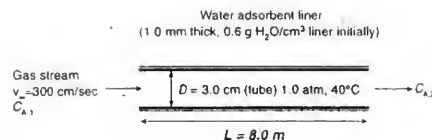
USE EQ. 30-18

$$k_c = \frac{D_{AB}}{D} (0.023) Re^{0.83} \left( \frac{\eta}{D_{AB}} \right)^{1/3}$$

SUBSTITUTING VALUES:

$$D_{AB} = 5.36 \times 10^{-5} \text{ cm}^2/\text{s} \quad (b)$$

30.19

H<sub>2</sub>O INTO AIR

$$T = 313 \text{ K} \quad P = 1 \text{ ATM}$$

$$p_A^0 = 55.4 \text{ mm Hg} = 0.0729 \text{ ATM}$$

$$\mu_{\text{AIR}} = 1.91 \times 10^{-4} \text{ g/cm.s}$$

$$\eta = 1.13 \times 10^{-3} \text{ g/cm}^3 \quad D = 0.169 \text{ cm}^2/\text{s}$$

$$D_{AB} = 0.240 \left( \frac{313}{298} \right)^{3/2} = 0.280 \text{ cm}^2/\text{s}$$

$$Sc = \frac{0.169}{0.280} = 0.60 \quad (a)$$

USUAL MASS BALANCE - SEE PROB 30.17

$$\int_{C_{A1}}^{C_{A2}} \frac{dC_A}{C_A^* - C_A} = \frac{4 k_c}{D U} \int_0^L dx$$

$$\ln \frac{C_A^* - C_{A1}}{C_A^* - C_{A2}} = \frac{4 L k_c}{D U} \quad (b)$$

$$Re = \frac{(3)(300)}{0.169} = 5325$$

$$\text{EQN (30-18): } k_c = \frac{D_{AB}}{D} (0.023) Re^{0.83} Sc^{1/3}$$

SUBSTITUTING VALUES...  $k_c = 2.24 \text{ cm/s}$ 

$$\ln \frac{C_A^* - C_{A1}}{C_A^* - C_{A2}} = \frac{4(800) 2.24}{3 \cdot 300}$$

$$\frac{C_A^* - C_{A1}}{C_A^* - C_{A2}} = 2.877$$

$$C_A^* = \frac{p_A^0}{RT} = \frac{0.0729}{(82.06)(313)} = 2.8 \times 10^{-6} \text{ mol/cm}^3$$

$$C_{A1} = yC = 0.01 \left[ \frac{1}{(82.06)(313)} \right]$$

$$= 3.89 \times 10^{-7} \text{ mol/cm}^3$$

30.19 CONTINUED -

SUBSTITUTING VALUES:  $C_{A2} = 2.8 \times 10^{-4} \text{ mol/cm}^3$

MASS OF  $\text{H}_2\text{O}$  ABSORBED -

$$m = 0.6(0.10)(\pi)(800)^3/8$$

$$= 25.13 \text{ g mol}$$

$$= (C_{A2} - C_{A1})V(\pi \frac{D^2}{4})t$$

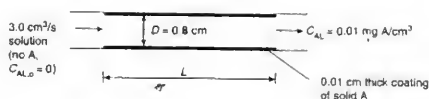
$$= (2.8 - 3.89)(10^{-7})(300)(\frac{\pi}{4})(3)^2 t$$

$$= 5.112 \times 10^{-3} \text{ g mol/s}$$

$$t = \frac{25.13}{5.112 \times 10^{-3}} = 4916 \text{ s}$$

$$1.366 \text{ h}$$

30.20



$$C_{AS} = 20 \text{ mg/cm}^3 \quad D = 0.02 \text{ cm}^2/\text{s}$$

$$C_{AL} = 0.01 \text{ " } \quad D_{AB} = 4 \times 10^{-5} \text{ "}$$

$$\rho_{\text{solid}} = 1.10 \text{ g/cm}^3$$

$$V = \frac{\dot{V}}{A} = \frac{(3)(4)}{\pi(0.8)^2} = 5.968 \text{ cm/s}$$

$$Re = \frac{(0.8)(5.968)}{0.02} = 238.7$$

{ LAMINAR }

$$Sc = \frac{D}{D_{AB}} = \frac{0.02}{4 \times 10^{-5}} = 500$$

$$\text{Eqn (30-19)} \quad k_c = \frac{D_{AB}}{D} (1.86) \left[ \frac{D Re Sc}{L} \right]^{1/3}$$

$$\Rightarrow k_c = 4.234 \times 10^{-4} \text{ L}^{-1/3}$$

30.20 CONTINUED -

~ USUAL MASS BALANCE -

$$\ln \frac{C_A^* - C_A^0}{C_A^* - C_{AL}} = \frac{4 k_c}{D V}$$

$$\ln \frac{20 - 0}{20 - 0.01} = 5.00 \times 10^{-4}$$

$$= \frac{4(4.234 \times 10^{-4}) L^{2/3}}{0.8(5.968)}$$

$$L = 1.67 \text{ cm}$$

30.21 WETTED WALL COLUMN

$$P = 1 \text{ ATM} \quad D_i = 5 \text{ cm}$$

$$T = 300 \text{ K} \quad L = 600 \text{ cm}$$

$$D = 0.157 \text{ cm}^2/\text{s} \quad p_A^0 = 0.035 \text{ ATM}$$

For  $\text{H}_2\text{O}$  IN AIR @ 298 K -

$$D_{AB} = \frac{260}{2} \left( \frac{300}{298} \right)^{3/2} = 0.131 \text{ cm}^2/\text{s} \quad (a)$$

$$V = \frac{\dot{V}}{A} = \frac{4000}{(\pi/4)(5)^2} = 203 \text{ cm/s}$$

$$Re = \frac{DV}{D} = \frac{(5)(203)}{0.157} = 6487$$

$$\text{Eqn (30-18)} \quad k_c = \frac{D_{AB}}{D} (0.023) Re^{0.83} Sc^{1/3}$$

$$Sc = \frac{0.157}{0.131} = 1.198 \quad k_c = 0.934 \text{ cm/s}$$

$$Sh = \frac{(0.934)(5)}{0.131} = 35.65 \quad (b)$$

$$k_G = \frac{k_c}{RT} = \frac{0.934}{(82.06)(300)} = 3.79 \times 10^{-5} \text{ g mol/cm}^2 \cdot \text{s} \cdot \text{ATM} \quad (c)$$

30,21 CONTINUED -

$$C_A^* = \frac{P_A}{RT} = \frac{0.035}{(82.10)(300)} = 1.42 \times 10^{-6} \text{ gmol/cm}^3 \quad (d)$$

USUAL MASS BALANCE FOR CYL.

$$\int_{C_{Ai}}^{C_{AL}} \frac{dC_A}{C_A^* - C_A} = \frac{4}{D} \frac{k_c}{U} \int_0^L dx$$

$$\ln \frac{C_A^* - C_{Ai}}{C_A^* - C_{AL}} = \frac{4L}{D} \frac{k_c}{U}$$

$$\ln \frac{1.42 \times 10^{-6}}{1.42 \times 10^{-6} - C_{AL}} = \frac{4(600)(0.934)}{(5)(203)}$$

$$C_{AL} = 1.263 \times 10^{-6} \text{ gmol/cm}^3$$

30,22 WATER WALL COLUMN CO<sub>2</sub>-H<sub>2</sub>O

$$T = 293 \text{ K} \quad \rho_L = 998.2 \text{ kg/m}^3$$

$$P = 2.54 \text{ ATM} \quad \mu_L = 993 \times 10^{-6} \text{ kg/m.s}$$

$$H = 25.5 \text{ ATM/(kg mol/m}^3)$$

$$L = 2 \text{ m} \quad \dot{m}_{H_2O} = 2 \text{ gmol/s}$$

$$D = 6 \text{ cm} \quad \dot{m}_{CO_2} = 0.5 \text{ "}$$

$$P_{O_2} = H C_{O_2}^* \quad C_{O_2}^* = \frac{2.54}{25.4} = 0.1 \text{ kg mol/m}^3 \quad (a)$$

$$Re_w = \frac{4 \dot{m}_w}{\pi D \mu_w} = \frac{4(2)(18)}{\pi(6)(993 \times 10^{-5})} = 769$$

Eqn (30-20) APPLIES

$$k_L = \frac{D_{AB}}{\delta} (0.433) Sc^{1/4} \left( \frac{g_L^3}{\rho_L^2} \right)^{1/6} Re_L^{0.4}$$

30,22 CONTINUED -

FOR CO<sub>2</sub> IN H<sub>2</sub>O @ 293 K

$$D_{AB} = 1.77 \times 10^{-9} \text{ m}^2/\text{s}$$

$$Sc = \frac{993 \times 10^{-6}}{(998.2)(1.77 \times 10^{-9})} = 562$$

$$\frac{g_L^3}{\rho_L^2} = \frac{(9.81)(2)^3}{[993 \times 10^{-6}/998.2]^2} = 79.2 \times 10^{12} \quad (b)$$

SUBSTITUTING VALUES:  $k_L = 2.686 \times 10^{-3} \text{ cm/s}$

USUAL MASS BALANCE:

$$\int_{C_{Ao}}^{C_{AL}} \frac{dC_A}{C_A^* - C_A} = \frac{4}{D} \frac{k_c}{U} \int_0^L dz$$

$$Re = 769 = \frac{DU}{\mu} \sim U = 0.0127 \text{ m/s}$$

$$\ln \frac{C_A^*}{C_A^* - C_A} = \frac{4(2)(2.686 \times 10^{-3})}{0.06(0.0127)}$$

$$\frac{C_A^*}{C_A^* - C_A} = 1.3258 = \frac{0.1}{0.1 - C_A}$$

$$C_A = 0.0246 \text{ kg mol/m}^3 \quad (c)$$

30,23 FALLING FILM TYPE TEOS (A) INTO H<sub>2</sub>

$$\dot{V}_L = 2000 \text{ cm}^3/\text{s} \quad T = 333 \text{ K}$$

$$D_i = 5 \text{ cm}$$

$$L = 2 \text{ m}$$

$$D_G = 1.47 \text{ cm}^2/\text{s}$$

$$D_{AB} = 1.315 \text{ cm}^2/\text{s}$$

$$P_A^0 = 2133 \text{ Pa}$$

$$U = \frac{\dot{V}}{A} = \frac{2000}{\frac{\pi}{4}(5)^2} = 101.9 \text{ cm/s}$$

30.23 CONTINUED -

$$Re = \frac{DU}{\mu} = \frac{(5)(101.9)}{1.47} = 346.5 \quad \{ \text{LAMINAR} \}$$

$$\text{Eqn (30-19)} \quad k_c = \frac{D_{AB}}{D} (1.86) \left[ \frac{D}{L} Re Sc \right]^{1/3}$$

$$Sc = \frac{1.47}{1.315} = 1.118$$

SUBSTITUTING VALUES:  $k_c = 1.042 \text{ cm/s}$

$$k_b = \frac{k_c}{RT} = \frac{1.042}{(82.06)(333)} = 3.813 \times 10^{-5} \frac{\text{g mol}}{\text{cm s atm}} \quad (a)$$

USUAL MASS BALANCE:

$$\int_{C_{A0}}^{C_{AL}} \frac{dC_A}{C_{AS} - C_A} = \frac{4}{D} \frac{k_c}{U} \int_0^L dx$$

$$\ln \frac{C_{AS} - C_{A0}}{C_{AS} - C_{AL}} = \frac{4L}{D} \frac{k_c}{U}$$

$$= \frac{4(200)}{5} \frac{1.042}{101.9} = 1.637$$

$$\frac{C_{AS} - 0}{C_{AS} - C_{AL}} = 5.1387$$

$$C_{AL} = 0.805 C_{AS}$$

$$C_{AS} = \frac{P_A^0}{RT} = \frac{2133}{(8.314)(333)} = 0.770 \text{ mol/m}^3$$

$$C_{AL} = 0.805(0.770) = 0.620 \text{ mol/m}^3 \quad (b)$$

AT BOTTOM OF COLUMN:  $C_{AS} - C_{AL} = 0.770 - 0 \text{ mol/m}^3$

AT TOP:  $C_{AS} - C_{AL} = 0.770 - 0.620$  "

30.23 CONTINUED -

$$(C_{AS} - C_{AL})_{L.M.} = \frac{0.770 - 0.150}{\ln \frac{0.770}{0.150}} = 0.380 \text{ mol/m}^3$$

$$N_A = k_c (C_{AS} - C_{AL})_{L.M.}$$

$$= \frac{1.042 (0.380)}{(100)^3} = 3.96 \times 10^{-7} \text{ mol/cm}^2 \cdot \text{s}$$

$$W_A = N_A A = (3.96 \times 10^{-7})(\pi)(5)(200)$$

$$= 1.24 \times 10^{-3} \text{ mol/s}$$

$$\dot{m}_{H_2}^0 = \frac{P_A^0}{RT} = \frac{(1.013 \times 10^5)(2 \times 10^{-3})}{(8.314)(333)}$$

$$= 0.0732 \text{ mol/s}$$

$$y_A = \frac{1.24 \times 10^{-3}}{1.24 \times 10^{-3} + 0.0732} = 0.0167 \quad (b)$$

$$\dot{m}_A = (1.24 \times 10^{-3})(208.33)$$

$$= 0.258 \text{ g/s} \quad (c)$$

30.24 WETTED-WALL COLUMN  
- ETHYL ACETATE (A) INTO AIR

$$U = 0.2 \text{ m/s} \quad T = 300 \text{ K} \quad P = 1 \text{ atm}$$

$$D = 0.05 \text{ m} \quad P_A^0 = 0.080 \text{ atm}$$

$$L = 10 \text{ m}$$

$$\nu_{\text{AIR}} = 1.569 \times 10^{-5} \text{ m}^2/\text{s}$$

$$D_{AB} = 0.0709 \left( \frac{300}{273} \right)^{3/2} = 0.0817 \text{ cm}^2/\text{s}$$

$$Sc = \frac{(1.569 \times 10^{-5})(100)^2}{0.0817} = 19.2$$

$$Re = \frac{DU}{\nu} = \frac{(0.05)(0.2)}{1.569 \times 10^{-5}} = 637$$

{ LAMINAR }

30,24 CONTINUED -

$$\text{Eqn (30-19): } k_c = \frac{D_{AB}}{D} (186) \left[ \frac{D}{L} Re Sc \right]^{-1/3}$$

SUBSTITUTING VALUES:  $k_c = 5.55 \times 10^{-4} \text{ m/s}$  (a)

THE USUAL MASS BALANCE YIELDS

$$\int_{C_{A0}}^{C_{AS}} \frac{dC_A}{C_{AS} - C_A} = \frac{4 k_c}{D} \int_0^x dy$$

$$\ln \frac{C_{AS} - 0}{C_{AS} - C_{AL}} = \frac{4 L k_c}{D U}$$

$$C_{AS} = \frac{p_A^0}{RT} = \frac{0.08}{(82.06)(300)} = 3.25 \times 10^{-6} \text{ gmol/cm}^3$$

$$= 3.25 \text{ gmol/m}^3$$

$$\ln \frac{3.25}{3.25 - C_{AL}} = \frac{4(10)}{0.05} \frac{5.55 \times 10^{-4}}{0.2}$$

GIVING  $C_{AL} = 2.90 \text{ gmol/m}^3$

$$\dot{m} = C_{AL} U A$$

$$= 2.90(0.2) \left( \frac{\pi}{4} \right) (0.05)^2 (3600)$$

$$= 4.1 \text{ gmol/h}$$

30,25 OZONE BUBBLED INTO H<sub>2</sub>O

$$T = 293 \text{ K} \quad V_{\text{TANK}} = 2 \text{ m}^3$$

$$p = 1 \text{ atm}$$

$$C_A = 4 \text{ gmol/m}^3 \text{ AFTER } 10 \text{ m}$$

$$H = 6.67 \times 10^{-2} \text{ atm/(gmol/m}^3)$$

30,25 CONTINUED -

FOR A WELL-MIXED PROCESS:

ALL OZONE IS DISSOLVED -

MASS BALANCE ON OZONE (A)

$$k_L a (C_A^* - C_A) = \frac{dC_A}{dt}$$

$$\int_0^{C_A} \frac{dC_A}{C_A^* - C_A} = k_L a \int_0^t dt$$

$$\ln \frac{C_A^*}{C_A^* - C_A} = k_L a t$$

$$p_A^* = H C_A^* = 6.67 \times 10^{-2} C_A^* = 1$$

$$C_A^* = 14.99 \text{ gmol/m}^3$$

$$\ln \frac{14.99}{14.99 - 4} = k_L a (10)(60)$$

$$k_L a = 5.14 \times 10^{-5} \text{ s}^{-1}$$

30,26

USING Eqn (30-21)  $Jo = 1.17 Re^{-0.415}$

$$Jo = \frac{k_c}{U_p} Sc^{2/3} = 1.17 Re^{-0.415}$$

$$k_c = 1.17 U_p Re^{-0.415} Sc^{-2/3}$$

AT  $T = 311 \text{ K}$   $D = 1.673 \times 10^{-5} \text{ m}^2/\text{s}$

$$D_{AB} = \frac{2.634}{1.013 \times 10^5} \left( \frac{311}{298} \right)^{3/2} = 2.772 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Sc = \frac{1.673 \times 10^{-5}}{2.772 \times 10^{-5}} = 0.60$$

30.26 CONTINUED -

$$Re = \frac{Dv}{\mu} = \frac{Dg}{\mu} = \frac{Dg}{\mu}$$

AT 311 K -  $\mu_{AIR} = 1.897 \times 10^{-5} \text{ Pa}\cdot\text{s}$

$$Re = \frac{(0.00571)(0.816)}{1.897 \times 10^{-5}} = 246$$

SUBSTITUTING VALUES:  $k_c = 0.120 \text{ m/s}$

$$k_g = \frac{k_c}{RT} = \frac{0.120}{(8.314)(311)} = 4.64 \times 10^{-5} \text{ gmol/m}^2\cdot\text{s}\cdot\text{Pa}$$

$$= 4.64 \times 10^{-8} \text{ kgmol/m}^2\cdot\text{s}\cdot\text{Pa}$$

$$= 4.70 \times 10^{-3} \text{ kgmol/m}^2\cdot\text{s}\cdot\text{ATM}$$

COMPARED WITH EXPERIMENTAL VALUE

$$\Delta = 0.28 \times 10^{-3} \text{ kgmol/m}^2\cdot\text{s}\cdot\text{ATM} \sim 6.33\%$$

METHOD 2:

$$Ej_0 = \frac{2.06}{Re^{0.575}} = E \frac{k_c}{U} Sc^{2/3}$$

$$Re = 246 \quad Sc = 0.6 \quad E = 0.75$$

$$k_c = \frac{2.06(U_p)Re^{-0.575}Sc^{-2/3}}{0.75}$$

$$U = G/\rho = \frac{0.816}{1.136} = 0.718 \text{ m/s}$$

SUBSTITUTING VALUES:  $k_c = 0.117 \text{ m/s}$

$$k_g = \frac{k_c}{RT} = 4.52 \times 10^{-8} \text{ kgmol/m}^2\cdot\text{s}\cdot\text{Pa}$$

$$= 4.58 \times 10^{-3} \text{ kgmol/m}^2\cdot\text{s}\cdot\text{ATM}$$

$\sim 6.32\%$  DIFFERENT FROM  
EXPERIMENT

30.27 FOR  $O_2$  TRANSFER

$$k_{La} = 300 \text{ h}^{-1}$$

FOR  $O_2-H_2O$  - Eqn: (24-52)

$$\frac{D_{AB}\mu_B}{T} = \frac{7.4 \times 10^{-8} (\phi_B M_B)^{1/2}}{V_A^{0.6}}$$

VALUES:  $\phi_B = 2.26 \quad M_B = 18 \quad T = 283 \text{ K}$

$$V_A = 7.4 \quad \mu_B = 1.45 \text{ cP}$$

$$D_{O_2-H_2O} = 2.77 \times 10^{-5} \text{ cm}^2/\text{s}$$

TABLE J.2  $D_{CO_2-H_2O} = 1.46 \times 10^{-5} \text{ cm}^2/\text{s}$

FILM THEORY -  $k_{La} \sim D_{AB}^1$

$$k_{La, CO_2} = k_{La, O_2} \left[ \frac{D_{CO_2-H_2O}}{D_{O_2-H_2O}} \right]^1$$

$$= 300 \left[ \frac{1.46 \times 10^{-5}}{2.77 \times 10^{-5}} \right] = 158 \text{ h}^{-1}$$

BOUNDARY-LAYER THEORY:  $k_{La} \sim D_{AB}^{2/3}$

$$k_{La, CO_2} = 300 \left[ \right]^{2/3} = 195.8 \text{ h}^{-1}$$

PENETRATION THEORY:  $k_{La} \sim D_{AB}^{1/2}$

$$k_{La, CO_2} = 300 \left[ \right]^{1/2} = 217.8 \text{ h}^{-1}$$

30,28 CO<sub>2</sub> INTO H<sub>2</sub>O IN PACKED BED

$$\dot{m}_w = 5 \text{ kg mol/m} \quad P = 2 \text{ ATM}$$

$$\dot{m}_{\text{CO}_2} = 1 \quad T = 293 \text{ K}$$

$$D = 0,25 \text{ m}$$

$$\rho_{\text{H}_2\text{O}} = 55,5 \text{ kg mol/m}^3$$

$$= 998,2 \text{ kg/m}^3$$

$$\mu_w = 993 \times 10^{-6} \text{ kg/m.s}$$

$$A = 25,4 \text{ ATM/(kg mol/m}^3)$$

$$\text{FROM (30-33)} \cdot \frac{k_a}{D_{AB}} = \alpha \left( \frac{L}{\mu} \right)^{1-n} Sc^{1/2}$$

$$\text{FOR 1-IN. PACHEL RINGS: } N = 100$$

$$n = 0,22$$

$$D_{AB} \sim \text{CO}_2 \text{ IN H}_2\text{O @ 293 K}$$

$$= 1,77 \times 10^{-9} \text{ m}^2/\text{s}$$

$$Sc = \frac{993 \times 10^{-6}}{(998,2)(1,77 \times 10^{-9})} = 562$$

$$A_x = \frac{\pi}{4} (0,25)^2 = 0,0491 \text{ m}^2$$

$$= 0,529 \text{ FT}^2$$

$$L = 5(18)(60) \frac{2,2}{0,529}$$

$$= 22500 \text{ km/h.FT}^2$$

$$D_{AB} = 1,77 \times 10^{-9} (0,3048)^2$$

$$= 6,86 \times 10^{-5} \text{ FT}^2/\text{h}$$

SUBSTITUTING VALUES:

$$k_a = 0,0566 \text{ s}^{-1} \quad (a)$$

MASS BALANCE FOR A

$$C_A^* = \frac{p_{AO}}{H} = \frac{2}{25,4}$$

$$= 0,075 \text{ kg mol/m}^3$$

$$C_{AL} = 0,95 C_A^*$$

$$= 0,95(0,075)$$

$$= 0,07125 \text{ kg mol/m}^3$$

MASS BALANCE -

$$k_a(C_A^* - C_A) = v \frac{dC_A}{dz}$$

$$\int_0^L \frac{dC_A}{C_A^* - C_A} = \frac{k_a L}{v} \int_0^L \frac{1}{C_A^* - C_A} dz$$

$$\ln \frac{C_A^*}{C_A^* - 0,95 C_A^*} = \frac{k_a L}{v}$$

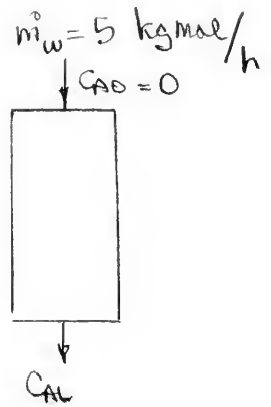
$$L = \frac{v}{k_a} \ln 20$$

$v$  (ASSUMING EMPTY X-SECTION)

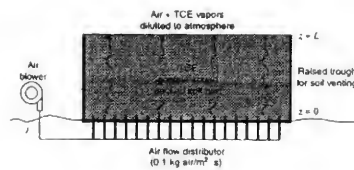
$$= \frac{\dot{m}}{\rho A} = \frac{5}{8A (60)(55,5)(\pi/4)(0,25)^2}$$

$$= 0,0306 \text{ m/s}$$

$$L = \frac{(0,0306)(\ln 20)}{0,0566} = 1,62 \text{ m} \quad (b)$$



30,29



TCE (A) IN AIR

$$D_p = 3 \text{ mm}$$

$$\epsilon = 0.5$$

$$T = 293 \text{ K}$$

$$G_B = 0.1 \text{ kg/m}^2 \text{ s}$$

$$p_A^0 = 58 \text{ mm}$$

$$\rho = 1,200 \text{ kg/m}^3$$

$$D_{AB} = 8.08 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\nu_B = 1.505 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Sc = \frac{1.505 \times 10^{-5}}{8.08 \times 10^{-6}} = 1.863$$

$$Re = \frac{D_p G}{\mu} = \frac{(0.003)(0.1)}{1.815 \times 10^{-5}} = 16.53$$

Eqn (30.23) APPLIES

$$\epsilon \frac{k_c}{U} Sc^{2/3} = 0.25 Re^{-0.31}$$

$$k_c = \frac{0.25 U Re^{-0.31}}{\epsilon Sc^{2/3}}$$

$$U = \frac{G}{\rho} = \frac{0.1}{1,200} = 0.083 \text{ m/s}$$

SUBSTITUTING VALUES:  $k_c = 0.011 \text{ m/s}$ 

(a)

MASS BALANCE FOR A:

$$k_c \frac{A}{V} (C_A^* - C_A) = U \frac{dC_A}{dz}$$

$$\int_0^{0.9C_A^*} \frac{dC_A}{C_A^* - C_A} = \frac{k_c A}{U V} \int_0^L dz$$

30,29 CONTINUED -

$$\ln \frac{C_A^*}{C_A^* - 0.9C_A^*} = \frac{k_c A}{U V} L$$

$$L = \frac{U V}{k_c A} \ln 10$$

$\frac{V}{A}$  IS VOLUME TO SURFACE  
AREA RATIO OF PARTICLES  
IN TOWER

FOR A SPHERICAL PARTICLE -

$$A = \pi D^2$$

OCCUPYING A SPACE WITH  $V = D^3$ 

IN TOWER -

$$\frac{V}{A} = \frac{D^3}{\pi D^2} = \frac{D}{\pi}$$

$$L = \frac{U}{k_c} \frac{D}{\pi} \ln 10$$

$$= \frac{0.083 (0.003)}{0.011 (\pi)} \ln 10$$

$$= \underline{\underline{0.0166 \text{ m} = 1.66 \text{ cm}}} \quad (b)$$

# CHAPTER 31

## 31.1 AERATION TANK w/ SPARGERS -

THIS PROBLEM IS SIMILAR TO  
EXAMPLE 2 IN SECTION 31.2.

FOR A WELL MIXED TANK - EQN (31-1)  
APPLIES & THE FINAL  $O_2$   
LEVEL IS DESCRIBED BY

$$C_A = C_A^* - (C_A^* - C_{A0}) \exp(-K_L a t)$$

$$\begin{aligned} \dot{V}_{AIR} &= 0.0078 \text{ m}^3/\text{s} \\ &= \frac{(0.0078)(60)}{(0.3048)^3} = 15 \text{ cfm} \end{aligned}$$

FOR 6 SPARGERS - & FIG 31.7  
@ 15 cfm & 15 FT DEPTH

$$\begin{aligned} K_L a &= K_L \frac{A}{V} = \frac{1200(6)}{V} \\ &= \frac{(1200)(6)}{10,000} = 0.72 \text{ h}^{-1} \end{aligned}$$

$$\begin{aligned} P_{AUL} &= \frac{P_{TOP} + P_{BOTTOM}}{2} \\ &= \frac{1 + [1 + 14.93(0.0295)]}{2} = 1.22 \text{ ATM} \end{aligned}$$

$$X_{O_2}^* = \frac{P}{H} = \frac{0.21(1.22)}{3.27 \times 10^4} = 7.83 \times 10^{-6}$$

$$C = \frac{1000}{18} = 55.56 \text{ g mol/L}$$

$$\begin{aligned} C_A^* &= (7.83 \times 10^{-6})(55.56) \\ &= 4.35 \times 10^{-4} \text{ g mol/L} \end{aligned}$$

SUBSTITUTING VALUES:

$$\text{FOR } t = 9000 \text{ s} = 2.5 \text{ h}$$

$$C_{O_2t} = 3.64 \times 10^{-4} \text{ g mol/L}$$

## 31.2 OZONE/H<sub>2</sub>O TREATMENT USING 8 SPARGERS

SYSTEM IS ANALOGOUS TO EXAMPLE 2 -

$$t = \ln\left(\frac{C_A^* - C_{A0}}{C_A^* - C_{At}}\right) \left(\frac{1}{K_L a}\right)$$

$$\text{FOR } \dot{V}_G = 17.8 \text{ m}^3/\text{h} = 4.9 \times 10^{-3} \text{ m}^3/\text{s} = 10.4 \text{ cfm}$$

$$\frac{1}{2} \text{ DEPTH} = 3.2 \text{ m} = 10.5 \text{ FT}$$

$$\text{FIG 31.7 GIVES } K_L \frac{A}{V} \approx 400 \text{ cfm}$$

$$K_L a = \frac{(400)(8)}{(80)/(0.3048)^3} = 1.132 \text{ h}^{-1}$$

BY PENETRATION THEORY:

$$\frac{K_L a_{O_3}}{K_L a_{O_2}} = \left[\frac{D_{O_2-H_2O}}{D_{O_3-H_2O}}\right]^{1/2} = \left[\frac{1.7 \times 10^{-5}}{2.14 \times 10^{-5}}\right]^{1/2} = 0.891$$

$$K_L a_{O_3} = (1.132)(0.891) = 1.01 \text{ h}^{-1} \text{ (a)}$$

$$\begin{aligned} P_{AUL} &= \frac{1 + (3.2)(0.0295)/0.3048 + 1}{2} \\ &= 1.155 \text{ ATM} \end{aligned}$$

$$P_{O_3} = 0.04(1.155) = 0.0462 \text{ ATM}$$

$$\begin{aligned} C_{O_3}^* &= \frac{0.0462}{0.0667} = 0.682 \text{ g mol/m}^3 \\ &= 0.682 \left(\frac{48}{1000}\right) = 32.7 \text{ mg/L} \end{aligned}$$

$$C_{At} = 0.15 \text{ g mol/m}^3 = 7.2 \text{ mg/L}$$

SUBSTITUTING VALUES:

$$t = 0.246 \text{ h} = 886 \text{ s} \quad \text{(b)}$$

### 31.3 WASTEWATER TREATMENT USING 10 SPRINGERS -

$$V = 425 \text{ m}^3 = 15000 \text{ FT}^3$$

$$\dot{V} = 7.08 \times 10^{-3} \text{ m}^3/\text{s} = 15 \text{ cfm}$$

$$\text{DEPTH} = 3.2 \text{ m} = 10.5 \text{ FT.}$$

ANALYSIS PARALLELS EXAMPLE 2.

$$t = \ln \left( \frac{C_{O_2}^* - C_{O_2,0}}{C_{O_2}^* - C_{O_2,t}} \right) \left( \frac{1}{K_L a} \right)$$

FIG (31.7)  $K_L a V = 800 \text{ FT}^3/\text{h}$

$$K_L a = \frac{(800)(10)}{15000} = 0.533 \text{ h}^{-1}$$

$$P_{\text{TOP}} = 1 \text{ ATM}$$

$$P_{\text{BOTTOM}} = 1 + (10.5)(0.0295) = 1.31 \text{ ATM}$$

$$P_{\text{AUG}} = 1.155 \text{ ATM}$$

$$P_{O_2} = 0.21 (1.155) = 0.2425 \text{ ATM}$$

$$X_{O_2}^* = \frac{P_{O_2}}{H} = \frac{0.2425}{3.27 \times 10^4} = 7.42 \times 10^{-6}$$

$$C_L = \frac{1000}{18} = 55.56 \text{ mol/L}$$

$$C_{O_2}^* = (7.42 \times 10^{-6})(55.56) = 4.12 \times 10^{-4} \text{ g mol/L}$$

$$t = \ln \left[ \frac{4.12 \times 10^{-4} - 8 \times 10^{-5}}{4.12 \times 10^{-4} - 2 \times 10^{-4}} \right] \left( \frac{1}{0.533} \right)$$

$$= 0.841 \text{ h} = 3028 \text{ S}$$

### 31.4 O<sub>2</sub> ABSORPTION USING 1 SPARGER

$$V = 28.3 \text{ m}^3 = 1000 \text{ FT}^3$$

$$\dot{V} = 7.08 \times 10^{-3} \text{ m}^3/\text{s} = 15 \text{ cfm}$$

$$\text{DEPTH} = 3.2 \text{ m} = 10.5 \text{ FT}$$

FROM ANALYSIS ACCORDING TO EXAMPLE 2

$$t = \frac{1}{K_L a} \ln \frac{C_{O_2}^* - C_{O_2,i}}{C_{O_2}^* - C_{O_2,t}}$$

$$C_{O_2,i} = 0.04 \text{ mmol/L}$$

$$C_{O_2,t} = 0.25 \text{ "}$$

FOR 10.5 FT DEPTH:

$$P_{\text{TOP}} = 1 \text{ ATM} \quad P_{\text{BOTTOM}} = 1 + (10.5)(0.0295) = 1.31 \text{ ATM}$$

$$P_{\text{AUG}} = 1.155 \text{ ATM}$$

$$P_{O_2} = 0.21 (1.155) = 0.2426 \text{ ATM}$$

$$X_{O_2}^* = \frac{P_{O_2}}{H} = \frac{0.2426}{3.27 \times 10^4} = 7.42 \times 10^{-6}$$

$$C = \frac{1000}{18} = 55.56 \text{ g mol/L}$$

$$C_{O_2}^* = (7.42 \times 10^{-6})(55.56) = 4.12 \times 10^{-4} \text{ g mol/L} = 0.412 \text{ mmol/L}$$

FOR  $t = 4 \text{ h}$  - SUBSTITUTION YIELDS

$$K_L a|_{O_2} = 0.208 \text{ h}^{-1} \text{ FOR 1 SPARGER (a)}$$

$$\frac{K_L a|_{H_2S}}{K_L a|_{O_2}} = \left[ \frac{D_{H_2S-H_2O}}{D_{O_2-H_2O}} \right]^{1/2} = \left[ \frac{1.4 \times 10^{-5}}{2.14 \times 10^{-5}} \right]^{1/2} = 0.809$$

$$K_L a|_{H_2S} = 0.809 (0.208) = 0.168 \text{ h}^{-1} \text{ (b)}$$

31.4 CONTINUED -

FOR  $H_2S$  - 10 SPARKERS,  $K_L a = 1.68 \text{ h}^{-1}$

$$V = 425 \text{ m}^3 = 15000 \text{ ft}^3$$

$$\text{DEPTH} = 10.5 \text{ FT} \quad t = 4 \text{ h}$$

$$y_{H_2S} = p_{H_2S} = C_{H_2S}^* = 0$$

$$t = 4 = \frac{1}{1.68} \ln \left( \frac{0 - C_{H_2Si}}{0 - C_{H_2St}} \right)$$

$$C_{H_2Si} = 0.03 \text{ mmol/l} \quad (C)$$

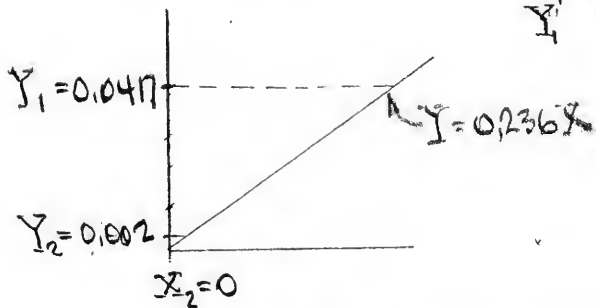
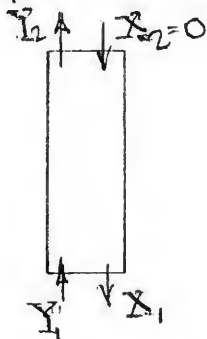
SOLVING:  $C_{H_2St} = 3.61 \times 10^{-4} \text{ gmol/l}$

31.5 COUNTERCURRENT ABSORPTION TOWER

$$X_1 = ?, X_2 = 0$$

$$Y_1 = \frac{0.04}{0.96} = 0.0417$$

$$Y_2 = \frac{0.002}{0.998} = 0.002$$



$$\left. \frac{L}{G} \right|_{\min} = \frac{Y_1 - Y_2}{X_1^* - X_2} = \frac{0.0417 - 0.002}{0.177 - 0} = 0.224$$

$$\left. \frac{L}{G} \right|_{\text{ACTUAL}} = (0.224)(1.5) = 0.336 \frac{\text{mol Solv}}{\text{mol C.G.}}$$

31.5 CONTINUED -

$$0.336 = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.0417 - 0.002}{X_1 - 0}$$

$$X_1 = \frac{0.0397}{0.336} = 0.118$$

$$X_1 = \frac{X_1}{1 + X_1} = \frac{0.118}{1.118} = 0.106$$

31.6 TCE STRIPPED FROM  $H_2O$  NA  
COUNTERCURRENT TOWER

FOR DILUTE STREAMS

$$X \approx x \quad Y \approx y$$

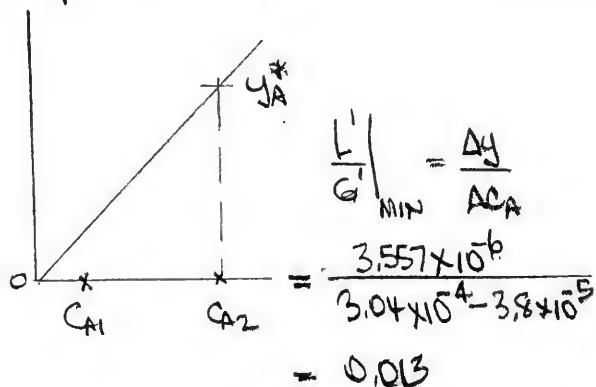
$$C_{A2} = \frac{40(1000)}{131.5} = 3.04 \times 10^{-4} \text{ mol/m}^3$$

$$C_{A1} = \frac{5(1000)(10^{-6})}{131.5} = 3.80 \times 10^{-5}$$



$$y_{A1} = 0$$

$$y_{A2}^* = \frac{H}{P} C_{A2} = \frac{11.7 \times 10^{-3}}{1} (3.04 \times 10^{-4}) = 3.557 \times 10^{-6}$$



$$\left. \frac{L'}{G'} \right|_{\text{ACTUAL}} = \frac{0.013}{3} = 4.33 \times 10^{-3} = \frac{y_{A2} - 0}{2.66 \times 10^{-4}}$$

$$y_{A2} = 1.15 \times 10^{-6}$$

$$p_{A2} = y_{A2} P = 1.15 \times 10^{-6} \text{ ATM}$$

31.7

Tower (a)

- COUNTERCURRENT

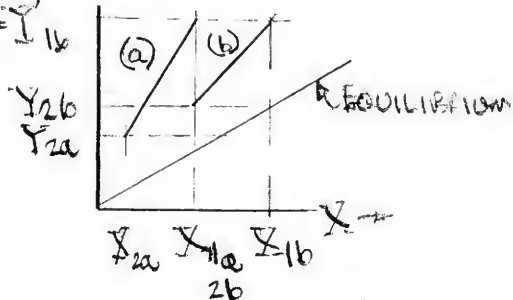
Tower (b)

- COUNTERCURRENT

$$Y_{1a} = Y_{1b} > Y_{2b} > Y_{2a}$$

$$X_{1b} > X_{1a} = X_{2b} > X_{2a}$$

$$Y_{1a} = Y_{1b}$$

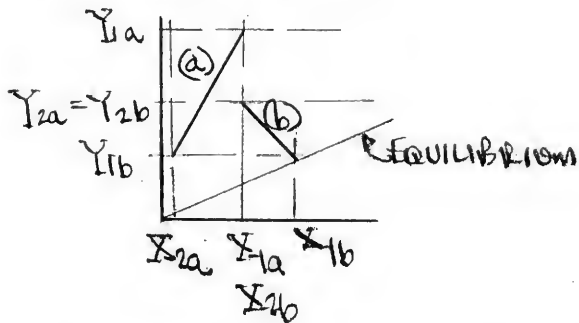


Tower (a) - COUNTERCURRENT

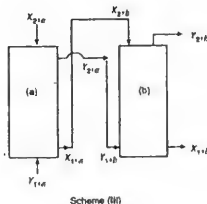
" (b) - COCURRENT

$$X_{1b} > X_{1a} = X_{2b} > X_{2a}$$

$$Y_{1a} > Y_{2a} = Y_{2b} > Y_{1b}$$



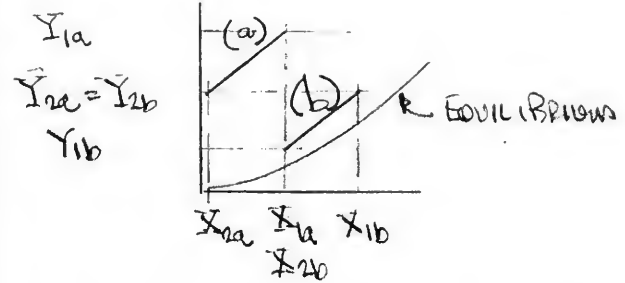
Both Towers ARE COUNTERCURRENT



$$X_{1b} > X_{1a} = X_{2b} > X_{2a}$$

$$Y_{1a} > Y_{2a} = Y_{1b} > Y_{2b}$$

31.7 CONTINUED

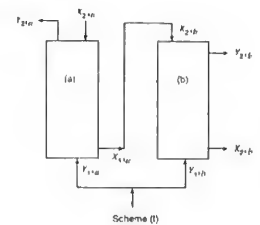
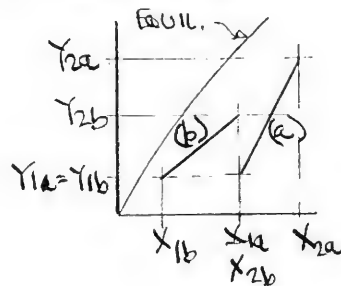


31.8 SAME FLOW SCHEMES AS IN PROB 31.7 EXCEPT PROCESSES ARE NOW DESORPTION/STRIPPING

BOTH ARE COUNTERCURRENT

$$X_{2a} > X_{1a} = X_{2b} > X_{1b}$$

$$Y_{1a} > Y_{2a} = Y_{2b} > Y_{1b}$$

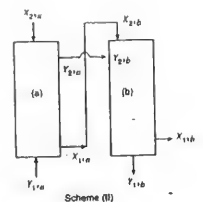
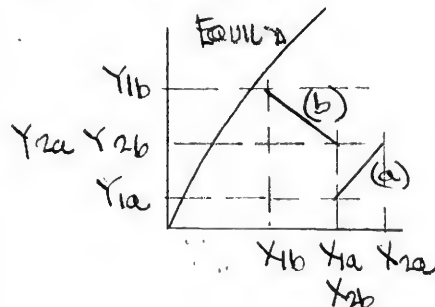


(a) COUNTERCURRENT

(b) COCURRENT

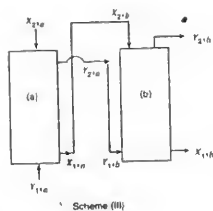
$$X_{2a} > X_{1a} = X_{2b} > X_{1b}$$

$$Y_{1b} > Y_{2a} = Y_{2b} > Y_{1a}$$



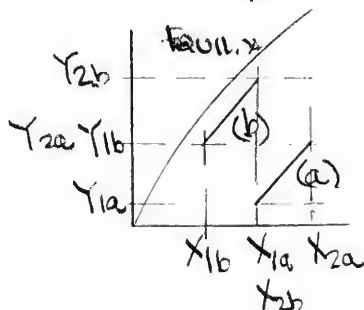
### 31.8 CONTINUED -

BOTH TOWERS ARE  
COUNTERCURRENT



$$X_{2a} > X_{1a} = X_{2b} > X_{1b}$$

$$Y_{2b} > Y_{2a} = Y_{1b} > Y_{1a}$$



### 31.9

$$G_1 = 136 \text{ mol/m}^2 \cdot \text{s}$$

$$\begin{aligned} G_s &= G_1(1 - y_1) \\ &= 136(0.95) \\ &= 129.2 \text{ mol air/m}^2 \cdot \text{s} \end{aligned}$$

$$\begin{aligned} L_s &= L_2 = \frac{3400}{18} \\ &= 188.9 \text{ mol H}_2\text{O/m}^2 \cdot \text{s} \end{aligned}$$

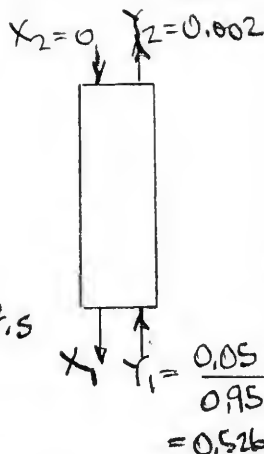
$$L_s(X_1 - X_2) = G_s(Y_1 - Y_2)$$

$$188.9(X_1 - 0) = 129.2(0.0526 - 0.002)$$

$$X_1 = 0.0346$$

$$X_1 = \frac{\bar{X}_1}{1 + X_1} = \frac{0.0346}{1.0346} = 0.033 \quad (a)$$

$$\frac{L_s}{G_s} \bigg|_{\text{ACTUAL}} = \frac{188.9}{129.2} = 1.462$$

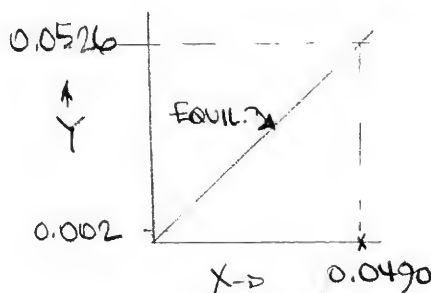


### 31.9 CONTINUED -

EQUILIBRIUM DATA:  $y = 1.075x$

$$Y = \frac{y}{1-y} \quad X = \frac{x}{1-x}$$

Y	y	X	x
0	0	0	0
0.0054	0.0054	0.005	0.005
0.0109	0.0108	0.01	0.0101
0.0220	0.0215	0.02	0.0204
0.0333	0.0323	0.03	0.0309
0.0449	0.0430	0.04	0.0417
0.0568	0.0538	0.05	0.0526



$$\frac{L_s}{G_s} \bigg|_{\text{MIN}} = \frac{0.0526 - 0.002}{0.0490 - 0} = 1.033$$

$$\frac{L_s/G_s|_{\text{ACT}}}{L_s/G_s|_{\text{MIN}}} = \frac{1.462}{1.033} = 1.415 \quad (b)$$

MASS BALANCE - REFERENCE IS TOY ~ (2)

$$G_s(Y - Y_2) = L_s(X - X_2)$$

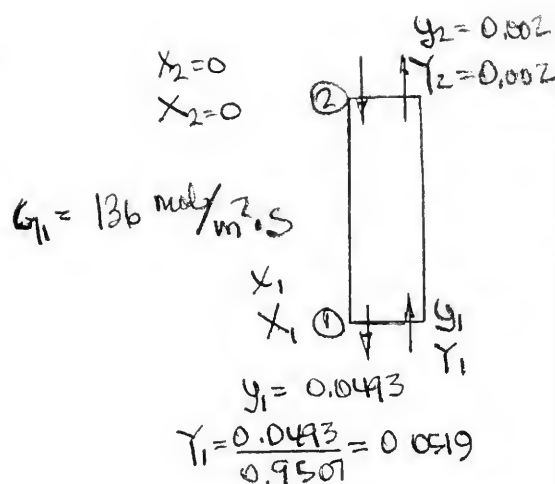
$$Y = \frac{y_2}{1 - y_2} = \frac{0.02}{0.98} = 0.0204$$

$$129.2(0.0204 - 0.002) = 188.9 X_2$$

$$X_2 = 0.01258$$

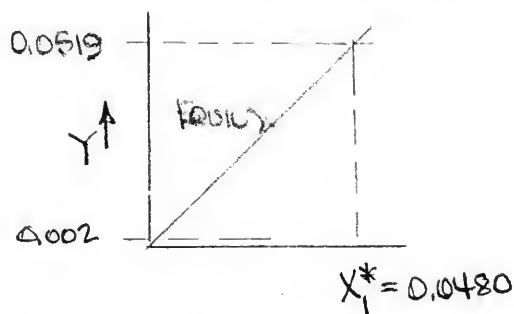
$$X_2 = \frac{0.01258}{1.01258} = 0.0124 \quad (c)$$

31.10



$$G_s = G_1(1 - y_1) = 136(0.9507) = 129.3 \text{ mol/m}^2 \cdot \text{s}$$

EQUILIBRIUM DATA - SEE TABLE FOR PROB 31.9



$$\left. \frac{L_s}{G_s} \right|_{\min} = \frac{Y_1 - Y_2}{X_1^* - X_2} = \frac{0.0519 - 0.002}{0.0480 - 0} = 1.0396$$

$$\left. \frac{L_s}{G_s} \right|_{\text{ACTUAL}} = 1.4(1.0396) = 1.455$$

$$= \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.0519 - 0.002}{X_1 - 0}$$

$$X_1 = 0.0345$$

$$\text{MOLAR FLUX OF NH}_3 = G_s(Y_{A1} - Y_{A2})$$

31.10 CONTINUED

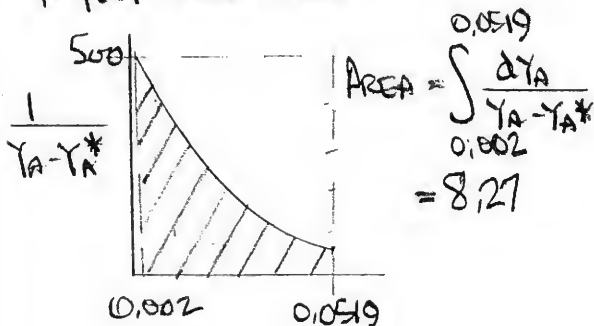
$$\begin{aligned} &= G_s(Y_{A1} - Y_{A2}) \\ &= 129.3(0.0519 - 0.002) \\ &= 6.45 \text{ g mol/m}^2 \cdot \text{s} \times \frac{17}{1000} \\ &= 0.1096 \text{ kg/m}^2 \cdot \text{s} \quad (a) \end{aligned}$$

$$\text{HEIGHT OF PACKING: } Z = \frac{G_s}{K_y a} \int_{0.002}^{0.0519} \frac{dY_A}{Y_A - Y_A^*}$$

SINCE INTEGRAND INFORMATION IS NOT IN ANALYTIC FORM - EVALUATION OF Z MUST BE DONE GRAPHICALLY OR NUMERICALLY -

$Y_A$	$Y_A^*$	$Y_A - Y_A^*$	$(Y_A - Y_A^*)^{-1}$
0.002	0	0.002	500
0.010	0.0057	0.0043	232.6
0.015	0.0095	0.0055	181.8
0.020	0.0132	0.0068	147.6
0.025	0.0170	0.0080	125.0
0.030	0.0208	0.0092	108.7
0.035	0.0247	0.0103	97.1
0.040	0.0284	0.0116	86.2
0.045	0.0321	0.0129	77.5
0.050	0.0358	0.0142	70.4
0.0519	0.0372	0.0147	68.0

A PLOT WILL YIELD

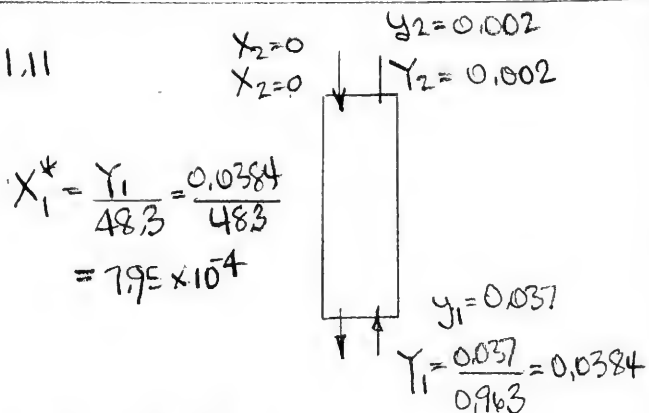


31.10 CONTINUED -

$$Z = \int_{0.002}^{0.0384} \frac{G_S}{K_y a (Y_A - Y_A^*)} dY_A$$

$$= \frac{129.3 (8.27)}{107} \approx 10 \text{ m}$$

31.11



$$\left. \frac{L_S}{G_S} \right|_{\min} = \frac{Y_1 - Y_2}{X_1^* - Y_2^*}$$

$$= \frac{0.0384 - 0.002}{7.95 \times 10^{-4}} = 45.8$$

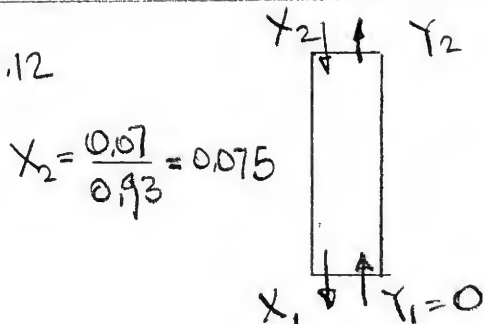
$$\left. \frac{L_S}{G_S} \right|_{\text{ACT}} = 1.5 (45.8) = 68.7 \text{ mol H}_2\text{O} / \text{mol C}_6\text{H}_6 \quad (a)$$

$$= \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.0384 - 0.002}{X_1 - 0}$$

$$X_1 = 5.30 \times 10^{-4}$$

$$X_1 = \frac{X_1}{1 + X_1} = 5.30 \times 10^{-4} \quad (b)$$

31.12



$$X_2 = \frac{0.07}{0.93} = 0.075$$

31.12 CONTINUED -

EQUILIBRIUM DATA

x	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14
Y	0.00	0.07	0.14	0.22	0.31	0.405	0.515	0.65

$$L'_S = 6.94 \text{ mol wash oil/s}$$

$$\text{Benzene in } L'_2 = 0.075 (6.94)$$

$$= 0.52 \text{ mol Benz/s}$$

$$\text{Bz to be removed} = (0.52)(0.85)$$

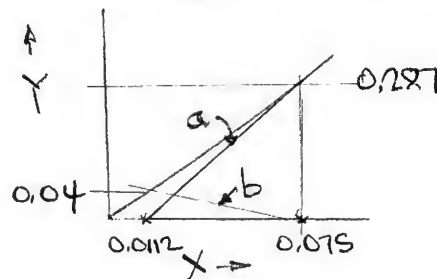
$$= 0.442 \text{ mol/s}$$

$$\text{Bz remaining in liquid} = 0.078 "$$

$$X_1 = \frac{0.078}{6.94} = 0.0112 \text{ mol Bz / mol W.O.}$$

FOR COUNTERCURRENT FLOW STREAMS:

$$\left. \frac{G_S}{L_S} \right|_{\min} = \frac{X_2 - X_1}{Y_2^* - Y_1} = \frac{0.075 - 0.0112}{0.287 - 0}$$



$$G_{S \min} = 0.222 (6.94) = 1.54 \text{ mol/s}$$

$$G_{S \text{ ACTUAL}} = 1.4 (1.54) = 2.16 \text{ mol/s} \quad (a)$$

COCURRENT FLOW: -

$$\left. \frac{G_S}{L_S} \right|_{\min} = \frac{X_2 - X_1}{Y_1^* - Y_2} = \frac{0.075 - 0.0112}{0.040 - 0} = 1.595$$

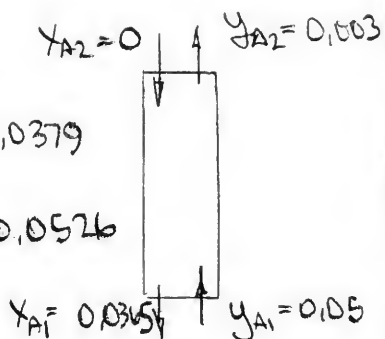
$$G_{S \min} = 1.595 (6.94) = 11.07 \text{ mol/s}$$

$$G_{S \text{ ACT}} = 1.4 (11.07) = 15.5 \text{ mol/s} \quad (b)$$

31.13

$$X_{A1} = \frac{0,0365}{0,9635} = 0,0379$$

$$Y_{A1} = \frac{0,05}{0,95} = 0,0526$$



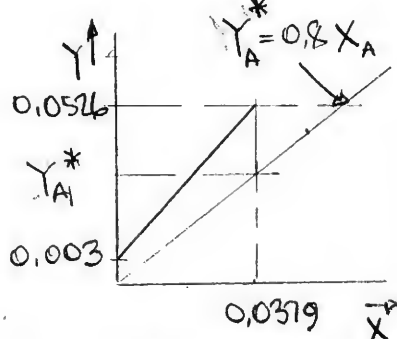
$$Y_{A2} = \frac{0,003}{0,997} \approx 0,003$$

$$G_1' = \dot{V} \frac{P}{RT} = \frac{(15/60)(1,013 \times 10^5)}{(8,314)(289)} = 10,54 \text{ g mol/s}$$

$$G_s' = G_1'(1 - y_{A1}) = (10,54)(0,95) = 10,01 \text{ g mol/s}$$

$$\frac{G_s'}{L_s} = \frac{X_{A1} - X_{A2}}{Y_{A1} - Y_{A2}} = \frac{0,0379 - 0}{0,0526 - 0,003}$$

$$L_s' = 13,1 \text{ g mol/s}$$



$$Y_{A1}^* = 0,8 X_{A1} = 0,8(0,0379) = 0,0303$$

$$Y_{A1} - Y_{A1}^* = 0,0526 - 0,0303 = 0,0223$$

$$Y_{A2} - Y_{A2}^* = 0,003 - 0 = 0,003$$

31.13 CONTINUED -

$$Z = \frac{G_s}{K_y a} \frac{Y_{A1} - Y_{A2}}{(Y_{A1} - Y_{A1}^*)_{L.M.}}$$

$$(Y_{A1} - Y_{A1}^*)_{L.M.} = \frac{0,0223 - 0,003}{\ln \frac{0,0223}{0,003}} = 0,0096$$

$$G_s = G_s' / A = \frac{10,01}{0,2} = 50,05 \text{ g mol/m}^2 \cdot \text{s}$$

$$Z = \frac{(50,05)(0,0223 - 0,003)}{(52)(0,0096)}$$

$$= 4,97 \text{ m}$$

31.14

$$X_{A2} = 0$$

$$X_{A2} = 0$$

$$y_{A2} = 0,003$$

$$Y_{A2} = \frac{0,003}{0,997} \approx 0,003$$

$$y_{A1} = 0,036$$

$$Y_{A1} = \frac{0,036}{0,964} = 0,0373$$

$$A = \frac{\pi}{4} (0,15)^2 = 0,0177 \text{ m}^2$$

$$L_1 = L_s = \frac{14,5}{0,0177} = 819,2 \text{ mol/m}^2 \cdot \text{s}$$

$$G_1 = \frac{8}{0,0177} = 452 \text{ mol/m}^2 \cdot \text{s}$$

$$G_s = G_1(1 - y_1) = 452(0,964) = 435,7 \text{ mol/m}^2 \cdot \text{s}$$

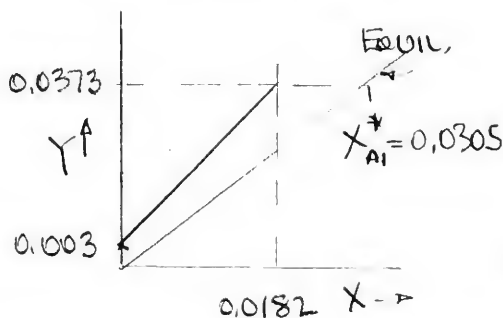
$$\frac{L_s}{G_s} \bigg|_{ACT} = \frac{819,2}{435,7} = 1,88$$

31.14 CONTINUED -

$$\frac{L_s}{G_s} = 1.88 = \frac{Y_{A1} - Y_{A2}}{X_{A1} - X_{A2}} = \frac{0.0373 - 0.003}{X_{A1} - 0}$$

$$X_{A1} = 0.0182$$

EQUILIBRIUM	x	mole NH <sub>3</sub>	0.00	0.0164	0.0252	0.0349	0.0445	0.0722
		mole NH <sub>3</sub> -free water						
DATA	y	mole NH <sub>3</sub>	0.00	0.021	0.032	0.042	0.053	0.080
		mole NH <sub>3</sub> -free air						



$$\left. \frac{L_s}{G_s} \right|_{\min} = \frac{Y_{A1} - Y_{A2}}{X_{A1}^* - X_{A2}} = \frac{0.0373 - 0.003}{0.0305 - 0} = 1.125$$

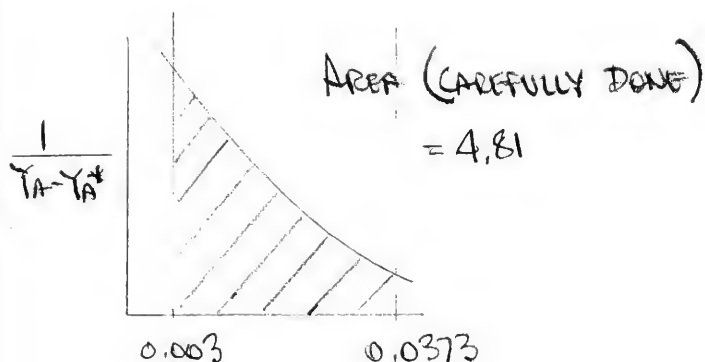
$$\frac{L_s/G_s|_{\text{ACT}}}{L_s/G_s|_{\min}} = \frac{1.88}{1.125} = 1.67 \quad (a)$$

USING GRAPHICAL INTEGRATION:

$$Z = \frac{G_s}{K_y a} \int_{0.003}^{0.0373} \frac{dY_A}{Y_A - Y_A^*}$$

$Y_{AG}$	$Y_A^*$	$Y_{AG} - Y_A^*$	$(Y_{AG} - Y_A^*)^{-1}$
0.003	0	0.003	333.3
0.010	0.0048	0.0052	192.3
0.015	0.0083	0.0067	149.2
0.020	0.0116	0.0084	119.1
0.025	0.0150	0.010	100.0
0.030	0.0183	0.0117	85.5
0.0373	0.023	0.0143	69.9

31.14 CONTINUED -



$$Z = \frac{435.7}{71} (4.81) = 29.52 \text{ m} \quad (b)$$

31.15 SAME SPECS & SYSTEM AS IN PROB 31.14

FROM PROB 31.14 SOLUTION:

$$G_s = 435.7 \text{ mol/m}^2 \cdot \text{s}$$

$$K_y a = 71 \text{ mol/m}^2 \cdot \text{s} \cdot \Delta Y_A$$

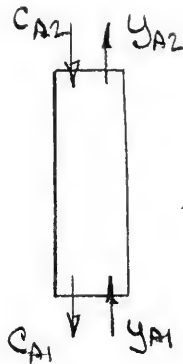
$$Y_{A1} - Y_{A1}^* = 0.0373 - 0.023 = 0.0143$$

$$Y_{A2} - Y_{A2}^* = 0.003$$

$$(Y_A - Y_A^*)_{L.M.} = \frac{0.0143 - 0.003}{\ln \frac{0.0143}{0.003}} = 0.0072$$

$$Z = \frac{G_s}{K_y a} \frac{Y_{A1} - Y_{A2}}{(Y_A - Y_A^*)_{L.M.}} = \frac{435.7 (0.0373 - 0.003)}{71 \cdot 0.0072} = 29.2 \text{ m}$$

31.16



$$C_{A2} = 0.0394 \times 10^{-3} \text{ g mol/l} \\ = 0.0394 \text{ mol/m}^3$$

$$C_{A1} = 0.0131 \times 10^{-3} \text{ g mol/l} = 0.0131 \text{ mol/m}^3$$

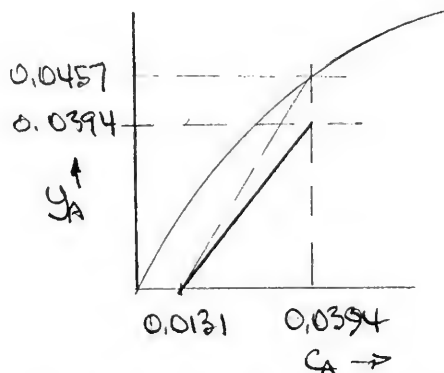
$$Y_{A1} = 0$$

$$Q'_L = \frac{5000(3.785)}{1000} = 18.92 \text{ m}^3/\text{h}$$

$$G' = \frac{\dot{V}_L'}{1.5} = \frac{18.92}{1.5} = 12.62 \text{ mol/h}$$

EQUILIBRIUM  
DATA →

$C_A$ , moles VOC/m <sup>3</sup>	0.014	0.0240	0.0349	0.0498
$Y_A$ , VOC	0.018	0.030	0.042	0.053



SINCE STREAMS ARE RELATIVELY DILUTE

 $Q'_L$  &  $G'$  ARE CONSTANT

$$Q'_L (C_{A1} - C_{A2}) = G' (Y_{A1} - Y_{A2})$$

31.16 CONTINUED -

$$\frac{Q'_L}{G'} \bigg|_{\min} = \frac{Y_{A2}^* - Y_{A1}}{C_{A2} - C_{A1}} = \frac{0.0457 - 0}{0.0394 - 0.0131} \\ = 1.738$$

$$G'_{\min} = \frac{18.92}{1.738} = 10.89 \text{ mol/h} \quad (a)$$

$$Q'_L (C_{A1} - C_{A2}) = G'_L (Y_{A1} - Y_{A2})$$

$$18.92 (0.0131 - 0.0394) = 12.62 (0 - Y_{A2})$$

$$Y_{A2} = 0.0394$$

$$\text{TOWER HEIGHT: } Z = \frac{\dot{V}_L}{K_L a} \frac{C_{A2} - C_{A1}}{(C_A - C_A^*)_{L.m.}}$$

$$(C_A - C_A^*)_2 = 0.0394 - 0.0305 = 9.0 \times 10^{-3}$$

$$(C_A - C_A^*)_1 = 1.31 \times 10^{-2} - 0 = 0.0131$$

$$(C_A - C_A^*)_{L.m.} = \frac{0.0131 - 0.009}{\ln \frac{0.0131}{0.009}} = 0.0109$$


$$\frac{Q'_L}{A} = \frac{18.92}{\frac{\pi}{4} (0.6)^2 (3600)} = 0.0186 \text{ m/s}$$

$$Z = \frac{0.0186 (0.0394 - 0.0131)}{0.01 (0.0109)}$$

$$= 4.49 \text{ m} \quad (b)$$

31.17 THE EXOTHERMIC REACTION WILL CAUSE THE TEMPERATURE IN THE TOWER TO INCREASE, WHICH, IN TURN, WILL CAUSE THE EQUILIBRIUM LINE TO SHIFT UPWARD. THE RESULT WILL BE A SMALLER DRIVING FORCE,  $Y_A - Y_A^*$  & A TALLER TOWER WILL BE REQUIRED RELATIVE TO ONE OPERATING ISO-THERMALLY.

31.18



$x_2 = 0$   
 $x_1 = 0.0305$   
 $x_2 = 0$   
 $y_2 = 0.003$   
 $y_1 = 0.05$   
 $y_2 = 0.003$   
 $y_1 = \frac{0.05}{0.95} = 0.0526$   
 $X_1 = \frac{0.0305}{0.9695} = 0.0315$   
 $G'_1 = \frac{\dot{V}P}{RT} = \frac{(0.236)(1.013 \times 10^5)}{(8.314)(293)} = 9.814 \text{ g mol/s}$   
 $G'_s = G'_1(1 - y_1) = (9.814)(0.95) = 9.323 \text{ g mol/s}$   
 $L'_s = \frac{G'_s(Y_1 - Y_2)}{X_1 - X_2} = \frac{9.323(0.0526 - 0.003)}{0.0315 - 0} = 14.68 \text{ g mol/s}$

31.18 CONTINUED -

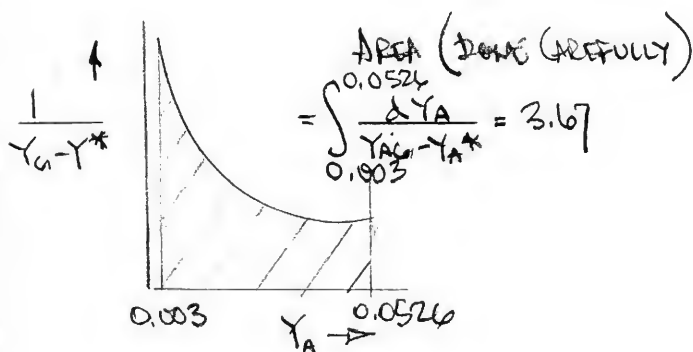
EQUILIBRIUM

x	moles mercaptan	0.00	0.01	0.02	0.03	0.04
moles mercaptan-free solvent						
y	moles mercaptan	0.00	0.0045	0.0145	0.0310	0.0545
moles mercaptan free air						

TOWER HEIGHT:  $Z = \frac{G_s}{K_y a} \int_{Y_2}^{Y_1} \frac{dY_A}{Y_A - Y_A^*}$

USING EQUILIBRIUM DATA PROVIDED:

$Y_6$	$Y^*$	$Y_6 - Y^*$	$(Y_6 - Y^*)^{-1}$
0.003	0	0.003	333.3
0.010	0.0016	0.0084	119.05
0.016	0.0038	0.0122	81.97
0.020	0.0057	0.0143	69.93
0.028	0.0100	0.0180	55.56
0.034	0.0140	0.0200	50.0
0.040	0.0193	0.0207	48.31
0.048	0.0281	0.0199	50.25
0.0526	0.0340	0.0186	53.76



$Z = \frac{9.323(3.67)}{(0.2)(40)} = 4.28 \text{ m}$

31.19 Same System & Feed Streams  
As in Prob 31.18

From Prob 31.18 Solution -

$$G'_1 = 9.814 \text{ g mol/s}$$

$$L'_S = 14.68 \text{ ''}$$

IN THIS CASE -

$$Z = 4.5 \text{ m} \quad Z_f = 155$$

1-IN. RASCHIG RINGS

$$\text{FOR GAS: } \Delta P/2 = 300 \text{ N/m}^2$$

PARAMETERS FOR FIG 31.25:

$$\frac{L'_1 \left[ \frac{P_G}{P_L - P_G} \right]^{1/2}}{G'_1 \left[ \frac{P_G}{P_L - P_G} \right]}$$

$$G'_1 = \frac{(9.814)(30.1)}{1000} = 0.295 \text{ kg/s}$$

$$L_1 = \frac{L_S}{1 - X_1} = \frac{14.68}{1 - 0.0315}$$

$$= 15.16 \text{ g mol/s}$$

$$L'_1 = \frac{(15.16)(180)}{1000} = 2.729 \text{ kg/s}$$

$$P_g = \frac{P}{RT} M = \frac{(1.013 \times 10^5)(30.1)}{(8.314)(293)(1000)}$$

$$= 1.252 \text{ kg/m}^3$$

$$P_L = 0.81(1000) = 810 \text{ kg/m}^3$$

SUBSTITUTING VALUES -

$$\frac{L'_1 \left[ \frac{P_g}{P_L - P_g} \right]^{1/2}}{G'_1 \left[ \frac{P_g}{P_L - P_g} \right]} = 0.364$$

31.19 CONTINUED -

FROM FIG 31.25 -

$$\frac{G'^2 C_g \mu_L^{0.1} J}{P_g (P_L - P_g) g_c} = 0.03$$

SUBSTITUTING VALUES

$$\mu_L = 0.0039 \text{ Pa.s}$$

$$J_c = 1$$

OTHERS ALREADY CALCULATED

$$G' = 0.592 \text{ kg/m}^2 \cdot \text{s}$$

$$\text{TOWER AREA} = \frac{G'_1}{G'} = \frac{0.295}{0.592}$$

$$= 0.498 \text{ m}^2$$


$$D = \left( \frac{0.498}{\pi/4} \right)^{1/2} = 0.796 \text{ m}$$

$$\text{OR } \sim 0.8 \text{ m}$$

31.20  $x_{A2} = 0$   
 $x_{A2} = 0$

$$y_{A2} = 0.004$$

$$Y_{A2} = \frac{0.004}{0.996} \approx 0.004$$



$$y_{A1} = 0.125$$

$$Y_{A1} = \frac{0.125}{0.875} = 0.143$$

FRACTION OF HCl REMOVED -

$$= \frac{G_S (Y_{A1} - Y_{A2})}{G_S (Y_{A1})} = \frac{0.143 - 0.004}{0.143}$$

$$= 0.972 \sim 97.2\% \text{ (a)}$$

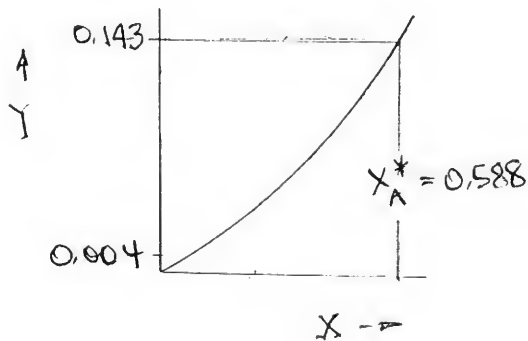
31,20 CONTINUED -

EQUILIB.

DATA →

DETERMINE EQUILIB. VALUES IN TERMS OF  $X_A, Y_A$

$Y_A$	$X_A$	$y_A$	$Y_A$
0,210	0,266	0,0023	0,0023
0,243	0,321	0,0095	0,0096
0,287	0,403	0,0215	0,0220
0,330	0,493	0,0523	0,0552
0,353	0,546	0,0852	0,0931
0,375	0,600	0,135	0,156
0,400	0,666	0,203	0,255
0,425	0,739	0,322	0,475



$$\left. \frac{L_S}{G_S} \right|_{\min} = \frac{Y_{A1} - Y_{A2}}{X_{A1}^* - X_{A2}} = \frac{0.143 - 0.004}{0.588 - 0} = 0.236$$

$$\left. \frac{L_S}{G_S} \right|_{\text{ACT}} = 0.236(1.64) = 0.387$$

$$= \frac{Y_{A1} - Y_{A2}}{X_{A1} - X_{A2}} = \frac{0.143 - 0.004}{X_{A1} - 0}$$

$$X_{A1} = 0.359$$

$$Y_{A1} = \frac{X_{A1}}{1 + X_{A1}} = \frac{0.359}{1.359} = 0.264 \quad (b)$$

31,20 CONTINUED -

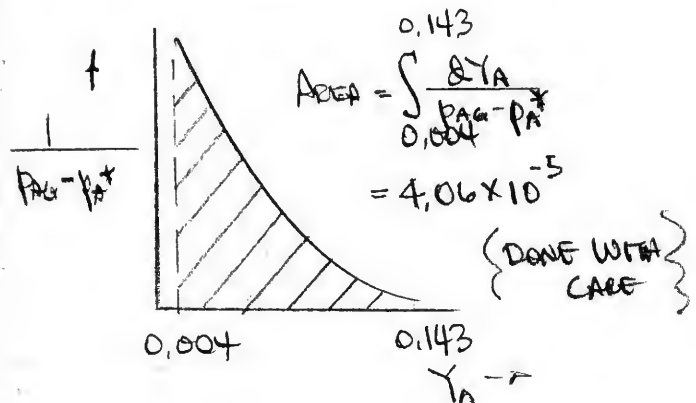
$$K_{Ga} = 8.8 \text{ kg mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

WITH  $K_{Ga}$  EXPRESSED IN THIS MANNER DRIVING FORCE MUST BE IN  $P_A - P_A^*$

$$Z = \frac{G_S}{K_{Ga}} \int_{0.004}^{0.143} \frac{dY_A}{P_{AG} - P_A^*}$$

$Y_{A1}$	$Y_A^*$	$y_{A1}$	$y_A^*$	$P_{AG}$
0,004	0	0,004	0	405
0,02	0,0005	0,0196	0,0005	1985
0,04	0,0010	0,0385	0,0010	3900
0,06	0,0015	0,0566	0,0015	5734
0,08	0,0019	0,0741	0,0019	7506
0,10	0,0021	0,0909	0,0021	9208
0,12	0,0060	0,1071	0,0060	10850
0,143	0,015	0,1250	0,0148	12662

$P_A^*$	$P_{AG} - P_A^*$	$(P_{AG} - P_A^*)^{-1} \times 10^4$
0	405	24.7
50.6	1935	5.17
101	3799	2.63
152	5582	1.79
192	7314	1.37
213	8996	1.11
608	10240	0.98
1499	11160	0.90



31.20 CONTINUED -

$$\dot{V} = 5 \text{ m}^3/\text{m}$$

$$G'_1 = \frac{\dot{V} P}{RT} = \frac{(5)(1.013 \times 10^5)}{(8.314)(293)(60)}$$

$$= 3.465 \text{ mol/s}$$

$$G'_2 = G'_1 (1 - y_{A1}) = (3.465)(1 - 0.125) \\ = 3.03 \text{ mol/s}$$

$$G_s = \frac{3.03}{(\pi/4)(0.6)^2} = 10.72 \text{ mol/m}^2 \cdot \text{s}$$

$$K_g a = 8.8 \times 10^{-8} \text{ kg mol/m}^3 \cdot \text{s} \cdot \text{Pa} \\ = 8.8 \times 10^{-5} \text{ mol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

SUBSTITUTING:

$$Z = \frac{10.72 (4.06 \times 10^{-5})}{8.8 \times 10^{-5}}$$

$$= \underline{\underline{4.94 \text{ m}}}$$